GAGE'S PRINCIPLES OF PHYSICS

GOODSPEED

J. M. Barnes.

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THE
PRINCIPLES OF PHYSICS

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REVISED BY

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GINN & COMPANY

BOSTON · NEW YORK · CHICAGO · LONDON

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The Athenæum Press
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PREFACE

The present volume is not a new work but a revision of the edition of 1895. Since the first appearance of the "Principles" in that year physical science has advanced with rapid strides, and even a whole new department of physics, Radio-activity, has developed; and while this subject cannot, of course, be treated in a book of this grade, yet some of the generalization of other departments must now be radically modified to be consistent with the new research.

While the scientific activity of this age has resulted in successive additions both to the theory of physics and to its applications, and has thus tended to make the ground covered by new text-books continually greater, an effort has been made to have the new edition even less voluminous than the old. This has been effected by omitting some sections which, in the experience of the reviser as a teacher, have been found of little value to students of an elementary course. It is needless to say that the omissions have been largely among the more remote application of principles and among the numerous individual experiments that the student was asked to perform. A sufficient number of these have been retained for illustration, it is thought, and space has been gained for the new material without increasing the size of the book.

In accordance with the new developments referred to, the work has been thoroughly revised and brought down to date during the past year. Parts of it have been rewritten and several new sections have been added. The two-course feature has been eliminated, and the whole of the matter made to conform with the requirements of the best high schools and college-preparatory

schools. It is thought also to be quite adequate for the required course in physics in such colleges and universities as pay special attention to broad culture and general education, without going into technical specialties.

The book is designed as a text for the student, to be used in connection with the class-room instruction consisting preferably of formal lectures illustrated experimentally, and informal recitations or conferences on the subject. The picture-book feature, somewhat evident in the former edition, has been made less prominent, a change which, it is believed, will bring the book more into accordance with modern methods of teaching.

A. W. G.

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PRINCIPLES OF PHYSICS

INTRODUCTION

FUNDAMENTAL UNITS OF MEASUREMENT

1. Accurate knowledge of physical phenomena is obtained only by the measurement of physical quantities.

A physical quantity is that attribute of anything which is measurable, as, for example, length, mass.

A measurement consists in finding the relative sizes of a definite quantity, called a *unit*, and that of the quantity to be measured. A unit which has become legalized, either by statute or by common usage, is called a *standard unit.*

The expression of a physical quantity consists of a statement of the *concrete unit* employed, e.g. gram, foot, second, etc., with a *number* prefixed. The numerical part is obtained by measurement.

2. The fundamental units, in terms of which all physical measurements are made, are those of *length*, *mass*, and *time*. These are fundamental in the sense that no one is derivable from others. The three entities which these units measure cannot be absolutely defined; but, fortunately, they are so familiar as to require no definition.

The fundamental unit of length in the metric system[1] is the *centimeter*, or the one-hundredth part of a meter. The *standard* meter is defined by law (1795) to be the shortest distance between

[1] The metric system is now generally employed in scientific work. The advantage of this system consists largely in the simplicity of the relations which exist between the standards of length and mass, and in the use of units each of which is some *decimal* multiple or submultiple of the others in the same series.

the ends of a certain platinum rod made by Borda and called *Mètre des Archives*, the temperature being 0° C. This rod is in the keeping of the Academy of Sciences at Paris.

The foot is one third of the British standard yard. The yard is defined by act of Parliament (1855) to be the distance between the intersections of the transverse lines on two gold plugs in a bronze bar deposited at the office of the Exchequer in London, the temperature of the bar being 62° F.

The fundamental unit of mass generally employed in science is the *gram*, or the one-thousandth part of the standard kilogram. This standard is a piece of platinum, called the *Kilogramme des Archives*, carefully preserved by the French government at Paris. It was originally intended to represent the mass of a cubic decimeter of pure water at the temperature of 4° C.

3. Mass; weight. By the *mass* of a body is meant the *quantity of matter* contained in the body. It is highly important to bear in mind that the idea implied in the term *mass* is quite distinct from that of *weight*. The weight of a body changes with its distance from the earth, while its mass remains the same. The weight of a body is the measure of the attraction between it and the earth, and is variable because the attraction of the earth differs at different places, but its mass is not affected by this attraction.

A kilogram of any substance is that quantity of the substance which, placed on a scale pan, would just balance in a vacuum the standard kilogram placed on the other pan.

The English unit of mass is defined by act of Parliament (1855) to be a piece of platinum marked "P.S., 1 lb.," denominated the Imperial Standard Pound Avoirdupois. It is deposited in the office of the Exchequer at London.

The process of measuring the mass of a body by balancing it with a body or bodies of known mass is called *weighing*. The process of weighing, as commonly understood, is essentially a comparison of masses. A set of masses, commonly called a set of weights, consists of a series of bodies having masses corresponding to the denominations marked upon them.

The process of measuring the mass of a body must not be confounded with the process of finding how heavy a body is, i.e. how great the attraction between it and the earth is, although both processes are, in common usage, called weighing. Not only are the things measured in the two cases entirely different, but the instruments that may be required are quite different. For example, a kilogram mass always has the same mass, and may, therefore, when used with the scale balance, be relied upon always and everywhere to measure accurately an equal mass of any body. But a kilogram mass when suspended from a spring balance may be found to be heavier in certain situations than in others; hence this instrument does not under all circumstances measure mass correctly. On the other hand, the scale balance will not detect any change in the heaviness of a kilogram mass; hence this instrument does not always measure heaviness correctly. For most practical purposes, however, these instruments may be used interchangeably, since at the same place masses are proportional to their weights.

The fundamental unit of time generally employed in scientific measurement is the *second*. The second is $\frac{1}{86400}$ of the mean solar day.

4. Derived units. The system of measurements in which all the units used in measuring physical quantities are derived from the three metric units given above is called the *centimeter-gram-second system*, or, briefly, the C.G.S. system. Such a system is an *absolute system* of units.

5. Dimensional equations. An equation showing the relation of the derived unit to the fundamental units is called a *dimensional equation*. The dimensional equation for any derived unit is deduced from the physical laws by which the unit is defined.

In any absolute system the unit of length is represented symbolically by [L], that of mass by [M], and that of time by [T]. Any derived unit may be represented by the product of certain powers of these symbols. Thus the unit of area $= [L^2]$.

6. Volume. By the *volume* of a body is meant the quantity of space it occupies. The unit of volume is the cubic centimeter,

and is defined as the volume of a cube the length of one side of which is one centimeter.

7. Matter, body, substance. Provisionally we may define *matter*[1] as that which occupies space. A *body* is any limited portion of matter, e.g. a lake, a tumbler, a desk, etc. Names of bodies should be carefully distinguished from names of *substances* which indicate merely the *kind* of matter of which the bodies are composed, such as water, iron, wood, etc.

8. Phenomena. Physical laws. By observation we learn, for example, that a piece of iron expands when heated, becomes luminous when heated to a sufficiently high temperature, and changes to a liquid when heated much more. Here are three distinct changes which heat may effect. Changes or events like these, and countless others which occur in nature, are called *physical phenomena.*

If, after many trials with iron from various sources and under varying conditions, we are able to state in general that the application of heat to iron under suitable conditions will be followed by these phenomena, such a generalized statement is called a *physical law. A physical law is an expression of a constant relation which has been discovered to exist between certain physical quantities.*

Physical laws, unlike statute and moral laws, do not *govern* events, but are generalized statements of the order of events. Iron does not expand in *obedience* to a law that "heat expands." We do not explain *why* a body falls to the earth by stating a law that "an unsupported body falls"; indeed, the cause of a fall has never been discovered, though every one of us has individually discovered the law just quoted.

[1] Matter is variously defined in scientific text-books according to the fancy of the authors. The definitions are, however, only provisional, serving merely the practical requirement of distinguishing between what is matter and what is not.

KINEMATICS

SECTION I

MOTION, VELOCITY, ACCELERATION

9. Kinematics treats of motions without reference to causes. *Motion* is a continuous change of position. The position of a particle of matter is determined by its direction and distance from some point of reference. A particle moves relatively to a given point when the straight line connecting it with the point changes either in *direction* or *length*. A particle is at rest relative to a given point when a straight line joining them changes *neither* in direction *nor* in length.

When a particle is spoken of as being in motion or at rest, some point is always expressed or understood, *relatively* to which the change or permanence of position is maintained. All motions with which we have to do, or which we can measure, are *relative*. Change of position with reference to a fixed point in space would be *absolute motion*. But we know of no fixed point. "There are no landmarks in space; one portion of space is exactly like every other portion, so that we cannot tell where we are" or in what direction, or how fast, we are going.

In ordinary language the phrase "a body at rest" means that the body does not change its position with reference to that on which it stands, as, for instance, the surface of the earth or the deck of a ship. It can mean nothing else, for both it and all points of the earth's surface are in rapid motion with reference to the sun and other heavenly bodies, and also with reference to the earth's axis.

10. Velocity. A point consumes time, longer or shorter, in its transit from one position to another. Rate of change of place (or time rate of displacement) of a body is called its *velocity*. Velocity is expressed by stating the number of units of distance traversed in a unit of time. For scientific purposes it is most frequently expressed in *centimeters per second;* for practical purposes it is often expressed in other units for convenience, as the velocity of a locomotive in *miles per hour*, of a rifle bullet in *feet per second*, etc. Observe that velocity is *distance per unit of time*, and cannot be correctly expressed in miles, feet, etc., alone.

Velocity involves also the idea of direction, and may change in both magnitude and direction. It is sometimes convenient to ignore the direction of a body's rate of displacement, in which case we use the term *speed*. Thus it is better to speak of the speed of the locomotive and the bullet when only the magnitude of the change in position is considered.

11. Constant and accelerated velocity. When a point moves in a straight line with unchanging rate its velocity is said to be constant. In case, however, of a continuous change of velocity, either in magnitude or direction, it is said to be *accelerated*. Finally, if this change of velocity is uniform, the point is said to have *constant acceleration*.

When the velocity increases, as in the case of a falling stone, its acceleration is said to be positive, or $+$; when the velocity decreases, as in the case of a stone thrown upward, its acceleration is said to be negative, or $-$.

Velocity is determined by dividing the distance traversed by the time consumed. If a body move l feet in t seconds, its velocity v is $\frac{l}{t}$ feet per second, or $v = \frac{l}{t}$. In case the velocity be accelerated, this result is to be regarded as the *average* velocity for that distance; and in the case of uniform motion the average velocity is the same as the *actual* velocity at every instant. It is evident that the actual velocity of a body whose motion changes can be given only at some definite instant or point in its journey.

It denotes the space which *would* be traversed in a unit of time, if at the given instant the velocity should become constant.

In the C.G.S. system the unit of velocity is that rate of displacement at which one centimeter is traversed in one second of time.

Change of velocity *per second* is called *acceleration*, and is represented by a. When a particle acquires velocity at a uniform rate the acceleration is said to be *constant*, and the motion *uniformly accelerated*. The acceleration of a body falling in a vacuum, and of a body projected vertically upwards in a vacuum, is practically constant; in the former case it is about 980 centimeters per second, in the latter case it is a negative acceleration of the same numerical value.

The *average* acceleration a of a particle in transversing a certain distance in a given time, and the rate of acceleration (also represented by the symbol a), provided the acceleration is constant, is found by dividing the entire change in speed v in a certain time by the time t taken in making the change, i.e. $a = \frac{v}{t}$, whence $v = at$. Thus, if the speed of a railroad train at a certain instant be 25 miles per hour, and half an hour hence it be 15 miles per hour, then the entire change of speed v is -10 miles per hour; hence the average acceleration, i.e. the acceleration if it were uniformly distributed throughout the 30 minutes, is $\frac{1}{3}\left(=\frac{-10}{30}\right)$ of a mile per minute. Again, if a stone falling freely acquire in 4 seconds a speed of 3920 centimeters per second, its acceleration is $\frac{3920}{4} = 980$ centimeters per second.

In the C.G.S. system the unit of acceleration is that acceleration in which a velocity of one centimeter per second is gained or lost each second.

SECTION II

UNIFORMLY ACCELERATED MOTION

12. If a particle be moving at a certain instant at a rate V, and its acceleration be $+a$, then its speed v at any time is expressed as follows:

at the initial instant, $v = V$;
at the end of the first second, $v = V + a \times 1$;
at the end of two seconds, $v = V + a \times 2$;
at the end of t seconds, $v = V + at$ (A).

The last, (A), is a general equation expressing the relation between the speed v at the end of any interval of time t, the original speed V, and the acceleration a. From this formula we derive the following law:

(1) *Change of speed due to uniform acceleration is equal to the product of the acceleration and the time.* Hence the change of speed is proportional to the acceleration and to the time occupied.

13. If the initial speed V be zero, i.e. if the particle start from a state of rest, the equation becomes $v = at$.

Since the speed of a particle starting from a state of rest increases from zero to at, the average speed must be $\frac{0 + at}{2} = \frac{1}{2}at$. At this rate, in the same time t it would traverse a distance S equal to $\frac{1}{2}at \times t = \frac{1}{2}at^2$ units; hence $S = \frac{1}{2}at^2$ (B). From this we derive the law:

(2) *The distance traversed in a given time by a particle starting from rest, and having uniformly accelerated speed, is one half the product of the acceleration and the square of the time.* Hence the entire distance traversed is proportional to the acceleration and to the square of the time.

If a particle, instead of starting from a state of rest, have an initial speed V, it would move in t seconds without acceleration a distance $V \times t$; to this distance must be added the distance

it moves in consequence of acceleration, in order to obtain the entire distance traversed in t seconds, and our formula becomes $S = Vt + \frac{1}{2}at^2$ (C).

QUESTIONS AND PROBLEMS

1. The speed of a particle at a certain instant is V and its acceleration is a. What will be its speed v after t seconds?

2. If the initial speed of a body be V, its acceleration a, and its final speed v, how long, t, was it in acquiring its final speed?

3. If a body having an initial speed V acquire in t seconds a speed v, what is its acceleration?

4. If a body move from rest with a uniform acceleration a, what space S will it traverse in t seconds?

5. If a body move from rest with an acceleration a, in what time t will it traverse the space S?

6. The speed of a particle at a certain instant is 20 cm. per second; its acceleration is 3 cm. per second. What will be its speed 10 sec. hence?

7. Suppose that the acceleration of the particle mentioned above is -2 cm. per second. What will be its speed 5 sec. afterwards?

8. (*a*) A body falls from rest; its speed increases each second 32.2 ft. per second. What is its speed at the end of the first second? (*b*) at the end of the tenth second? (*c*) at the end of half a second?

9. If the initial speed of a body be 5 cm. per second, its final speed 25 cm. per second, and its acceleration 2 cm. per second, what was the time consumed in acquiring the final speed?

10. A bullet is projected vertically upward with an initial speed of 161 ft. per second. What will be its speed at the end of the third second?

11. How long will the bullet named in the last problem rise?

12. What speed will the bullet have at the end of the sixth second, and in what direction will it be moving?

13. (*a*) How many centimeters will a body fall from a state of rest in 1 sec.? (*b*) in 2 sec.? (*c*) in 10 sec.?

14. A stone thrown vertically downward is given an initial speed of 40 m. per second. How far will it descend in 10 sec.?

15. (*a*) A bullet is projected vertically upward with an initial speed of 225.4 m. per second. How long will it rise? (*b*) How far will it rise?

16. How long will it take a body to fall 1030.4 ft. from rest?

17. (*a*) A body falls during 1.5 sec. What is its final speed? (*b*) How far does it fall?

18. A body falls 297.6 ft. in 4 sec. What was its initial speed?

19. What initial speed must be given a body that it may rise 6 sec.?

SECTION III

COMPOSITION AND RESOLUTION OF VECTORS

14. Graphical representation of vectors. It has already been mentioned that the term velocity may include more than simply the statement of the distance passed over per second, or the speed. Velocity implies the direction of the motion as well, and belongs to the class of quantities known as *vectors*. Any quantity which is completely known when its magnitude and direction are known is a vector quantity, and may be represented by a straight line, the direction of that line indicating the direction of the vector and its length indicating the magnitude of the vector.

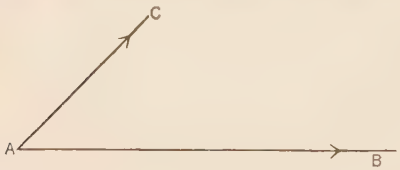


FIG. 1

If a person wishes to describe the rectilinear motion of a particle, he must tell two things — (1) the direction of the motion, and (2) its speed. These two elements completely describe the vector quantity, velocity, represented graphically by a straight line as above. Thus the line AB (Fig. 1), 5 centimeters long, drawn horizontally from left to right, may represent an eastward velocity of, say, 5 kilometers per hour. With the same scale, then, AC would denote a velocity of 2.5 kilometers per hour in a northeasterly direction.

15. Composition of simultaneous motions. If by any means a particle has two or more separate and independent motions communicated to it simultaneously, and if the motions imparted be themselves constant in velocity, the result of their concurrence is a single motion in a straight line with a single speed. This is illustrated somewhat imperfectly in the following manner. With the handle A in the position shown in Fig. 2, push the frame BC to the right. This frame carries a pencil D, whose point presses the paper below, and as the frame advances, the line ab is traced

upon the paper, graphically representing the motion of the pencil. If, when the pencil point is at a and the frame is at rest, the string G be pulled, the pencil will trace the line ac at right angles to ab. Now these two independent motions may be communicated to the pencil *simultaneously* by fastening the string E

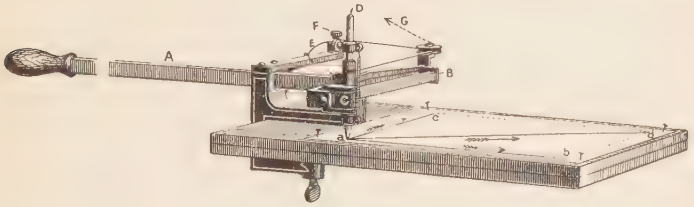


FIG. 2

to the binding screw F and pushing forward the handle A. The pencil point will not move in either of the lines ab or ac, but its motion will be intermediate between the two, and it will trace the line ad. This single motion, which is the result of the concurrence of two motions, is called their *resultant;* and the two motions, with regard to the resultant, are called its *components.*

The distance ad is traversed in exactly the same time that the distance ab would be traversed if the pencil had no other motion, the handle A being pushed forward with the same speed in both cases; likewise the distance ad is traversed in the same time that the distance ac is accomplished, when the string is simply pulled over the pulley G with the same speed, and has no other motion. The lines ab, ac, and ad represent not only the distances traversed in the several directions, but also the magnitudes and directions of the respective velocities. For example, if the velocity be constant and the pencil reach successively, at the end of equal intervals of time, the points m'', n'', and d (Fig. 3), then am'', $m''n''$, and $n''d$ represent its velocities in the successive intervals, and am, mn,

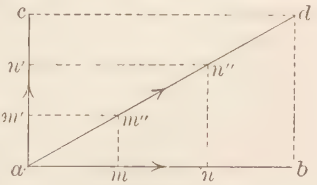


FIG. 3

and nb represent the velocities for the same intervals in the direction ab; and am', $m'n'$, and $n'c$ the velocities in the direction ac.

If points c and d and d and b are joined by dotted lines, we have a parallelogram of which the line ad, representing the resultant, is a diagonal. Hence to find the resultant of two simultaneous velocities when they make an angle with each other, the rule is: *Construct a parallelogram whose adjacent sides represent the two velocities, and the diagonal which lies between these adjacent sides represents their resultant.*

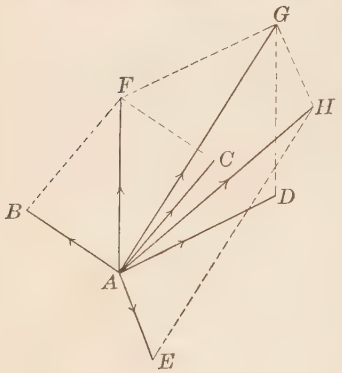


FIG. 4

16. When more than two components are given, find the resultant of any two of them, then of this resultant and a third, and so on until every component has been used. For example, let the several velocities imparted to a particle be represented by the lines AB, AC, AD, and AE (Fig. 4). The resultant of AB and AC is AF; the resultant of AF and AD is AG; and that of AG and AE is AH, which represents the resultant of the four velocities.

When two components are at right angles to each other, it is evident that we may obtain the magnitude of the resultant by finding the square root of the sum of the squares of the two components.

In case a particle has several velocities imparted to it, all in the same direction, their resultant is the sum of all. If some are opposite others, one of the two directions is considered as positive and the opposite one as negative, and + and − signs being prefixed to the numerical values, the algebraic sum of these values is the resultant.

In the foregoing discussion velocity has been chosen as a familiar and typical vector quantity. All the propositions must be understood as entirely general and true for all vectors.

17. Resolution of a vector into components. Any vector may be resolved into two or more vectors. Let AB (Fig. 5) represent the velocity of a particle. Draw a line AC to represent, either arbitrarily or according to the conditions of the problem, one of the required components. Connect B and C; draw AD parallel to BC, and DB to AC, and thus complete a parallelogram of which AB is a diagonal. The two adjacent sides AC and AD represent two component velocities of the particle; in other words, a particle having a velocity represented by the line AB has at the same time velocities represented in magnitude and direction by the lines AC and AD.

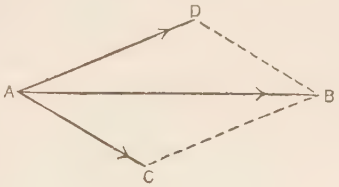


FIG. 5

QUESTIONS AND PROBLEMS

1. (*a*) If a ship move east at the rate of 10 mi. an hour and a person on deck walk towards the bow at the rate of 2 mi an hour, what is the resultant of these two velocities? (*b*) With reference to what has he this velocity?

2. (*a*) Suppose the person mentioned above walk aft at the rate of 2 mi. an hour. What will be the resultant of these two velocities? (*b*) Prefix suitable signs to the numbers given, and represent the addition which gives the resultant.

3. (*a*) A particle moves simultaneously northward with velocity a, and southward with velocity b. What is the resultant of these velocities? (*b*) How do you interpret the resultant if $a > b$? (*c*) How, if $b > a$? How, if $a = b$?

4. Suppose the person mentioned above walk directly north across the deck at the rate of 4 mi. an hour. What will be his resultant velocity?

5. Suppose the person walk northeast at the rate of 4 mi. an hour. What will be his resultant velocity? [In drawing the parallelogram of velocities, represent the component velocities to some scale, e.g. 1 cm. = 1 mi.; then, having completed the parallelogram and having drawn the diagonal which represents the resultant, measure the latter, and the result will express, on the scale chosen, the resultant velocity required.]

6. Suppose an attempt be made to row a boat at the rate of 6 mi. an hour directly across a stream flowing at the rate of 10 mi. an hour. Determine the velocity of the boat.

7. A vessel sails south-southeast, i.e. 22.5° east of south, at the rate of 14 mi. an hour. Determine its southerly and its easterly velocity.

8. Represent graphically, to scale, a speed of 100 ft. per second, and resolve this speed into two components which shall have an angle of 45° between them.

9. Represent graphically, all in different directions, speeds which a particle may have at a given instant, as follows: 20 ft., 30 ft., 15 ft., and 25 ft. per second. Determine its apparent velocity.

18. Composition of constant with accelerated velocity. Experience teaches that a body, e.g. a stone, projected in a horizontal direction moves not in a horizontal path, but in a path intermediate between a horizontal and a vertical one, showing that

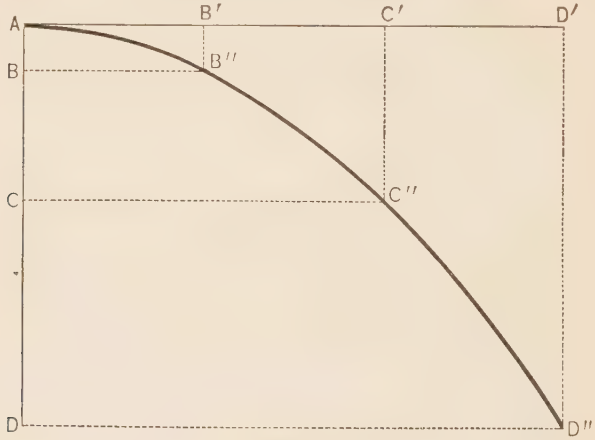


FIG. 6

its velocity is composed of a horizontal and a vertical component. Its horizontal velocity (if the resistance of the air be disregarded) is constant and its vertical velocity is uniformly accelerated. Let AB (Fig. 6) represent the vertical component of the distance traversed during the first second; then BC and CD will represent the vertical distances during the second and third seconds respectively. Let AB', $B'C'$, and $C'D'$ represent successive horizontal distances during the same three periods. Then it is evident by a combination of these two motions that

the body will pass from A to B'' during the first second, from B'' to C'' during the second second, and from C'' to D'' during the third second. The body traverses a curvilinear path shaped like a *parabola*, as shown in the figure. In practice, the resistance of the air would modify the nature of the curve somewhat, so that its real path is a peculiar curve known in the science of gunnery as a *ballistic curve* or *trajectory*.

It should be borne in mind that one of the component velocities of a particle moving in a curvilinear path is always accelerated.

Imagine a body to be projected obliquely upward at an angle of 45°; by a diagram represent arbitrarily its vertically downward accelerated motion and its obliquely upward constant motion for three seconds, and determine the actual path traversed by the body during this time.

SECTION IV

KINDS OF MOTION

19. Motion of translation and rotation. Up to the present time we have considered generally only the motion of points or particles. These, as we have seen, can move in either straight or curved lines, and their motion may be either uniform or variable. In the case of an extended body, however, still another classification of motions is necessary, namely, motion of translation and motion of rotation.

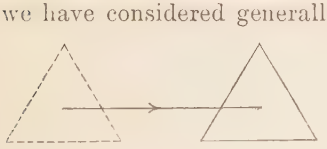


FIG. 7

In pure motion of *translation* all the points of a body move with the same speed and simultaneously in the same direction (Fig. 7); for example, the motion of an elevator or a piston in the cylinder of a stationary steam engine. When the points of a body describe arcs of circles, having its fixed centroid for a common center, the motion is one of pure *rotation* (Fig. 8); for

FIG. 8

example, the motion of a clock wheel or of a top. All possible varieties of motion may be produced by the combination of translation and rotation (Figs. 9 and 10); for example, the motions of the planets, of a ball thrown from the hand, or of a carriage wheel along a road.

When a body rotates only, every particle in the body describes a circle around some line, which is the *axis* of rotation.

The speed of a point far from the axis is greater than that of a point nearer the axis, and, generally, the speed of a point is proportional to its distance from the axis; hence the expression "speed of a rotating body" is meaningless.

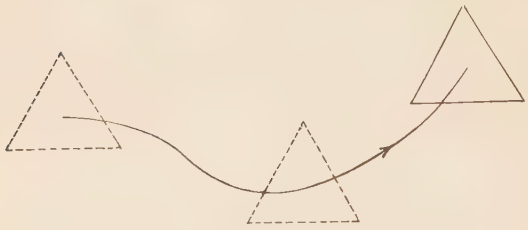


FIG. 9. Translatory and curvilinear motion

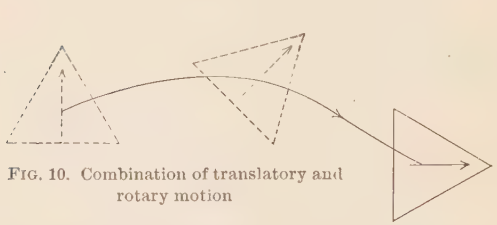


FIG. 10. Combination of translatory and rotary motion

20. Angular velocity. We may speak, however, of the *angular* velocity of the rotating body, which is the same for all points in the body. Angular velocity is *rate of rotation*, and is measured by the angle through which the rotating body turns in any given unit of time.

The unit angle in terms of which angular velocity is measured is called a *radian*, and is the angle which is subtended by an arc equal in length to the radius.

It is customary, then, to express angular velocity in radians per second, and it is numerically equal to the speed of a point one unit distant from the axis. The Greek letter ω (pronounced ō-měg'-à) is chosen to represent angular velocity.

Now since the linear speed of any point in a rotating body is proportional to its distance r from the axis, it must be the product of the angular velocity of the body multiplied by the distance of the point from the axis, i.e. $v = r\omega$.

It remains to develop a formula for finding the value of ω in terms of the time of a complete rotation. It is known that the ratio between the circumference of a circle and its diameter is about $3\frac{1}{7}$, and is usually represented by the Greek letter π. Now if the body make one rotation in T seconds, the angular velocity is equal to the total angle, 2π radians, divided by the time T, or $\omega = \dfrac{2\pi}{T}$ radians per second.

21. Rectilinear and curvilinear motion. Besides change in speed or rate of motion there may be a change in *direction* of motion. When a particle moves in a constant direction, i.e. in a straight line, as in the case of any freely falling mass, its motion is said to be *rectilinear*. But if its motion constantly changes in direction, i.e. at every point, as is the case of every particle in a rotating wheel except points on its axis, its motion is said to be *curvilinear*. It is evident that the direction of a motion in a curvilinear path can be given only at some specified instant, and, furthermore, that direction can be represented only by a straight line, for a curved line is a line composed of an infinite number of directions. Let A (Fig. 11) represent a body mounted on a cardboard sector SS', which is rotated about the axis C in the direction indicated by the arrow. The body will move in the circular path $ADEF$. The straight line AB will indicate the direction of the motion at every point,

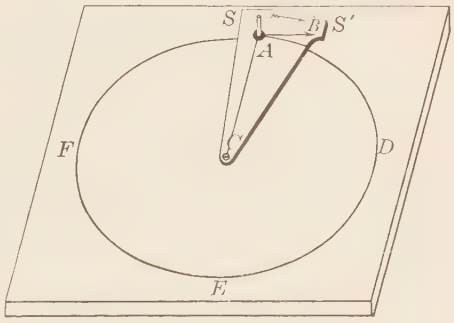


FIG. 11

but it will be seen that this line changes its direction constantly. At whatever point the body may be at any instant, the line AB, which shows the direction of the motion, is always tangent to the curve at that point.

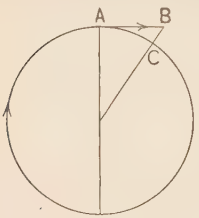


FIG. 12

If a particle move in a circular path, its motion at every instant is the resultant of a tangential motion and a centripetal (toward the center) motion. If when it passes point A (Fig. 12) its tangential velocity be represented by AB, its centripetal velocity may be represented by BC, because at the end of the unit of time in which it would reach B if it were moving in a straight line, it is found to be not at B, but at some other point C, nearer the center by the distance BC.

QUESTIONS AND PROBLEMS

1. A body makes ten rotations per second. What is its angular velocity?
2. What is the actual speed of a point in this body 10 in. from the axis of rotation?
3. (*a*) What is the angular velocity of the earth's rotation? (*b*) Does its angular velocity vary at different latitudes? (*c*) Do the actual speeds of points on the earth's surface differ at different latitudes?
4. What kind of motion is that of the earth in its orbit?
5. Why is it meaningless to speak of the velocity of rotation of a body?
6. (*a*) What motions have the wheels of a carriage drawn straight along a level plane? (*b*) What motion has the carriage?
7. Compare the several velocities of the small front wheels of the carriage with those of the larger hind wheels.
8. If a wheel make 200 revolutions per minute, what is its angular velocity?

SECTION V

22. Harmonic motion. Besides the motions of translation and rotation already considered, a third kind of motion must be studied somewhat in detail, as it plays so important a part in subjects to follow, notably sound and radiation.

If a lead bullet A (Fig. 13) be suspended by a thread and set swinging in a horizontal circular path, the motion is practically uniform, and, when viewed directly from above or below, appears circular. But if the motion of the bullet be viewed by the eye placed on the same level with it, it seems to travel to and fro in a straight line, but with varying speed. It is seen to move slowly near the ends of its path and more rapidly in the middle. The motion of the bullet as now viewed is virtually the projection of uniform circular motion on a diameter.

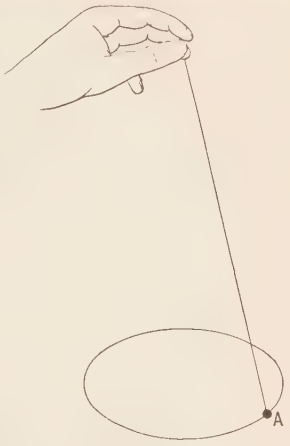


FIG. 13

Thus, let Fig. 14 represent a particle moving with uniform speed in the circle $ADMN$, as in the case of the swinging bullet. A line drawn from this particle, at the instant it passes the point A in the circle, perpendicular to the diameter MN intersects it at the point a. While the particle moves to B, C, D, E, F, and M, this intersecting point moves to b, c, d, e, f, and M. Although the speed of the moving particle is uniform, moving over the equal spaces AB, BC, CD, etc., in equal intervals of time, the speed of the intersecting point is variable, moving in the same intervals of time respectively through the unequal spaces ab, bc, cd, etc. The speed of this point is greatest as it passes the center O of the circle, diminishes toward the extremities of the diameter MN, and is momentarily

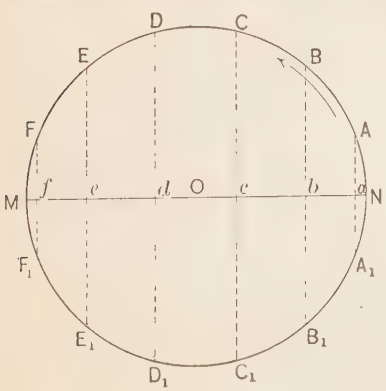


FIG. 14

zero at the points M and N. Such a motion as that, forward and backward along the line MN, executed in equal periods of time, is called *harmonic motion.*

It is the kind of motion executed by the vibrating prongs of a tuning fork, by the points of a stretched string when emitting a musical sound, by particles of air when traversed by sound waves, and very nearly by a pendulum swinging in a short arc. A like motion occurs in the ether when traversed by light waves or electrical waves, and in every elastic medium when set in vibration; hence its intimate relation with many branches of physics.

As the motion repeats itself at regular intervals, it is said to be *periodic.* The time occupied by a particle in executing a single complete harmonic motion, i.e. from M to N and back, is called the *period.* The period is evidently the time occupied by one complete revolution in the circle of reference, as N, D, M, C_1, N. When the body appears to move from left to right its motion is said to be *positive;* and when from right to left, *negative.* The extent of the vibration on either side of its middle point, as OM or ON, is called the *amplitude* of the motion. The distance of a moving particle from the middle point at any instant, as Oc, is called its *displacement,* and points M and N are *points of greatest elongation.* The position of the particle at any instant is denoted by its *phase,* which is defined as the fraction of a period since the particle last passed through O in the positive direction.

23. Composition of harmonic motions. Harmonic motions in the same or different directions may be compounded according to the same laws as uniform motions, accelerations, etc. We consider first the composition of harmonic motions of the same period. The resultant of two or more harmonic motions of the same period and in the same direction is a harmonic motion of the same period as that of the components and having an amplitude equal to the sum of the amplitudes of the several component motions. This assumes that the phases are the same. If, however, a particle be subjected to two harmonic motions along the same line, but differing in phase by half a period, the resultant

will be a motion with an amplitude equal to the difference of the amplitudes of the components; that is, the particle will remain at rest if the component amplitudes be equal.

If the two motions be in the same phase or in opposite phases, i.e. having a difference of phase of $\frac{1}{2}$ period, and at right angles to each other, the amplitude of the resultant will be represented by the diagonal of a rectangle constructed upon those lines as adjacent sides which represent the amplitudes of the component motions. Thus in Fig. 15, let AB and CD represent two component harmonic motions at right angles to each other, the center of the circle of reference being at O. Then EG or FH will represent the resultant motion if the phases are alike or opposite.

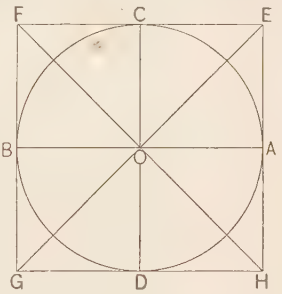


Fig. 15

If the difference of phase be only $\frac{1}{4}$ period, then the resultant motion will be a circle, or an ellipse, according as the component amplitudes are equal or not. If the difference of phase be a fraction of a period not as simple as $\frac{1}{4}$ or $\frac{1}{2}$, then the resultant motion in general will be an ellipse with its center at O, the position of its major axis depending on the relation of the component amplitudes and on the difference of phase.

The principles given above may be verified experimentally by the use of an approximately simple pendulum. Partly fill a small glass funnel (Fig. 16) with fine dark writing sand and suspend it from a frame above by means of a string, say, 1 meter long. It can be made to oscillate at will in any direction, and the sand falling upon a horizontal surface, e.g. a large sheet of white paper or pasteboard, leaves a tracing of the motions of the pendulum. The position of rest of the bob being at O (Fig. 15), give it an impulse with the hand in the direction OA sufficient, say, to carry it to A. We may, for convenience, divide the complete motion into four parts, each of $\frac{1}{4}$ period, e.g. from O to A,

A to O, O to B, and B to O. A similar impulse may be given to the bob in any other direction, for example, in the direction OC. The motions of such a pendulum are approximately harmonic. The directions OA and OC will be considered as positive, and their opposites, OB and OD, negative.

24. We will now examine four cases: (1) While the bob is oscillating in the path AB, gently tap it with the hand in the direction OC at the instant it passes O in the positive direction; the bob will now move along the line OE, the diagonal of the square on OA and OC. In this case the component motions are in the same phase, since both are as if impulses were given at O and in positive directions.

FIG. 16

(2) While the bob is moving through O toward A, let the second impulse be in the direction OD; the resultant motion is now in the direction OH, and the bob oscillates along the line FOH without change of period. In this case the phases are opposite, differing by $\frac{1}{2}$ period.

(3) Let the impulse be in the direction OC, but applied when the bob is at A instead of at O. The motion now is in a circle, the direction being anti-clockwise, i.e. opposite to the direction in which the hands of a clock move. Here the component BA is $\frac{1}{4}$ period ahead of the component DC.

(4) If the impulse be given at A in the direction OD, the resultant will be a circle still, but traversed in the opposite direction, the component BA being now $\frac{1}{4}$ period behind the component DC.

Observe that circular motion may be regarded as compounded of two harmonic motions at right angles to each other, the phases differing by $\frac{1}{4}$ period.

If the component amplitudes be not equal, the resultant in the case of the same or opposite phases will be in a straight line, — the diagonal of a rectangle, not of a square. With the difference of phase of $\frac{1}{4}$ period, the circle becomes an ellipse with its axes coincident with the directions of component motions.

25. Composition of harmonic motions of different periods. The motion resulting from the composition of two harmonic motions of different periods is more or less complicated according to the ratio of the periods. We have already discussed the case where the ratio is 1 : 1. The next simplest ratio is 1 : 2, and the following simple arrangement may be used to illustrate.

Suspend from two staples in a horizontal bar a loop of string about 2 meters long. Slip the end of the loop through a small clamp, which may be adjusted at any height O (Fig. 17). Adjust so that $OP = \frac{1}{4}\ CP$. Cause the pendulums OP and CP to oscillate in planes at right angles to each other. As CP is 4 OP, the period of CP is twice that of OP (§ 69). By following the directions given in the last experiment, the resulting motions may be observed and studied. Fig. 18 represents the path of P when the phase differences are respectively 0, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ period.

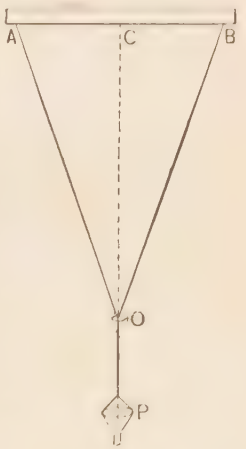


FIG. 17

Should the ratio be not exactly 1 : 2, the different curves will gradually change from one to another. The result of compounding harmonic motions of other period ratios may be studied in a similar manner by slipping the clamp up or down and thus changing the relative lengths of the two pendulums. The figures described on the paper by the falling sand are exceedingly intricate and interesting.

26. Harmonic curve. Let the harmonic motion of a particle be executed in the line AB (Fig. 19), and at the same time let the particle travel with uniform speed from left to right. ACB is the circle of reference, which is divided into fourteen parts of equal length by the points A, C, D, E, etc. Lines drawn through these points at right angles to the line AB determine the points on this line, viz. c, d, e, etc., which define the positions of the particle, so far as determined by the harmonic motion, at equal

intervals of $\frac{1}{14}$ of the period. Lines drawn parallel to AB divide the space into equal intervals, which represent the distances traversed by the particle from left to right during each $\frac{1}{14}$ of a period. Now combining the motion AM with Ac, MN with cd, NP with de, etc., we get points *1*, *2*, *3*, etc., which represent the

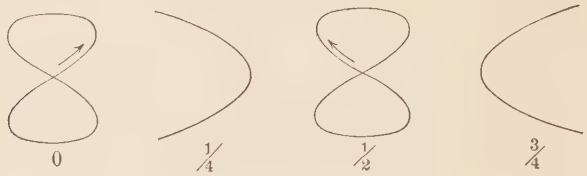


FIG. 18

actual positions of the particle after successive intervals of $\frac{1}{14}$ of a period. If these points be joined by a gently curving line, there results a characteristic curve called a *harmonic curve.*

This curve may be produced experimentally by filling the funnel pendulum (Fig. 16) with fine sand and swinging it just above a strip of paper moving uniformly in a direction at right angles to the plane of the pendulum.

In this case the harmonic curve is clearly the composition of harmonic motion with uniform rectilinear motion at right angles to it.

SECTION VI

27. Wave motion. Referring to Fig. 19, let us suppose that at the upper ends of the vertical lines were once located particles, and that these particles started to move harmonically, but each one $\frac{1}{14}$ of a period behind the next ahead. After a time equal to two periods the particles will be found to occupy positions *1*, *2*, *3*, etc., as noted in the figure, which, we have just seen, lie on a harmonic curve. It is by just such consecutive vibratory motions of particles of matter that waves are propagated in elastic media. A wave, then, in its simplest form consists in propagating the

shape of a harmonic curve by the successive harmonic motions of a line of material particles, the phases differing by the same amount from particle to particle.

28. Classification of waves. Waves are usually classified as (1) *tranverse* waves, in which the harmonic motion is at right angles to the direction of propagation of the wave; (2) *longitudinal*

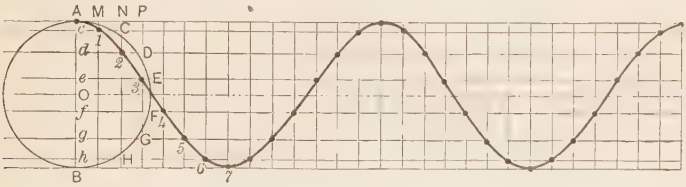


FIG. 19

waves, in which the vibrations of the particles are parallel to the direction of wave propagation; and (3) a combination of these two, in which the particles move in ellipses.

Fig. 19 is a diagram of a transverse wave. Sound waves are examples of longitudinal waves, and the small waves on the otherwise undisturbed surface of a body of water are examples of the third class. In the last case the paths of the particles are approximately circles, and Fig. 20 shows the shape of the wave.

FIG. 20

29. Let *A*, *B*, *C*, *D*, etc. (Fig. 20), represent a series of water particles lying in the same straight line. Just below, the same particles are represented as moving simultaneously, each in a circular path, in a vertical plane. Particle *B* is just $\frac{1}{8}$ period behind *A*, *C* the same interval behind *B*, and so on. A line drawn through the particles in their several positions at the same instant gives the *wave line* in this case. As long as the particles

continue to move in their respective circles, so long will a *wave form* traverse the series of particles. If a person is favorably placed so that he can observe a series of water waves passing him, he will perceive that floating blocks of wood move in approximately circular paths, never moving more than a small distance from certain points about which they oscillate. The motions of the blocks represent rather imperfectly the curvilinear paths in which the particles composing a surface of water move while that body is agitated by waves. To the observer ridges and furrows of *water* appear to move along the expanse, but objects floating upon the surface are not carried along by them, which shows that the appearance is a deception, and that the body of water is traversed only by wave *forms*.

DYNAMICS

MOLAR DYNAMICS

SECTION I

FORCE. MOMENTUM

30. *Dynamics* is the science which treats of the action of force. It will be treated under three heads: (1) Molar Dynamics; that is, the dynamics of solids and fluids, including the study of sound waves; (2) Molecular Dynamics, including heat; (3) Ether Dynamics; that is, radiation, including light and electricity.

For present purposes, at least, we may regard the term *physics*[1] merely as a generic term which includes all these branches. Hence *physics is the science which treats of the dynamics of masses, molecules, and the ether.*

31. Force. When a body at rest is set in motion, or one which is in motion is accelerated (positively or negatively), or when a moving body is deflected from a straight course, experience teaches us that there is always a cause, and we have also learned to apply to this cause the name *force.* We have also learned that when a body is under the influence of a force which tends to cause a change of motion in that body, another force or its equivalent must act on the same body if a change of motion is to be prevented.

We get our primitive idea of force from the sense of muscular exertion which we experience when, by personal effort, we put

[1] Physics is often defined as the Science of Matter and Energy, since "In the physical universe only matter and energy exist independently of our senses and reason" (Tait).

bodies in motion or stop bodies that are in motion. We transfer this conception by analogy to a change of motion observed in any body, and attribute this change to an interaction between that body and some other body, animate or inanimate. This interaction is always a *pull* or a *push*, and is accordingly called an *attractive* or a *repellent* force. It is evident that there can be no pull or push except between at least two bodies or two parts of the same body, i.e. there is no such thing as a one-sided pull. In other words, when there is a pull or a push there are at least two bodies pulled or pushed, and it is only for the sake of convenience in speech that we are permitted to say that one body pulls and the other is pulled.

We are now prepared for a definition of force. *Force is an interaction between two bodies (or two parts of the same body), causing or tending to cause a change in the motion of each, either in direction or in magnitude:* or, more simply, *force is that which tends to modify motion.*

It should be observed that the above conveys no idea of what the real nature or essence of force is, for of this we are quite ignorant. Indeed, *we know of the existence of force only by its effects:* hence an idle force, i.e. a force producing no effect, is an absurdity.

32. Action and reaction. Force is always dual, inasmuch as it is always oppositely directed upon two bodies. By a conventionality of speech we say that one of the two bodies *acts* upon the other, and the latter *reacts* upon the former. Later on it will be shown that the reaction is always equal to the action.

The wings of a bird act upon the air, giving a certain portion of it a rearward motion; the air reacts upon the wings, giving the bird a forward motion. The bat strikes the ball, imparting to it an acceleration; the ball reacts upon the bat, giving it a negative acceleration.

33. The subject of pure motion (kinematics) has already been briefly treated, and it now becomes necessary to study the relation which the *mass* of the moving body has to the phenomena of

motion. This is a department of dynamics and is called *kinetics*. If a body have a speed of 100 centimeters per second, the character of the motion, as such, is the same whether the mass of the body be 10 grams or 100 kilograms. The dynamics of the case, however, would be quite different. It requires far more *force* to produce the given speed in 100 kilograms than in 10 grams; in fact, the force required is exactly proportional to the mass.

34. Momentum. From the above consideration we are led to associate both the mass and the velocity of a body, and the product of these two quantities, namely mv, is called *momentum*.

A large mass moving slowly has great momentum, but the same mass will have twice the momentum if its velocity be doubled; again, a small mass moving swiftly has great momentum, but the momentum is increased in proportion as the mass is increased.

If the motion of a mass of 1 kilogram having a velocity of 1 meter per second is taken as a unit of momentum, then a mass of 5 kilograms moving with the same velocity would have a momentum of 5; and if the latter mass should have a velocity of 10 meters per second, its momentum would be $5 \times 10 = 50$. Hence *the numeric of momentum is found by multiplying the number of units of mass by the number of units of velocity.*

If the mass and the velocity both be unity, the momentum will also be unity; and the unit of momentum, which has received no special name, may be defined as the momentum of a unit mass moving with unit velocity, and momentum may be defined as *rate of mass displacement.*

Since momentum is a quantity which has direction, it may be compounded and resolved like motions and velocities.

By experiment, we learn that a given force acting for two units of time produces twice the velocity that it does in one unit of time, and that the velocity which a given force produces is proportional to the time it acts; hence momentum, when the force is constant, is equal to the product of the force and the time, i.e. ft; or $mv = ft$. If the force be not constant, then the

momentum must be computed from the *average* force acting. The product mv signifies that the momentum of a body depends on its mass as well as its velocity. The product ft signifies that the momentum imparted to a body depends upon the time t during which a force acts as well as upon the intensity f of the force. We infer from the above equation that a definite force acting upon any mass for a given time will generate in it a speed whose magnitude is inversely as the mass.

35. Impulse. The product of the time during which a force acts and its mean intensity is called the *impulse* of the force. This term is usually restricted to a force acting for a short time, as in a blow given to a ball by a bat. There is no propriety in asking, "With what force does the bat hit the ball?" The inquiry may be with reference to the average pressure which it exerts on the ball, or, more likely, with reference to the impulse of the force, i.e. the product of the mean intensity and the time it acts. It has just been shown that $ft = mv$; hence we infer that the impulse of a force is measured by the momentum produced.

36. With a *given* momentum, then, even though small, a body may exert a considerable force in being stopped, providing the time of stopping be *short* enough; in other words, the force of a blow depends on the character of the body struck. Imagine a 5-gram bullet fired into a board with a speed of 1000 meters per second. The C.G.S. unit of momentum is evidently that of 1 gram moving with a speed of 1 centimeter per second. Therefore the bullet has, on striking the board, a momentum of 5×10^5 units. Now suppose the board to be of soft pine and that the bullet sinks in quite a distance and takes $\frac{1}{100}$ of a second in coming to rest. Then, since $ft = mv$, or $f = \frac{mv}{t}$, the value of f will be $\frac{5 \times 10^5}{\frac{1}{100}} = 5 \times 10^7$ dynes (§ 41). If, however, the board be of oak, the bullet will be brought to rest much quicker, say, in $\frac{1}{1000}$ of a second. In this case the force will be $\frac{5 \times 10^5}{\frac{1}{1000}} = 5 \times 10^8$ dynes, or ten times as great as before.

37. Sand blast. Even very small bodies require time to gain or surrender a definite velocity. The sand blast, employed for engraving figures on glass, furnishes a fine illustration of this fact. A box of fine quartz sand is placed in an elevated position, from which a long tube extends vertically downwards. The plate of glass to be engraved is covered with a thin layer of melted wax. When this wax is cool the design is sketched into it with a sharp-pointed instrument, leaving the glass exposed only where the lines are traced. The plate is then placed beneath the orifice of the tube and exposed to a shower of sand. The velocity of the sand grains is not at its maximum at the start, but is constantly accelerated till they reach the plate, where, in turn, it is gradually given up. The wax, on account of its yielding nature, gradually brings the sand grains to rest; but the glass, notwithstanding its hardness, cannot stop them quite at its surface, and therefore it suffers a chipping action. Thus the soft wax protects from the action of the falling sand all parts except those intended to be cut. A still greater force is generally given to the sand by steam blown through the tube. For this reason the apparatus is called a sand blast. Hard metals like steel are engraved in the same manner. Yet the hand may be held in the blast several seconds without injury.

QUESTIONS AND PROBLEMS

1. What agent is the immediate cause of motion?

2. What distinction do you make between velocity and momentum?

3. Upon what does the momentum given to a ball fired from a gun by the expanding gases depend?

4. Inasmuch as equal forces are exerted for the same length of time by the gases on the ball and the gun, how will the momenta communicated to each compare?

5. If there be 25 lb. of matter in the gun and 1 oz. ($\frac{1}{16}$ lb.) in the ball, and if the gun acquire a maximum speed of 3 ft. per second, what, at that instant, is the speed of the ball?

6. Can any body be put in motion in no time? (Demonstrate from formula $ft = mv$.)

7. Compare the momentum of a car weighing 50 T., moving 10 ft. per minute, with that of a lump of ice weighing 5 cwt., at the end of the third second of its fall.

8. With what velocity must a boy weighing 25 kg. move to have the same momentum that a man weighing 80 kg. has when running at the rate of 10 km. per hour?

9. Since $ft = mv$, to what is change of momentum proportional?

10. If the same force act for the same length of time upon bodies having different masses, to what will the velocities produced be proportional?

11. Two boats of unequal masses are brought together by pulling on a rope. (*a*) Resistance being disregarded, how will their momenta at any given instant compare? (*b*) How will their velocities at the same instants compare?

12. If the motion of the moon in its orbit were to cease, the moon and the earth would approach each other. The mass of the earth is about 80 times that of the moon. What part of the whole distance between them would the moon move before collision?

SECTION II

MEASUREMENT OF FORCE

38. Force tends to produce acceleration. Thus the force of gravity causes bodies to fall with accelerated velocity; it also transforms into a retarded velocity the otherwise constant velocity of a body projected upward. *A constant force acting upon a free body*, i.e. *a body which encounters no resistances, always produces a uniformly accelerated motion.* This is best illustrated by the fall or ascent of a body in a vacuum, the body being meantime acted on only by the constant force of gravity.

39. Force of gravitation; weight. It has been said that the best way of defining gravitation is to "let a stone drop." In this phenomenon we discover evidence of the action of a force between masses; this force is called *gravitation*, or *gravity*, when the action is between the earth and some other mass. Inasmuch as by the action of this force the earth and stone are brought together, this force has been assumed to be, and is universally spoken of as, an attractive force; but the probability is that the earth and stone do not draw each other together, but are brought together

through the agency of some surrounding medium, and the action is quite as likely to be a push as a pull. *The term "weight" signifies the magnitude of the force of gravity which exists between any body and the earth.* It is usually determined by measuring the pressure which gravity causes the body weighed to exert upon a supporting body, e.g. on a scale pan, or the distortion which it produces in the supporting body, e.g. the elongation of the spring in the spring balance. The units generally employed are the *pound* and *kilogram*, and are called the *gravitation units* of force. All forces may be measured in the same units. To say that a man pulls a boat with a force of one hundred pounds is equivalent to saying that he pulls with a force that is equal to the force which acts between the earth and a body having a mass of one hundred pounds. A force of one pound, then, is an abbreviated expression for a force equal to the weight of one pound of matter.

40. Absolute measurement of force by direct observation of acceleration and mass. If a force f be applied to a certain mass m for a given time, a certain momentum is generated in the mass. If the same force be applied to a greater mass for the same time, it will move, as has been shown, with as many times less velocity as the mass is times greater; but the *product* of the mass and the velocity, i.e. the momentum, is the same. In other words, the same force acting for the same length of time on free bodies having different masses may be measured by the change of momentum generated by it in a second, since this is constant and depends on nothing but the force; that is, $f = ma$, in which f represents any constant force acting on any mass m, a the acceleration, and $ma \left(= \frac{mv}{t}\right)$ the rate of change of momentum. Force is therefore appropriately defined as the *time rate of change of momentum.*

41. Absolute unit of force. *A unit of force* in the absolute system is that which will give to a unit of mass a unit of acceleration. The absolute unit of force (in the C.G.S. system) is called a *dyne*, and is *that force which is capable of giving to a gram mass*

an acceleration of one centimeter per second; in other words, it is a constant force of the requisite intensity to impart in every second to a gram mass a velocity of one centimeter per second.

Any constant force which produces in a mass of m grams an acceleration of a centimeters per second must be equal to $m \times a$ dynes, i.e. $f = ma$. In physics the letter g is generally used instead of the letter a to denote the acceleration due to the force of gravity. By exact measurement the acceleration produced by the force of gravity on free bodies, i.e. in a vacuum, is found to be, in the latitude of Boston at the level of the sea, 980.4 centimeters per second. Hence the force of gravity acting on a mass of one gram must be (substituting, in the equation above, W (weight) for f, and g for a) $W = mg = 1 \times 980.4 = 980.4$ dynes.[1] Consequently it requires a force of 980.4 dynes to support, i.e. prevent from falling, a mass of one gram; or the weight of a gram mass at sea level in latitude 42° is 980.4 dynes. A dyne[2] is therefore about $\frac{1}{980}$ of the weight of a gram mass. In the gravitation system the weight of a gram mass is a gram force; hence 1 gram force = 980 dynes approximately. Gravitation units in grams force are readily changed into dynes by multiplying in general by the value of g at any place.

From the foregoing statements it appears that the weight of the gram mass varies with locality, e.g. Boston or Paris, so that the value of a gravitation unit of force, e.g. a gram force, is variable. This want of definiteness constitutes a serious objection to the gravitation system when great accuracy is required. The value of the dyne is definite, absolute.

42. Expression for the mass of a body. Since $W = mg$, $m = \frac{W}{g}$; that is, mass is measured by its weight in dynes or poundals,[3] divided by the acceleration in feet or centimeters per second

[1] The equation $W = mg$ expresses the fact that the number of dynes acting on a given mass is g times the number of grams mass in that body.

[2] A dyne is a very small force. In expressing a force of considerable magnitude the *megadyne* (a million dynes) is commonly used.

[3] A poundal is that force which is capable of giving to a pound mass an acceleration of one foot per second.

produced by gravity. Although W and g vary with latitude and elevation, they vary proportionally; hence the ratio $\frac{W}{g}$, i.e. the mass, does not change, but is constant for the same body.

43. Measurement of force by counterbalancing. The household instrument called a spring balance is, strictly speaking, a *dynamometer*, i.e. a force measurer. It contains a spiral spring carrying an index which moves over a scale. If a unit of mass, e.g. one pound or one kilogram, be hung upon the spring, the latter is lengthened by a certain definite quantity. If, grasping the ring in one hand and the hook in the other, you lengthen the spring by a muscular pull as much as it was lengthened by the force of gravity acting on the mass, the inference is that the muscular force which you exert is equal to the force of gravity exerted on the mass; hence the spring balance measures all forces in gravitation units. A spring balance might, however, be graduated in dynes so as to measure force in absolute units.

The pound and the gram are primarily units of mass. A pound force is *a force equal to the weight of one pound of matter on the earth's surface.*

44. Two systems of measurement of force. We have found in the foregoing discussions that there are two methods of measuring force: one, specially adapted to measuring balanced forces, called the *statical* or *gravitation* system; the other, specially adapted to measuring unbalanced forces, called the *kinetic* or *absolute*[1] system; though a force, whether balanced or unbalanced, may always be measured by either system. The gravitation system is so called because by it forces are compared with the force of gravity as a standard. The two methods of measuring force give rise to *two systems of units*, called respectively the *gravitation* and the *absolute* systems, either one of which is easily convertible into the other, as shown above.

[1] Measurements of force in the absolute system are attended with serious practical difficulties in the way of observation of the acceleration produced, yet the absolute units are almost indispensable in very many scientific calculations, especially in electricity and magnetism.

QUESTIONS AND PROBLEMS

1. A constant force acts on an otherwise freely moving body in a direction opposite to that in which it is moving. How is the body's motion affected thereby? Give an illustration.

2. How does a gravitation unit of force differ from an absolute unit?

3. Would a spring balance graduated in grams in Paris (g. = 980.9), and sensitive to the smallest changes, answer for weighing, i.e. determining the exact masses of bodies, in Boston (g. = 980.4)?

4. What kind of motion does a constant force produce on a free body? How has this been shown?

5. On what condition will equal forces produce equal accelerations?

6. Suppose that you fill a box with sand, place it on a toy cart, pull the cart by a string with a constant force along a smooth floor for a certain number of seconds, and observe the acceleration given the load (cart, box, and sand); then remove the sand and replace it with lead shot. How could you tell, by pulling the load with the same force as before, when it has the same mass as the former load?

7. (*a*) Has the same mass equal weights in Paris and Boston? (*b*) How sensitive must a spring balance be to show any difference?

8. Show that a spring balance is, strictly speaking, a force measurer and not a mass measurer.

9. (*a*) When we speak of a force of one pound, what do we mean? (*b*) When we speak of a force of one dyne, what do we mean? (*c*) When we speak of a mass of one pound, what do we mean?

10. (*a*) If one mass is four times another, how many times as much force is necessary to produce the same acceleration in the former as in the latter? (*b*) How many times greater is the force of gravity acting on a mass of one hundred pounds than on a mass of one pound? (*c*) If a hundred-pound iron ball and a one-pound iron ball be let drop from the same height at the same instant, which ought to reach the ground first?

11. A body weighing 4 g. is moving with an acceleration of 12 cm. per second. What is the force acting?

12. A body acted on by a force of 100 dynes receives an acceleration of 20 cm. per second. What is its mass?

13. A body of mass 30 g. is moved by a constant force of 50 dynes. What is its acceleration?

14. What acceleration will a force of 20 dynes produce on a mass of 10 g.?

15. What velocity will a force of 20 dynes acting on 1 kg. impart to it in 5 min.?

16. (*a*) What is the weight in dynes of a mass of 1 kg. in Boston? (*b*) How many more dynes does it weigh in Paris?

17. A constant force of 20 dynes acts on a mass of 5 g. and gives it a velocity of 500 cm. per second. How many seconds does it act?

18. How is the value in dynes of a gram weight at any locality determined?

19. How much is the momentum of one gram changed by gravity in one second when falling freely in Boston?

20. Why will a bullet fired at an open door pass through it without moving it perceptibly, while a gentle push of the hand moves it an appreciable distance?

21. Explain why the weight of a body is not a perfect measure of its mass.

SECTION III

COMPOSITION AND RESOLUTION OF FORCES

45. Graphical representation of force. A force is defined when its *magnitude, direction,* and *point of application* are given. Hence we may represent forces graphically by straight lines whose lengths bear to one another the same relation as do the numerics of the forces, while the directions of these lines indicate the directions of the forces, and the points from which the lines are drawn indicate the points of application. Thus, on a scale of 1 centimeter = 1 kilogram the line.

FIG. 21

AB (Fig. 21) represents a force of 3.2 kilograms acting toward the right, with its point of application at *A*; and the line *DE* represents a force of 2 kilograms acting parallel to the first, with its point of application at *D*.

46. Composition of forces acting in the same line; equilibrium of forces; balanced and unbalanced forces. Insert two stout screw eyes in opposite ends of a block of wood. Attach a spring balance to each eye. Let two persons pull on the spring balances at the same time, with equal force, as shown by the indexes, but in opposite directions. The block does not move. One force just neutralizes the other, and the result, so far as any movement of the block is concerned, is the same as if no force acted on it.

When one force opposes in any degree another force, each is spoken of as a *resistance* to the other. Let f represent the number of pounds of any given force, and let a force acting in any given direction be called *positive*, and indicated by the plus (+) sign; and let a force when acting in an opposite direction to the force which we have denominated positive, be called *negative*, and indicated by the minus (−) sign. Then if two forces $+f$ and $-f$ acting on a body at the same point or along the same line are equal, they are said to be *balanced*, and the result is that no change of motion is produced.

Viewed algebraically, $+f-f=0$; or, correctly interpreted, $+f-f \doteq$ (is equivalent to) 0, i.e. no force. In all such cases there is said to be an *equilibrium of forces*, and the body is said to be in *a state of equilibrium*.

A force that produces equilibrium with one or more forces is called an *equilibrant*.

If one of the forces be greater than the other, the excess is spoken of as an *unbalanced force*, and its direction is indicated by one or the other sign, as the case may be. Thus, if a force of $+8$ pounds act on a body toward the east, and a force of -10 pounds act on the same body along the same line, then the unbalanced force is -2 pounds; i.e. the result is the same as if a single force of 2 pounds acted on the body toward the west. Such an equivalent force is called a *resultant*. *A resultant force is a single force that may be substituted for two or more forces and produce the same result that the simultaneous action of the several forces would produce.*

The resultant of any number of forces acting in the same straight line is equal to the algebraic sum of the forces. An equilibrant of several forces is equal in magnitude to their resultant, but opposite in direction. The process of combining several forces so as to find their resultant is called *composition of forces*. The forces combined are called the *components*. The converse operation, that of finding component forces which shall have the same effect as a given force, is called *resolution of forces*.

47. An unbalanced force always produces acceleration. A body acted on by an unbalanced force cannot be at rest. That branch of dynamics which treats of the relation of force to the motion which it produces is called *kinetics*, and that branch which treats of equilibrium of forces is called *statics*.

Equilibrium is often maintained by the reaction of a surface with which the body acted on is in contact. A simple illustration is that of a body supported on a horizontal surface, as of a table. Here the reaction caused by the compression of the material of which the table is composed is equal to the weight of the body.

48. Pressure; tension. A balanced force does not produce acceleration, but causes either a *pressure* or a *tension*. A force exerts pressure when it tends to *compress* or shorten in the direction of its action the body on which it acts; for examples, pressure exerted on the springs of a carriage, on air when it is compressed in an air gun, etc. A force causes tension when it tends to *lengthen* in the direction of its action a body on which it acts. A body thus subjected to a force tending to elongate it is said to be in a *state of tension*, the stress to which it is subjected is called *tension*, and the strength to resist being pulled apart is called its *tensile strength*.

QUESTIONS AND PROBLEMS

1. Explain the use of a line to represent force.

2. (*a*) When a force of 100 lb. is represented by a line 5 in. long, what is the scale? (*b*) What force will a line $\frac{1}{4}$ in. long represent on the same scale?

3. (*a*) Represent on a scale of $\frac{1}{4}$ in. = 1 lb. the resultant of forces of 5 lb. and 7 lb. acting in the same direction. (Always place arrowheads in lines representing forces, to indicate the direction of the forces.) (*b*) Show, by points *A*, *B*, and *C* placed in the line, the components of this resultant. (*c*) Represent the same two forces acting in opposite directions at the same point *A*. (*d*) How will you represent the resultant of these two opposing forces?

4. Three men, A, B, and C, pull on a rope in the same direction with forces respectively of 50 lb., 60 lb., and 70 lb. A is nearest the end of the rope, B next, and C next. (*a*) What is the tension of the rope between

A and B? (*b*) between B and C? (*c*) A man, D, just beyond C, pulls with a force of 75 lb. in the opposite direction. With what force must a man, E, pull that there may be equilibrium? (*d*) When there is equilibrium, what is the tension of the rope between C and D? (*e*) How great must be the tensile strength of the rope between C and D? (*f*) Write the equation showing the algebraic addition of the forces in case of equilibrium.

SECTION IV

COMPOSITION OF PARALLEL FORCES. MOMENTS OF FORCES

49. Composition of parallel forces acting in the same direction and in the same plane. *AB* (Fig. 22) represents a rod lying on a table with three strings loosely looped around it in such a manner that they may be slid along the rod. The strings are all stretched in parallel directions in a plane parallel with the top of the table and dynamometers are attached to the free ends. (Great care must be taken in the manipulation to keep the three strings exactly parallel.) The dynamometers register the tensions in the several strings, i.e. the forces applied through them to the rod.

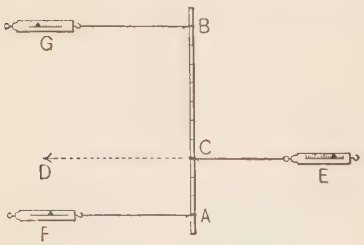


FIG. 22

Observe (1) when there is equilibrium the dynamometer *E* registers as much as those of *F* and *G* added together. This would be true if more than two forces were applied in the same direction as *AF* and *BG*. But the force applied at *C* is the equilibrant of the other forces, and this is equal to their resultant acting in the direction *CD*. (2) The point of application of the resultant (or equilibrant) is between the points of application of the components. (3) This point is nearer the greater force. (4) The distance of this point from the smaller force is as many times greater than its distance from the larger force as the larger force is times the smaller force. For example, if *AF* be

14 pounds and BC 6 pounds, i.e. if the two forces be as 7 : 3, then distances CA and CB will be as 3 : 7. In other words, the component forces are said to *vary inversely as*, or to be *inversely proportional to*, their distances from their resultant. These observations are summarized as follows: *The resultant of two parallel forces in the same direction is equal to their sum; and the distances of their points of application from the point of application of the resultant vary inversely as the intensities of the components.*

COROLLARY. *The condition of equilibrium is that the algebraic sum of all the forces (positive and negative) must be zero.*

When more than two forces act on a body in the same plane and in the same direction, the resultant of any two of them (and its point of application) is found, then the resultant of this resultant and a third force, and so on until all have been used.

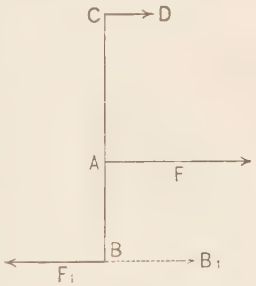


FIG. 23

50. Composition of two unequal parallel forces acting in opposite directions. Let F and F_1 (Fig. 23) be parallel forces acting in opposite directions on CB, F being the greater. The force F may be resolved into two forces: one, represented by BB_1, equal and opposite to F_1; the other equal to $F - F_1$, and represented by the line CD. But the forces F_1 and BB_1 are in equilibrium, leaving an unbalanced force at C equal to $F - F_1$. This, then, is the resultant R of the forces F and F_1, i.e. $R = F - F_1$. But $BB_1 : CD = AC : AB$, and, by composition,

$$BB_1 + CD : BB_1 = AC + AB : AC,$$

or

$$F : F_1 = CB : CA.$$

Hence *the resultant of two unequal parallel forces acting in opposite directions is equal to their difference, and acts outside of both in the direction of the greater component; and the distances of its point of application from the points of application of the two forces are inversely proportional to their intensities.*

51. Dynamical couple. If F and F_1 (Fig. 23) be equal, the magnitude of the resultant, being equal to the difference of the components, is zero, i.e. they have no resultant. *Two equal forces applied to the same body in parallel and opposite directions not in the same line constitute what is called a couple.* The effect of a couple is to produce *rotation*, but no motion of translation. The value of a couple will be determined later on.

52. Moment of a force. The value of a force to produce rotation around a given axis is called its *moment* with reference to that axis. The axis is, of course, always a line at right angles to the plane of rotation. Point C (Fig. 24) may represent the extremity of the axis about which AB is supposed to rotate. The perpendicular distance (CA or CB) from the axis of rotation to the line of direction in which a force acts (AD or BE) is called the *arm* or *leverage* of the force.

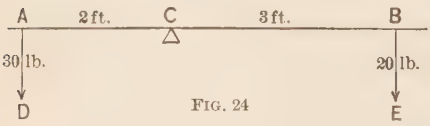


FIG. 24

The moment of a force is measured by the product of the intensity of the force by the arm. For example, the moment of the force AD (Fig. 24) is expressed numerically by the number $(30 \times 2 =) 60$, and the moment of BE is $(20 \times 3 =) 60$. By definition the line AC (Fig. 25) is the arm of the force P, and BC of the force Q.

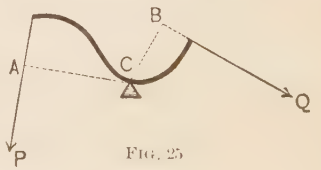


FIG. 25

53. Equilibrium of moments. The moment of a force is said to be *positive* when it tends to produce right-hand rotation, i.e. in the direction in which the hands of a clock move, and *negative* when its tendency is in the reverse direction. *If two forces act at different points of a body which is free to rotate about a fixed point, they will produce equilibrium when the algebraic sum of their moments is zero.* Thus the moment of the force applied at A (Fig. 24) is algebraically -60. The moment of the force applied at B in an opposite direction is

accordingly + 60. Their algebraic sum is zero; consequently there is equilibrium between the moments, and no tendency to rotation.

When more than two forces act in this manner there will be equilibrium if the sum of all the positive moments be equal to the sum of all the negative moments. Thus, the sum of the positive moments acting about point D (Fig. 26) is

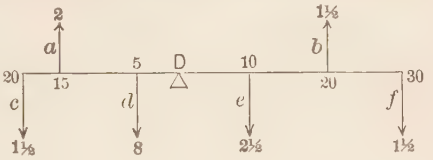


FIG. 26

$$(f)45+(e)25+(a)30=100;$$

the sum of the negative moments acting about the same point is

$$(c)\,30+(d)\,40+(b)\,30=100;$$

the two sums being equal, the moments are in equilibrium.

54. Moment of a couple. The moment of a couple, or its value in producing rotation, is equal to the sum of the moments of its two components around the axis of rotation. Let F and F_1 constitute a couple whose arm is AB (Fig. 27). To find the rotating value of the couple, let P be the axis of rotation; then the moments of F and F_1 relatively to P are $F \times AP$ and $F_1 \times BP$. The total resultant moment of the two forces is

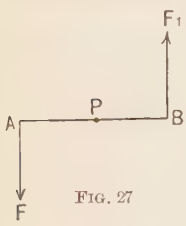


FIG. 27

$$(F \times AP)+(F_1 \times BP), \text{ or (since } F=F_1)\, F \times AB.$$

Therefore the moment of any couple is the product of either force by the perpendicular distance between them.

QUESTIONS AND PROBLEMS

1. Two parallel forces of 8 lb. and 12 lb. act in the same direction respectively at points A and B, 12 in. apart. Find the magnitude and position of their resultant.

2. The smaller of two parallel forces having the same direction is 5 in. from the resultant. What is the distance of the resultant from the other force?

3. Two men carry a weight of 100 lb. suspended from a pole 15 ft. long; each man is 18 in. from his end of the pole. Where must the weight be attached in order that one man may bear $\frac{3}{4}$ of it?

4. Take from the last problem the number of pounds supported by each man and the respective distances of each from the weight, and make an inverse proportion which shows the relation that must exist between these quantities.

5. How can a force of 4 lb. be made to produce equilibrium with a force of 12 lb.?

6. Draw a line 2 in. long. Represent on a scale of $\frac{1}{4}$ in. = 1 lb. a force of 8 lb. applied at a point A $\frac{1}{4}$ in. from one end of the line and at right angles to it. Take for the axis of rotation a point B $\frac{3}{4}$ in. from the same end of the line. From point C, $\frac{1}{4}$ in. from the other end of the line, draw a line which will represent a force that will produce equilibrium with the first force, and therefore prevent rotation.

7. Repeat the work of the last problem, except that the force applied at A shall act obliquely on the line.

8. Can a single force produce equilibrium with a couple?

9. (*a*) A plank weighing 40 lb. is placed across a log so as to be balanced. A boy weighing 60 lb. sits on one end of the plank. Where shall another boy weighing 90 lb. sit that he may balance the first? (*b*) What pressure will be exerted upon the log?

10. Two horses harnessed abreast are plowing. How can you arrange that one horse shall pull only two thirds as much as the other?

11. The maximum muscular force which a certain man can exert is 200 lb. With what leverages can he raise a stone weighing 1 T.?

12. How can pressure be multiplied indefinitely?

SECTION V

CENTER OF MASS, OR CENTROID

55. Center of mass defined. Let Fig. 28 represent any body of matter, — a stone, for instance. Every particle of the body is acted upon by the force of gravitation. The weights of all the particles form a set of parallel forces acting vertically downward, the resultant of which equals their sum and has the same direction as its components. The resultant passes through a definite point in whatever position the body may be, and this point is called its *center of mass*, or *centroid*. *The centroid of a body is*

therefore the point of application of the resultant of all these forces; and for practical purposes in the case of parallel forces *the whole mass of the body may be supposed to be concentrated at this point.*[1] By the *place* or *location* of a body mathematicians mean that point where its center of mass is situated.

FIG. 28

It is evident that *in whatever position a body may be placed, the resultant of the weights of all its particles passes through its centroid.* Hence, *to support a body* (i.e. to prevent its falling), *the supporting force, or equilibrant, — the resultant of several supporting forces, — must act in a line through the centroid of the body and vertically upward.* A *vertical line* is any straight line passing through the centroid of the earth. In this line up and down are directions from and toward the earth's centroid.

Let G in the figure represent the centroid of the stone. For practical purposes, then, we may consider that the force of gravitation acts only at this point, and in the direction GF. If the stone fall freely, this point cannot deviate from a vertical path, however much other points of the body may rotate about this point during its fall. Inasmuch, then, as the centroid of a falling body always describes a definite path, a line (GF) that represents this path, or the path in which a body supported tends to move, is called the *line of direction.* It may be defined as a straight line in which lie the centroid of the body and the centroid of the earth.

To support any body, then, it is only necessary to provide a support for its centroid. The supporting force must be applied somewhere in the line of direction. The difficulty of poising a book, or any other object, on the end of a finger consists in keeping the support under its centroid, i.e. in the line of direction.

[1] The expression "center of mass" does not necessarily signify that point occupying a central position among the particles of a body, but a point where, for convenience in many dynamical problems, we may consider all the mass (or inertia) to be concentrated.

Whether a body having no other support than that applied at its base will stand or fall depends upon whether or not its line of direction falls within its base. The base of a body is not necessarily limited to that part of the under surface of a body that touches its support. For example, place a string around the four legs of a table, close to the floor: the rectangular figure bounded by the string is the base of the table.

The centroid of any symmetrical body of homogeneous material, i.e. of uniform density, coincides with its geometrical center, as, for example, the middle point of a material straight line; that point on a straight line joining the vertex to the middle of the base of a triangle situated at a distance from the vertex equal to two thirds the length of the line; the geometrical center of any polygon, sphere, or circular cylinder.

56. Method of finding the centroid of a body. Imagine a string to be attached to a potato by means of a tack, as in Fig. 29, and to be suspended from the hand. When the potato is at rest there is an equilibrium of forces, and the centroid must be somewhere in the line of direction *an*; hence, if a knitting needle is thrust vertically through the potato from *a*, so as to represent a continuation of the vertical line *oa*, the centroid must lie somewhere in the path *an* made by the needle. Suspend the potato from some other point, as *b*, and a needle thrust vertically through the potato from *b* will also pass through the centroid. Since the centroid lies in both the lines *an* and *bs*, it must be at *c*, their point of intersection. It will be found that, from whatever point the potato is supported, the point *c* will always be vertically under the point of support. On the same principle the centroid of any body is found. But the centroid of a body may not be coincident with any particle of the body; for example, the centroid of a ring, a hollow sphere, etc.

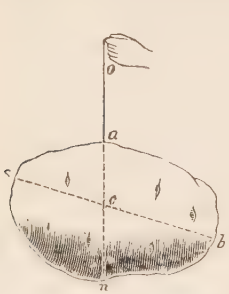


FIG. 29

57. Three states of equilibrium. That a body acted on solely by the force of gravitation may be in equilibrium, it is necessary and sufficient that a vertical line through its centroid shall pass through the point or surface by which it is supported. The weight of a body is a force tending downward; hence *a body tends to assume a position such that its centroid will be as low as possible.*

A body is said to be in *stable* equilibrium if its position is such that any motion except one of translation would raise its centroid, since in that event it would tend to return to its original position. On the other hand, a body is said to be in *unstable* equilibrium when a slight disturbance would lower its centroid, since it would not then tend to return to its original position.

A body is said to be in *neutral* or *indifferent* equilibrium when it rests equally well in any position in which it may be placed. A sphere of uniform density, resting on a horizontal plane, is in neutral equilibrium, because its centroid is neither raised nor lowered by a change of base. Likewise, when the support is applied at the centroid, as when a wheel is supported by an axle, the body is in neutral equilibrium.

It is evident that *if the centroid be below the support*, as in the case of a ring hung on a peg, *the equilibrium must be stable*; but a body may be in stable equilibrium, though its centroid be above the point of support.

It is difficult to balance a lead pencil on the end of a finger; but by attaching two knives to it, as in Fig. 30, the centroid may be brought below the support, and it may then be rocked to and fro without falling.

Fig. 30

58. Degree of stability. The ease or difficulty with which bodies supported at their bases are overturned varies with the height to which their centroids must be raised to overturn them. The letter c (Fig. 31) marks the position of the centroid of each of the four bodies, A, B, C, and D. If any one of these bodies be overturned, its centroid must have passed through the arc ci and have been raised through the height ai. By comparing A with B,

and supposing them to be of equal weight, we learn that *of two bodies of equal weight and height of centroid, the centroid of that body which has the larger base must be raised higher, and that body is, therefore, overturned with greater difficulty.* A comparison of *A* and *C*, supposing them to be of equal weight, shows that *when two bodies have equal bases and weights, the body having its centroid higher is more easily overturned.* *D* and *C* have equal masses, bases, and heights; but *D* is made heavy at the bottom, and this *lowers its centroid and gives it greater stability.*

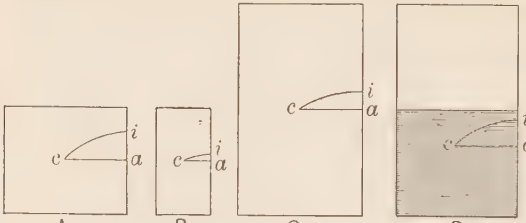


FIG. 31

QUESTIONS AND PROBLEMS

1. Where is the centroid of a box?
2. Why is a pyramid a very stable structure?
3. What is the object of ballast in a vessel?
4. State several ways of giving stability to an inkstand.
5. (*a*) In what position would you place a cone on a horizontal plane, that it may be in stable equilibrium? (*b*) that it may be in neutral equilibrium? (*c*) that it may be in unstable equilibrium?
6. In loading a wagon, where should the heavy luggage be placed? Why?
7. Why are bipeds slower in learning to walk than quadrupeds?
8. How will a man rising in a boat affect its stability?
9. Which is more liable to be overturned, a load of hay or a load of stone of equal weight?
10. Draw a triangle and find its centroid.
11. What attitude does a man assume when carrying a heavy load on his back? Why?

FIG. 32

12. Explain the difference in the behavior of a ball and of a cube, when placed on a plane slightly inclined.

13. What position do bodies floating in air or in water take?

14. (*a*) Explain how the toy horse in Fig. 32 stands upon the platform without falling off. (*b*) Explain how the toy may rock upon its support without falling off.

SECTION VI

COMPOSITION OF FORCES ACTING AT ANGLES WITH ONE ANOTHER

59. When the handle *A* (Fig. 2) is pushed forward there is applied to the pencil a force which may be represented in magnitude and direction by the line *ab*; at the same time the pencil is pulled by a force which may properly be represented by the line *ac*. The pencil, however, moves in the line *ad*, which is a diagonal of a parallelogram constructed on the lines *ab* and *ac*. It is evident that a single force might be applied to the pencil with the same effect that the two forces produce. Obviously, if a single force were to move the pencil in the line *ad*, it must have the direction of this line. It remains to ascertain whether the diagonal line *ad* represents the *magnitude* of the resultant.

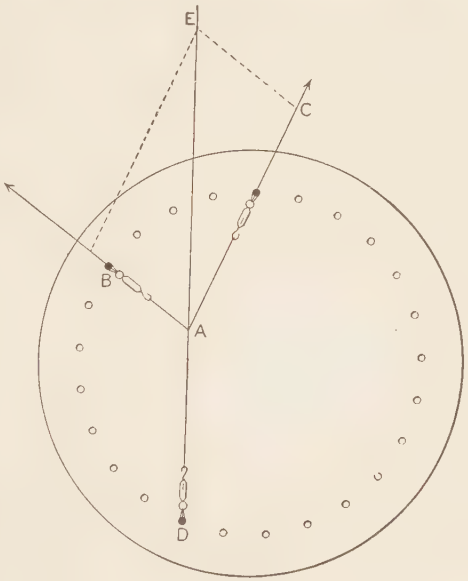


FIG. 33

Evidently if this diagonal does represent the resultant, then the same diagonal with the direction reversed will represent the equilibrant of these forces. We put the matter to an experimental test with other apparatus.

Insert pegs in any three holes of the circle [1] in the top of the table (Fig. 33). Join these by threads attached each to a spring balance, as shown in the figure. Stretch the balances so as to indicate any desired pull in each of the threads. Place under the threads a sheet of white paper. Locate on the paper the common point of application A of the three forces. Draw lines AB, AC, and AD, to represent the directions in which the forces act. Since the point A does not move, it is evident that the three forces are in equilibrium, and that any one of them is the equilibrant of the other two. Select any one for an equilibrant (e.g. AD) and extend it in an opposite direction from A, representing (on some suitable scale) a force AE equal to and opposite to the force AD, as indicated by the dynamometer D. On the same scale lay off distances AB and AC, representing the magnitudes of the forces acting in the directions of these lines. The line AD is, by definition, the resultant of AB and AC. Connect E with C and B. The figure, if the work be done with care, will be found to be a *parallelogram*. The diagonal EA represents the magnitude of the equilibrant of the forces AB and AC, and the same line with the direction reversed (i.e. AE) represents the resultant.

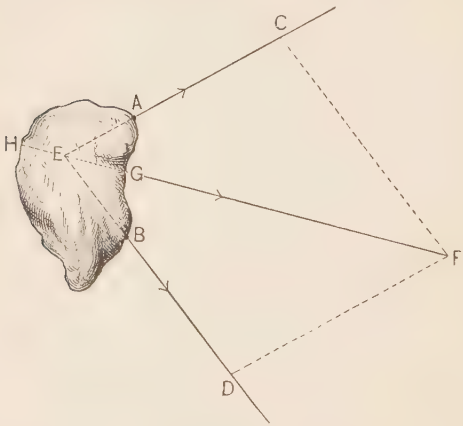


FIG. 34

[1] There is no significance in the fact that the holes are arranged in a circle.

60. Parallelogram of forces. *If two forces applied at a point be represented in magnitude and direction by the adjacent sides of a parallelogram, their resultant will be represented in magnitude and direction by the diagonal which passes through that point.*

This proposition is applicable whether the forces act on a particle or on a rigid body, provided they lie in the same plane. Thus, let two forces applied at points A and B of a stone (Fig. 34) act in the directions AC and BD respectively. The direction of the resultant must pass through E, the point where the lines of direction of the given forces, produced backwards, intersect. If, now, the lines EC and ED be laid off to represent the relative intensities of the forces, the diagonal EF of the parallelogram constructed thereon will represent their resultant, and its point of application may be G or any other point in the line GH.

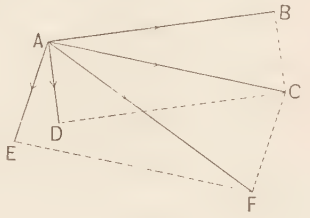


FIG. 35

61. Composition of more than two forces in the same plane. *When more than two components are given, find the resultant of any two of them, then of this resultant and a third, and so on till every component has been used.* Thus, in Fig. 35, AC is the resultant of AB and AD, and AF is the resultant of AC and AE, i.e. of the three forces AB, AD, and AE.

Generally speaking, *a motion may be the result of any number of forces.* When we see a body in motion we cannot determine by its behavior how many or what forces have concurred to produce its motion.

62. Triangle of forces. Since in Fig. 33 $BE = AC$, the three forces which are represented in the parallelogram by the lines AB, AC, and AE are also represented by AB, BE, and AE, three sides of a triangle ABE.

Hence, *if two forces are represented in magnitude and direction by two sides of a triangle, the third side will represent their resultant. Similarly, if any number of forces applied at a point*

are represented by all the sides of a polygon but one, the remaining side will represent their resultant. Thus the forces AB, AD, and AE (Fig. 35) are represented respectively by the sides $A'B'$, $B'C'$ ($= AD$), and $C'F'$ ($= AE$) of a polygon $A'B'C'F'$ (Fig. 36) that is completed by the side $A'F'$ ($= AF$), which represents the resultant of the three forces.

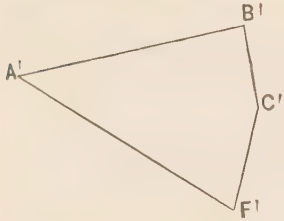


FIG. 36

63. Parallelopiped of forces. *If three forces not in the same plane are applied at a point, they will form three edges of a parallelopiped; and that diagonal of this solid which is concurrent with these edges will represent the resultant of these forces.* It will be readily seen that the resultant of the forces AB, AD, and AC (Fig. 37) is represented by the diagonal AE.

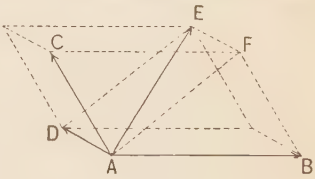


FIG. 37

64. Resolution of forces. Assume that a ball has an acceleration in a certain direction AC (Fig. 38), and that one of the forces that produces this acceleration is represented in intensity and direction by the line AB. What must be the intensity and direction of the other force? Since AC is the resultant of two forces acting at an angle to each other, it is the diagonal of a parallelogram of which AB is one of the sides. From C draw CD parallel and equal to BA, and complete the parallelogram by connecting the points B and C and A and D. Then, according to the principle of composition of forces, AD represents the intensity and direction of the force which, combined with the force AB, would produce the acceleration AC. The component AB being given, no other single force than AD will satisfy the question.

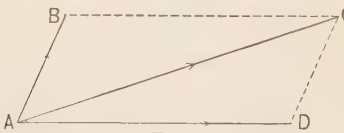


FIG. 38

Had the question been, What forces can produce the acceleration AC? an infinite number of answers might be given. In a like

manner, if the question were, What numbers added together will produce 50? the answer might be $20 + 30$, $40 + 10$, $20 + 20 + 10$, and so on, *ad infinitum;* but if the question were, What number added to 30 will produce 50? only one answer could be given.

It is often necessary to resolve a force in order to ascertain the effective force in a certain direction. Thus, when boat sails are exposed obliquely to the wind, the pressure effectual in moving the boat is only a component of the whole force of the wind. The line *af* (Fig. 39) represents the force of the wind acting on the sail *cd* at the point *a*. Resolving this force, we obtain the components *2*, normal to the sail, and *1*, parallel to it (a useless component called a tail wind). The boat does not move in the direction of the pressure on its sail because it is more easily moved lengthwise than breadthwise. Hence the normal pressure may be resolved into two components, one *4* along the direction of least resistance, i.e. the direction of easy motion, the other *3* at right angles to it. The latter component does tend to cause a slow broadside motion called leeway, but this may be partly counteracted by a deep keel or a centerboard, so that the boat will sail approximately along the line *ab*.

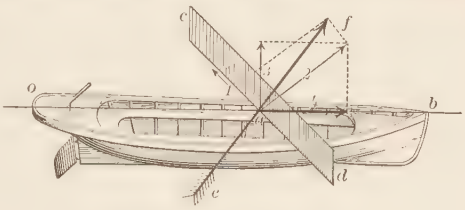


FIG. 39

QUESTIONS AND PROBLEMS

1. Draw upon paper pairs of lines making about the same angles with each other as AB and AC in the four diagrams (Fig. 40), and having about the same directions. Assign arbitrarily numerical values to each component, drawing to scale, and find the direction and the numerical value of the resultant of each pair of components.

2. (*a*) Find the intensity of the resultant of two forces acting at an angle of 45°. (*b*) Find the intensity of their resultant when they act at an angle of 150°. (The pupil will require either a pair of dividers or a protractor. He will do well to learn to use both in measuring angles.)

3. (*a*) A heavy rock rests upon a smooth plane; two men, A and B, pull the rock by means of ropes attached to it, A with a force of 100 lb., B with a force of 150 lb. If A pull toward the north and B toward the south, what will be the resultant? (*b*) If A pull toward the east and B toward the south, what will be the resultant? (*c*) In the last case, if the easterly acceleration at a certain instant is 10 ft. per minute, what is the southerly acceleration at the same instant? (*d*) In what direction should they pull the rock to give

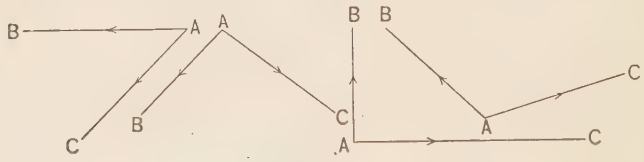


FIG. 40

it the maximum acceleration? (*e*) If A pull it 25° S. of E., in what direction and with what force may B pull it that the resultant may be directly east? (*f*) Give a different answer to the last question.

4. On a scale of 1 cm. = 1 kg., represent a force of 5 kg. acting northward at a point A.

5. On a scale of 1 cm. = 1 kg., represent forces of 4 kg., 6 kg., and 8 kg., acting simultaneously on point A in directions N., N.E., and S.E. respectively. Find their equilibrant.

6. A ship is sailing N.N.E. at the rate of 12 knots per hour. Find its northerly and easterly velocities.

7. Find both by construction (of parallelogram) and by calculation the intensity of two equal forces, acting at right angles to each other, that will support a weight of 15 lb.

8. A sailor climbs a mast at a uniform rate of 5 ft. a minute while the vessel moves forward at the rate of 15 ft. a minute. What is his actual velocity?

9. On a scale of $\frac{1}{4}$ in. = 10 lb., represent a force of 80 lb. Resolve this force into two forces one of which shall act at an angle of 30° with the given force. Determine the numerical intensities of each of the components.

10. Show by construction that a northeast wind may be made up of a north and an east wind, each $\frac{1}{\sqrt{2}}$ of the actual velocity of the wind.

11. If two lines AB, CA represent two forces acting at a point A, the one toward and the other from it, show how to find the resultant.

12. Find the resultant of two equal forces of P lb., the angle between them being 120°.

13. Two rafters, making an angle of 60°, support a chandelier weighing 90 lb. What is the pressure along each rafter?

SECTION VII

NEWTON'S THREE LAWS OF MOTION

65. Laws of motion. The science of dynamics rests on certain fundamental principles termed the *laws of motion*, first clearly stated by Newton in the *Principia* more than two centuries ago, and *verified by universal experience*. The laws as given in this text-book are as originally enunciated by Newton, with very slight verbal modifications in conformity to modern terminology.

First Law. *A body at rest remains at rest, and a body in motion continues to move with constant speed in a straight line, unless acted upon by some external unbalanced force.*

This law may be paraphrased as follows: A body under the action of no force, or of balanced forces, is either at rest or in uniform motion; if it be at rest it will remain at rest, and if it be in motion its motion will be in none other than a straight line, and its speed will never change.

Motion unobstructed is perpetual. Is perpetual motion possible? has been often asked. The answer is simple, — yes, more than possible; *it is even necessary, if no force interfere to prevent*. An example of this may be seen in the motions of the planets. On our earth we have no instances, for resistances such as friction, resistance of the air, etc., are continually opposed to all movements of terrestrial bodies. On this account we find that force is required to perpetuate the motion of all bodies with which we deal, and we fall readily into the fallacy that force is necessary to *maintain* motion, which the first law distinctly contradicts.

The clause "unless acted upon by an external force" virtually states that "all matter is inert," i.e. that bodies of matter are utterly incapable of putting themselves in motion or stopping themselves; the inability is called *inertia*. Inertia may be defined as *that property of matter in virtue of which external force is required to produce a change in momentum*. It is the sole unalterable property of matter.

The terms *mass* and *inertia* are often used interchangeably to denote a quantity proportional to the unbalanced force required to produce a given change in the velocity of a body in a given time. It is known that all bodies unobstructed by the air fall with the same speed irrespective of their masses or inertia. But to produce equal acceleration in equal times requires forces proportional to the masses; it follows, then, that at *the same locality weight and mass (or inertia) are proportional.* Hence we compare masses by comparing their weights.

The somewhat vague yet common expressions "to overcome inertia" and "to destroy inertia" mean to produce a certain change of momentum, and may signify either an increase or a decrease of it.

Second Law. *Change of momentum is in the direction in which the unbalanced force acts, and is proportional to its intensity and to the time during which it acts.*

It will be seen that this law (except as regards direction) is contained in the formula $mv = ft$ (§ 34), which has already been developed. This formula virtually asserts that where there is no force there is no change of momentum, i.e. if $f = 0$, $ft = 0$. Hence the first law of motion is a deduction from the second.

This law declares, by implication, (1) *that an unbalanced force in a given time always produces exactly the same change of momentum regardless of the mass of the body:* that *an unbalanced force never fails to produce a change of momentum. Hence any force, however small, can move any body of however great mass:* for example, a child can move a body having a mass equal to that of the earth, provided only that this body be free to move. Moreover, the *quantity* of momentum that the child can generate in this immense body in a given time is precisely the same as that which he could generate by the exertion of the same force for the same length of time on a body having any other mass, small or great. Momentum is the product of mass into velocity; so, of course, as the mass is large, the velocity acquired in a given time will be correspondingly small. The instant the child begins

to act, the immense body begins to move. Its velocity, infinitesimally small at the beginning, would increase at an almost infinitesimally slow rate, so that at the same rate it might be years before its motion would become perceptible.

It is easy to see how persons may get the impression that very large masses are immovable except by very great forces. The erroneous idea is acquired that bodies of matter are capable of resisting the tendency of forces to cause motion, and that the greater the mass the greater the resistance. The fact is, that

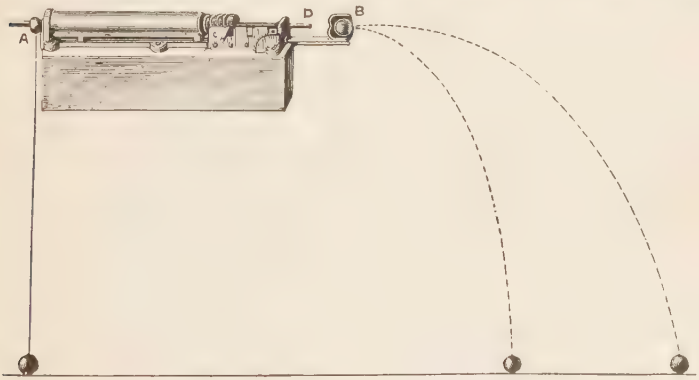


FIG. 41

no body of whatever mass can resist motion; in other words, *a body free to move cannot remain at rest under the slightest unbalanced force.* But as *time* is always required to generate change of momentum, there arises thence a deceptive appearance of resistance or holding back.

This law declares, by implication, (2) that *a force acting on a body in motion produces just the same effect as if it were acting on the same body at rest*, for no reference is made in the law to the *state* of the body acted upon.

Fig. 41 represents a simple device for demonstrating the independence of two forces and of the motions that each produces. The construction and its action are obvious.

Draw back the rod *D* towards the left, and place the detent pin *c* in one of the slots. Place one of the brass balls on the projecting rod, and in contact with the end of the instrument, as at *A*. Place the other ball in the short tube *B*. Raise the apparatus to as great an elevation as practicable, and place it in a perfectly horizontal position. Release the detent pin, and the rod, propelled by the elastic force of the spring within, will strike the ball *B*, projecting it in a horizontal direction. At the same instant that *B* leaves the tube and is free to fall, the ball *A* is released from the rod and begins to fall. Both balls are seen and heard to strike the floor at the same instant, which shows that the horizontal motion which one of the balls has does not affect the time of its fall, i.e. does not modify the effect of the force of gravity.

The law implies (3) that *if two or more forces act on a body, each produces its own change of momentum in its own direction independently of the others.* It declares, what we have previously learned, that the operation of compounding forces is just the same as that of compounding motions, which the several forces tend to produce in the same time; hence the apparatus (Fig. 2) illustrates either the composition of motions or the composition of the forces by which the motions are produced.

Third Law. *To every action there is an equal and opposite reaction.*

Previous to the announcement of the laws of motion our studies were such as to prepare us both to understand and accept them. We learned that there are always two bodies or two parts of the same body oppositely affected by every force. When the double aspect of a force, i.e. its mutual action between two portions of matter, is considered, it is customary to speak of the force as a *stress*. Illustrations of stress are tension in a stretched rubber band and pressure exerted between two bodies in contact when compressed. All force is of the nature of a stress, and the third law of motion virtually declares that *every action between two bodies is a stress.* When the effect of the action upon only one

of the two bodies is under consideration, the action is commonly spoken of as a force.

It remains to show that action and reaction are *equal*. That they are equal is deducible from the first law, for if they were unequal, then, when there was an action between two parts of the same body, there would not be equilibrium, that is, there would be an unbalanced force, which would cause the body to move with accelerated velocity,—a thing which is explicitly contradicted by the first law.

If action and reaction were not equal, there might be a possibility that a person might raise himself by pulling on the soles of his feet or the hair of his head; that a vessel might be propelled in a calm by blowing against its sail with a powerful bellows located on the deck; that a person sitting in a wagon might give himself a ride by pressing his feet against the dasher; that a person might advance, i.e. move his centroid without having the earth beneath him; that a bird might fly without the external air to act on.

In case the two bodies are free from the action of resisting forces the law implies that the momenta generated by the action and reaction are equal.

The application of this law is not always obvious. Thus, an apple falls to the ground in consequence of an action between the apple and the earth. The motion of the earth toward the apple is imperceptible, but this is because the mass of the earth is enormously greater than that of the apple, and its velocity, for an equal momentum, is proportionately less.

QUESTIONS AND PROBLEMS

1. (*a*) Why does not a given force, acting the same length of time, give a loaded car as great a velocity as an empty car? (*b*) After equal forces have acted for the same length of time upon both cars, and have given them unequal velocities, which will be the more difficult to stop?

2. (*a*) The planets move unceasingly. Is this evidence that there are forces pushing or pulling them along? (*b*) None of their motions are in straight lines. Are they acted upon by external forces?

3. A certain body is in motion. Supposing that all hindrances to motion and all external forces be withdrawn from it, how long will it move? Why? In what direction will it move? Why? What kind of motion will it have? Why?

4. Explain how rotating lawn sprinklers are kept in motion.

5. When you leap from the earth, which receives the greater momentum, your body or the earth?

6. When you kick a door rock why does snow or mud on your shoes fly off?

7. If a man in a boat move it by pulling on a rope at one end, the other end being fastened to a post, how is the boat put in motion? Would it move either faster or slower if the other end were fastened to another boat free to move, the man exerting the same force?

8. A 20-g. bullet leaves a gun of mass 4 kg. with a speed of 300 m. per second. What is the maximum speed of the gun's recoil?

9. Suspend two balls of soft putty, and hence inelastic, of equal mass, A and B (Fig. 42). Draw A to one side and let it fall so as to strike B. Both balls will then move on together. What momentum will they have compared with A's momentum just before it strikes B?

10. What will be the momentum of each ball after A strikes B, compared with A's momentum when it strikes B?

11. How will the velocity of A and B compare with A's velocity when it strikes B?

12. Raise A and B equal distances in opposite directions, and let them fall so as to collide. Both balls will instantly come to rest after collision. Show that this result is consistent with the third law of motion.

13. For the inelastic putty balls substitute ivory billiard balls, which are highly elastic. Let A strike B. Then B goes on with A's original velocity, while A is brought to rest. Show that this result is consistent with the third law of motion.

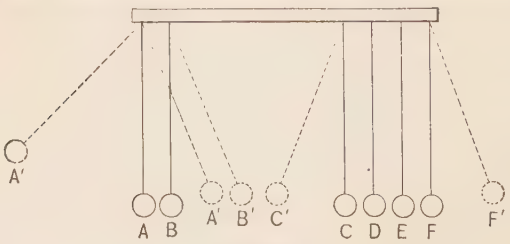


FIG. 42

14. Suspend very near together four ivory balls, C, D, E, and F (Fig. 42). Let C strike D. D receives all of C's momentum, instantly communicates it to E, and E to F. F, having nothing to which to communicate the momentum, moves with C's original velocity. Trace the actions and reactions throughout.

15. What would happen if the four balls were inelastic?

16. A shell at rest bursts into two parts, the smaller being one third of the whole. What is the ratio of the initial velocities of the parts?

17. (*a*) Can any body, animate or inanimate, by any action confined to itself, i.e. between component parts of itself, put itself in motion or stop itself? (*b*) How can a body put itself in motion?

18. Why do not heavy bodies fall faster in the air than light ones?

19. A stone falls from the top of a railway carriage which is moving at the rate of half a mile a minute. Disregarding the resistance of the air, find what horizontal distance and what vertical distance the stone will have passed through in one tenth of a second.

SECTION VIII

APPLICATIONS OF THE LAWS OF MOTION

66. Curvilinear motion. Motion is *curvilinear* when its direction changes at every point. But according to the first law of motion every moving body proceeds in a straight line unless compelled to depart from it by some external force. Hence curvilinear motion can be produced only by an external force acting continuously upon the body at an angle to the straight path in which the body tends to move, so as constantly to change its direction. In case the body moves in a circle, this force acts at right angles to the path of the body, or towards the center of motion; hence this deflecting force has received the name of *central force.*

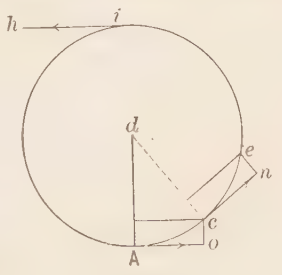


FIG. 43

Thus, suppose a ball at *A* (Fig. 43), suspended by a string from a point *d*, to be struck by a bat in a manner that would cause it to move in the direction *Ao*. At the same time it is restrained from taking that path by the tension of the string, which operates like a force drawing it toward *d*. It therefore takes an intermediate course. At *e* its motion is in the direction *en*, in which path it would move but for the string, in accordance with the first law of motion. Here, again, it is compelled to take an intermediate path. Thus, at every point, the tendency of the moving body is to preserve the direction it has at that point, and

consequently to move in a straight line. The only reason it does not so move is that it is at every point forced from its natural path by the pull of the string. But if, when the ball reaches the point *i*, the string be cut, the ball, having no force operating to change its motion, continues in the direction in which it is moving at that point, i.e. in the direction *ih*, which is tangent to its former circular path.

67. Magnitude of central force. Cause a ball to revolve around your hand by means of a string attached to it and held in the hand. Observe closely every phase of the operation. First you make a movement as if to project the ball in a straight line. Immediately you begin to pull on the string to prevent its going in a straight line. Under the continuous influence of these two forces the ball acquires great speed in a short time. You may now cease to exert any projecting force, and simply keep the hand still; but as the ball has acquired a motion, and all motion tends to be in a straight line, you are still obliged to exert a pulling force to deflect the ball from its path. Observe that, as the velocity of the ball is retarded by the resistance of the air, the pulling or deflecting force which you are obliged to employ rapidly diminishes.

To satisfy yourself that the ball tends to move in a straight line, let go the string or cut it, and the ball immediately moves off in a straight line, or simply perseveres in the direction it had at the instant the string was cut. Observe that the ball appears while rotating to be pulling your hand; but you know that all the force concerned originates in yourself, and that this apparent pull on the part of the ball is only the effect of the *reaction* of the force which you exert on the ball. This reaction is erroneously called *centrifugal force*. There is no centrifugal force. The only force exerted is the central force, which is of such a magnitude as to change the direction of the momentum just fast enough to keep the body moving in a circle.

Every revolving body affords an example of central force and centrifugal tendency. Hence we say that every revolving body

tends to fly away from the center (not radially, however, but tangentially), and a central force is required to keep the body in its circular path.

When you swing the ball about your hand you discover that the force of the pull increases with the velocity, and more rapidly than the velocity. Careful observations have determined that for bodies revolving in circular orbits *the central force varies as the mass of the body, as the square of its velocity, and inversely as its distance from the center.*

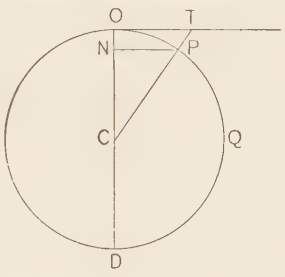


FIG. 44

Let a point move uniformly in the circular path OPQ (Fig. 44), traversing the distance OP in time t; then $OP = vt$ (1). If P be very near O, the deflection TP from a straight line, due to the central force, is approximately equal to ON. If a be the acceleration towards the center due to this force, $ON = \frac{1}{2}at^2$ (2). But by geometry $\overline{OP}^2 = ON \cdot OD$. Comparing (1) and (2), we get $v^2t^2 = \frac{1}{2}at^2 \cdot 2r$, or $a = \frac{v^2}{r}$. Since $f = ma$, we get $f = m\frac{v^2}{r}$.

The farther a point is from the axis of motion of a rigid body, the farther it has to move during a rotation; consequently the greater its velocity. Hence bodies situated at the earth's equator have the greatest velocity, due to the earth's rotation, and consequently the greatest tendency to fly off from its surface. The effect of this is to neutralize, in some measure, the force of gravity. It is calculated that a body weighs about $\frac{1}{289}$ less at the equator than at either pole, in consequence of the greater centrifugal tendency at the former place. But 289 is the square of 17; hence, if the earth's velocity were increased seventeen-fold, objects at the equator would weigh nothing, i.e. the centrifugal tendency would be equal to their weight.

The attraction between the sun and the earth causes these bodies to move in curvilinear paths, performing what are called

annual revolutions. Were it not for this mutual attraction (and the attraction of the other celestial bodies), the motion of both these bodies would be eternally in straight lines, but in consequence of their mutual attraction both rotate about a point C (Fig. 45), which is the centroid of the two bodies considered as one (as if connected by a rigid rod). Strictly speaking, the earth does not revolve around the sun any more than the sun revolves around the earth, but both rotate about their common centroid, which is really within the mass of the sun, though the figure is drawn otherwise for clearness. If both bodies had equal masses, the centroid and the center of motion would be halfway between the two bodies; but as the mass of the earth is less than that of the sun, so its velocity and the distance traversed are proportionally greater.

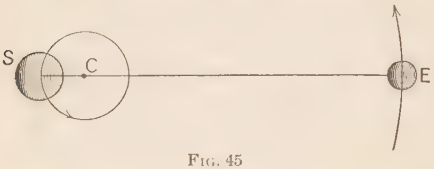


FIG. 45

68. Apply the frame T (Fig. 46) to any rotating apparatus R (Fig. 47) so that it may be rotated about its axis d. The rod c passes through the balls a and b loosely so that the latter are free to slide along the rod. The two balls are connected by a string so that they are compelled to rotate as one body or one *system* of bodies. The mass of a is twice that of b. Rotate the system and show that there is equilibrium in the system only when the center of b is twice as far from the axis of rotation as the center of a. How does this verify the above law? While there is rotation, is there tension in the string connecting the balls? What is the cause of an action between the balls? Ball a pulls ball b. What is the effect of this pull on b? What is the effect of the reaction on a? Is there a similar action between the sun and earth in their annual revolutions? By what

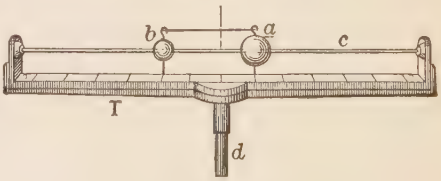


FIG. 46

name is the action known? If the sun or the earth were instantaneously annihilated, state what would happen to the other body

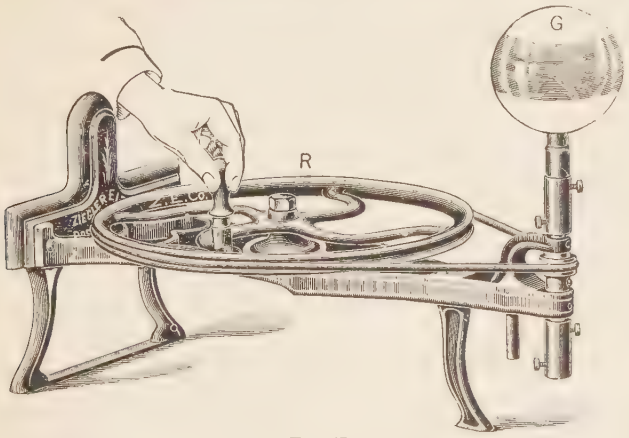


FIG. 47

if it were left entirely free, i.e. if its motion were not affected by other bodies in the universe.

Suspend a skein of thread *a* (Fig. 48) by a string and cause it to rotate by the device shown in Fig. 47; it assumes the shape of the oblate spheroid *a'*. Mount a glass globe *G* (Fig. 47) about one tenth full of colored water, and rotate. The liquid gradually leaves the bottom, rises, and forms an equatorial ring within the glass. This illustrates the probable method by which the earth, on the supposition that it was once in a fluid state, assumed its present spheroidal state. Pass a string through the longest diameter of an onion *c*, and cause it to rotate; the onion gradually changes its position so as to rotate on its shortest axis. The chain *b* will, when rotated as indicated, rise and take a position in a horizontal plane in the form of a ring, rotating about the center of the circle formed.

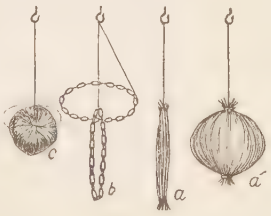


FIG. 48

It can be demonstrated mathematically, as well as experimentally, that *a freely rotating body is in stable equilibrium only when rotating about its shortest possible axis;* hence the tendency of a rotating body to take the several positions above noted.

QUESTIONS AND PROBLEMS

1. (*a*) What is the cause of the stretching force exerted on the rubber cord when you swing a return ball about your hand? (*b*) Suppose that you double the velocity of the ball. How many times do you increase this stretching force?

2. In what way can the tension in the string (Fig. 46) be so much increased as to break it?

3. Why do wheels and grindstones, when rapidly rotating, tend to break and the pieces to fly off?

4. On what does the magnitude of the pull between a rotating body and its center of motion depend?

5. (*a*) Explain the danger of a carriage being overturned in turning a corner. (*b*) How many fold is the tendency to overturn increased by doubling the velocity of the carriage?

6. Account for the *curvilinear* orbits of the planets.

7. How are their motions in their orbits and around their axes *maintained?*

8. In what way should the rails be laid so as to neutralize the centrifugal tendency of a railroad train going around a curve?

9. State and explain the posture of a bicycle rider in turning a curve.

10. In what way is the weight of terrestrial bodies nullified in some degree by the earth's motion?

11. A circus rider going around a ring inclines inward so that the line of direction of his body falls without his base. How is he supported?

SECTION IX

THE PENDULUM

69. Definition. Any body whatever suspended from a point and capable of swinging under the action of its weight is a *pendulum.* A *simple pendulum* would be a *sizeless mass* at the end of a *weightless,* inextensible cord, the other end of which is fixed. The nearest practical approximation to this would be a small

platinum sphere suspended by the finest silk fiber strong enough to support it. In all practical pendulums known as *compound pendulums* the mass is distributed in a more or less complicated way.

For the following suggestive experiments lead bullets at the ends of fine cords, as in Fig. 49, serve very well as simple pendulums. *B* and *C* are of the same length. Draw each to one side but to different heights, so that *B* may swing through a short arc, and let both drop at the same instant. *C* moves much faster than *B*, and completes a longer journey at each swing, but both complete their swing, or vibration, in about the same time.

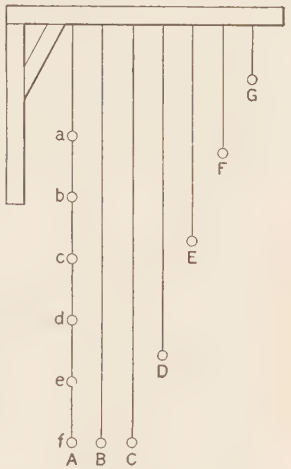


FIG. 49

Hence (1) *the time occupied by the vibration of a pendulum is independent of the length of the arc.* Of only very small arcs may this law be regarded as practically true. The pendulum requires a somewhat longer time for a long arc of vibration than for a short one, but the difference becomes perceptible only when the difference between the arcs is great, and then only after many vibrations. This property of oscillating in equal times whatever the amplitude of the swing is called *isochronism*, and anything thus vibrating is said to vibrate isochronously.

Set all the balls swinging. Only *B* and *C* swing together; the shorter the pendulum the faster it swings. Make *B* 1 meter long, and *F* $\frac{1}{4}$ meter long. Watch in hand, count the vibrations made by *B*. It completes just about 60 small vibrations in a minute; in other words, it "beats seconds." A pendulum, therefore, to beat seconds must be 1 meter long (more accurately, in the latitude of Boston at sea level 0.9935 meter, or 39.117 inches). Count the vibrations of *F*; it makes 120 vibrations in the same time that *B* makes 60 vibrations. Make *G* one ninth the length of *B*; the

former makes 3 vibrations while the latter makes 1, and consequently the time of vibration of the former is one third that of the latter.

Hence (2) *the time of one vibration of a pendulum varies as the square root of its length.*

The isochronism of the pendulum is utilized in the measurement of time, i.e. in subdividing the solar day into hours, minutes, and seconds. The office of the pendulum in clocks is to regulate the rate of motion of the works. The balance wheel replaces the pendulum in watches and some clocks.

70. Determination of the acceleration of gravity. The time of vibration is less at a place where the force of gravitation is greater because the accelerating force for the same mass is greater, and hence the pendulum will move faster.

Hence it is apparent that by determining the time of vibration of a pendulum of the same length at different distances from the center of mass of the earth, e.g. at the top and bottom of a mountain, or at sea level at different latitudes, the relative value of g at these places, i.e. the acceleration produced by gravitation, may be ascertained. We have already learned that the acceleration at the same locality is the same for all bodies regardless of their mass.

By experiments too difficult for ordinary school work it has been ascertained that (3) *the time of vibration of a pendulum varies inversely as the square root of the force of gravitation* (upon which the value of g depends).

To sum up the above three laws of the pendulum, we have the formula

$$T = 2\pi\sqrt{\frac{l}{g}}, \text{ whence } g = \frac{4\pi^2 l}{T^2},$$

in which l = length of pendulum, and T = time of one complete vibration to and fro in seconds.

This formula may be found derived in many more advanced works on physics.

At the poles of the earth the length of a seconds pendulum is 99.62 centimeters, and $g = 983.2$ centimeters per second. At the equator $l = 99.10$ centimeters; $g = 978.1$ centimeters per second.

71. The compound pendulum. A (Fig. 49) represents six lead balls about 15 centimeters apart strung on a stiff wire, and the combination comprises a compound pendulum. If A and B be set vibrating together, A will swing quicker than B, though their lengths are equal.

A reason for this is at once apparent. Since all of the six simple pendulums of which A is composed, except f, are shorter than B, each of the shorter five tends to vibrate quicker than f, and would do so but for the stiff wire which detains it. The result, therefore, of this combined effort is a period of swing considerably shorter than that of the simple pendulum B. From a continuance of the same line of reasoning with reference to A, it is apparent that there must be some point between a and f whose motion is neither quickened nor checked by the combined action of the balls above and below it, and where, if a single ball were placed, it would make the same number of vibrations in a given time that the compound pendulum does. Shorten pendulum B and find the required point. This point is called the *center of oscillation.*

Every compound pendulum is equivalent to a simple pendulum whose length is equal to the distance between the center of oscillation and the point of suspension of the compound pendulum. Inasmuch as the distance between the point of suspension and the center of oscillation determines the rate of vibration, whenever the expression "length of pendulum" is used it must be understood to mean this distance. This distance is often called the *radius of oscillation* of the compound pendulum.

The simplest kind of a compound pendulum is a uniform straight rod hung at one end, as AB (Fig. 50). Suppose it to be 1 meter long, and let one of the simple pendulums of Fig. 49 be adjusted as to length so as to have the same period of vibration as the rod. If AC be this length, then C will be the center of oscillation, and in this case will be located at two thirds of AB from A.

Now attach a pound mass, say, at the lower end of AB. Its period will be increased. If this mass is moved up the rod, the period will be lessened, and the theory and practice of regulating an ordinary pendulum clock is obvious.

If now a hole be bored through the rod at its center of oscillation C, and if, without the extra mass, the rod be reversed and suspended at C, the pendulum is now apparently shortened, and we naturally expect that its vibrations will be quicker than when suspended from A. But the part BC now vibrates in opposition to the part CA, rising as it sinks and sinking as it rises. This tends to check the rapidity of the vibrations of CA, and it is found that the pendulum vibrates in the same time when suspended from C as when suspended from A. *The point of suspension and the center of oscillation are interchangeable without changing the period.*

FIG. 50

72. Center of percussion. If the rod AB be held between the fingers at A and allowed to hang vertically, it will be found on striking it with another rod at various places both above and below C, that when struck anywhere *except at* C a disagreeable jar will be felt at A; when struck *at* C, however, no jar whatever is felt at A. In this case C is called the *center of percussion*, and, as is seen, it is coincident with the center of oscillation. It is the point where a blow, given or received, is most effective and produces the least stress upon the support or axis of motion. The baseball player soon learns at what point on his bat he can deal the most effective blow to the ball and at the same time feel the least tingle in his hands.

73. Demonstration of the earth's rotation on its axis. Foucault's experiment. If a ball and string pendulum be set in vibration in a certain plane, then, by virtue of the first law of motion, it will continue to vibrate in the same plane even if the string is twirled so that the ball rotates on its axis. If the pendulum be suspended from the ceiling of a cabin in a vessel and the vessel be turned completely around, the plane of vibration will not be

changed. Now if a graduated circle be placed just beneath the ball as the vessel turns about, the ball will cross the circle at different points and will *appear* to be changing its plane of vibration; but it is evident that this appearance is deceptive, and that the graduated circle must turn around with the vessel and also that the pendulum is merely pointing out the angular motion of the vessel. In the same manner, if a pendulum were suspended at one of the poles of the earth, in twenty-four hours every meridian of the earth would be brought beneath it, and although it would not meanwhile change its plane of vibration, it would appear to move from east to west, or in opposite direction to that of the earth, at the rate of 15° per hour. At the equator there would be no change. Between the equator and the pole the change per hour would vary from 0° to 15° according to the latitude. With a heavy metal ball and a wire as small as will support it, and very long (Fig. 51), one may successfully repeat this celebrated experiment by which Foucault demonstrated the motion of the earth on its axis.

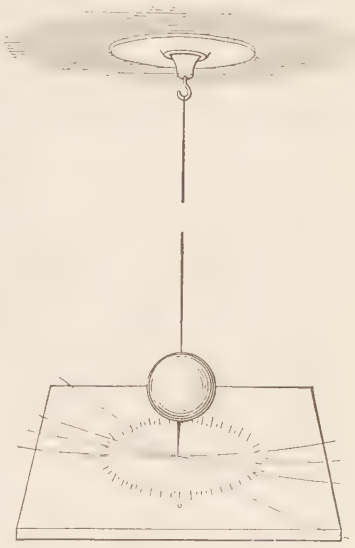


FIG. 51

QUESTIONS AND PROBLEMS

1. What is the length of a pendulum that beats half seconds? quarter seconds? that makes one vibration in two seconds? that makes two vibrations per minute?

2. Where is the center of percussion in a hammer or ax?

3. At what point (disregarding the length and mass of the striker's arm) should a blow be dealt with a bat of uniform dimensions when held in the hand at one extremity ?

4. What change in the location of the center of percussion is produced by making one end of a bat heavier than the other ?

5. Which end of a bat, the heavier or the lighter, should be held in the hands ? Why ?

6. One pendulum is 20 in. long and vibrates four times as fast as another. How long is the other ?

7. (*a*) What effect on the period of vibration of a pendulum has the weight of its bob ? (*b*) What effect has the length of the arc ? (*c*) What affects the period of vibration of a pendulum ?

8. How can you quicken the vibration of a pendulum threefold ?

9. A clock loses time. What change in the pendulum ought to be made ?

10. Two pendulums are 4 and 9 ft. long respectively. While the short one makes one vibration, how many will the long one make ?

11. The number of vibrations made by a given pendulum in a given time varies as the square root of the acceleration of gravity. Acceleration of gravity at any place is expressed by the value of g. (*a*) If at a certain place a pendulum 39.09 in. long make 3600 vibrations in an hour, and the value of g be 32.16 ft., what is the acceleration at a place where the same pendulum makes 3590 vibrations in the same time ? (*b*) Which of the two places is nearer the centroid of the earth ?

SECTION X

WORK AND ENERGY

74. Work. Whenever a force causes a change of motion or maintains motion against resistance it is said *to do work*. A force may act for an indefinite time without doing work; for example, a person may support a stone for a time and become weary from the continuous application of force to prevent its falling, but he does no work on the stone because he effects no change of motion or position. A force to do work *must effect a change of position*. *Force* and *distance* are essentials of work. Force without motion does no work; motion without force results in no work. Let the person supporting the stone exert a little more force, — the stone will rise and work will be done. *An unbalanced force always does work*, since motion is sure to ensue.

The body that moves another body is said to do work upon it, and the body moved is said to have work done upon it.

When the heavy weight of a pile driver is raised, work is done upon it; when it descends and drives the pile into the earth, work is done upon the pile, and the pile in turn does work upon the matter in its path.

75. Energy. By the *energy of a body* is meant "its capacity for doing work" (Maxwell). Energy is measured by the quantity of work which the body possessing it is capable of doing; hence the unit of work is also the unit of energy. The act of doing work consists in a transfer of energy from the body doing work to the body on which work is done; or it consists in a transformation of one kind of energy into another kind, as when the pile driver strikes the pile and the pile is forced into the earth. Here a part of the energy in each act is transformed into *heat*, which we shall learn, farther on, is *molecular energy*. Work, therefore, may be defined as *the act of transmitting or transforming energy.*

"We are acquainted with matter only as that which may have energy communicated to it from other matter, and which may in its turn communicate energy to other matter. Energy, on the other hand, we know only as that which in all natural phenomena is continually passing from one portion of matter to another" (Maxwell).

76. Kinetic and potential energy. Experience teaches that every moving body can impart motion to another body, and therefore can do work upon it; hence *every moving body possesses energy.* The energy which a body possesses in consequence of its motion is called *kinetic* (motion) *energy*. It is a property of a moving mass only. It is capacity for doing work possessed by a mass *in virtue of its motion.*

The kinetic energy of a body projected upward diminishes as the body rises and finally becomes *nil*, but it is not lost, for it is regained as the body falls. While rising, its energy becomes *stored up* in virtue of its higher *position*. Energy in store, i.e.

not in an active state, is called *potential energy*. It is the capacity for doing work possessed by a mass *in virtue of its position being such that it is possible for it to move, and in virtue of the existence of a stress which tends to move it*. Hence, the energy is convertible into kinetic energy without the agency of any additional work except to remove obstacles to the conversion. Potential or positional energy implies force, or a tendency to motion, as truly as kinetic energy implies motion.

The following are illustrations of energy in the potential state:

(1) A stone lying on the ground is in a sense devoid of energy. Raise it and place it on a shelf; in so doing you perform work upon it. As you look at it lying motionless upon the shelf, it *appears* as devoid of energy as when lying on the earth. Attach to it one end of a cord and pass this over a pulley, winding a portion of it around the shaft connected with a sewing machine, lathe, or other convenient machine. Suddenly withdraw the shelf from beneath the stone. The stone moves; it communicates motion to the machinery, and you may sew, turn wood, etc., with the energy given to the machine by the stone in falling.

The work done on the stone, or the energy transmitted to it in raising it, was not lost; it was recovered while the stone was descending. There is a very important difference between the stone lying on the ground and the stone lying on the shelf: the former is powerless to do work; the latter can do work. Both are alike motionless, and you can see no difference, except an *advantage* that the latter has over the former in having a *position* such that it *can move*. What gave it this advantage? Work. *A body, then, may possess energy due merely to* ADVANTAGE OF POSITION, *derived always from work performed upon it.* We see, then, that energy may exist in either of two widely different states: it may exist in bodies by virtue of their *actual motion*, or it may exist in bodies by virtue of their having an *opportunity* to move, as in the case of the stone lying on the shelf.

Possibly some will object and say that the work done is performed by gravity, and not by the stone's energy; that if this force should

cease to exist, the stone would not move if the shelf were removed, and consequently no work would be done. All this is very true, and it is likewise true that when the stone is on the ground the same force of gravity is acting, but can do no work simply because the position of things is such that the stone cannot move. The energy which the stone on the shelf possesses is due to the fact that its *position* is such that it *can* move, and that there is a *stress* between it and the earth which will cause it to move. Both advantage of position and stress are necessary, but the former is attained only by work performed. The force of gravity is employed to do work, as when mills are driven by falling water; but the water must first be raised from the ocean bed to the hill-side by the work of the sun's heat. The elastic force of springs is employed as a motive power, but this power is due to an advantage of position which the molecules of the springs have first acquired by work done upon them.

We are as much accustomed to store up energy for future use as to store up provisions for winter consumption. We store energy when we wind up the spring or weight of a clock, to be doled out gradually in the movements of the machinery. We store it when we bend the bow, or raise any body above the earth's surface.

(2) Matter may possess potential energy in virtue of chemical separation and chemical affinity, and the potential energy is a measure of the work done in effecting the separation; for example, the entire value of coal consists in its potential energy, which was stored up by the work performed through the agency of the sun's energy in separating the carbon of carbon dioxide from the oxygen. Gunpowder possesses, in a dormant state, energy sufficient to do a quantity of work, as in blasting, which would take many laborers a long time to do.

A body possesses potential energy when, in virtue of work done upon it, it occupies a position of advantage, or its constituent particles occupy positions of advantage, so that the energy expended can be recovered at any time by the return of the body to its original position, or by the return of its particles to their original positions.

77. Relation of energy to force and matter. Our discussions lead us to conclude that *energy is a condition of matter, due either to its motion or to its relation to other matter, in virtue of which the matter is capable of doing work.* Force may be regarded as "the measure of the tendency of energy" to transfer or "to transform itself" (Tait).

"By a convenient form of speech a given force is said to act upon a given body and to impart to it a given acceleration. It must be constantly borne in mind, however, that a force is not a physical entity, and can never be measured until we already know, absolutely or by comparison, the mass acted upon and the acceleration imparted to it" (Daniell). "Force is a mere phantom suggestion of our muscular sense" (Tait). "Energy has its price, force has not."

Energy is never found except in association with matter; hence matter may be defined as the vehicle or receptacle of energy. The first law of motion affirms that matter is simply passive, inert.

78. Practical units of work and energy. Inasmuch as a body's capacity to do work is dependent wholly upon the work which has been done upon it, it is evident that both work and energy may be measured by the same unit. One practical unit adopted is *the work done or energy imparted in raising one pound through a vertical height of one foot.* It is called a *foot-pound.* The metric unit is the work done or energy imparted in raising 1 kilogram a vertical height of 1 meter, and is called a *kilogrammeter.* The kilogrammeter is equivalent to 7.2331 foot-pounds. Since the work done in raising 1 pound 1 foot high is 1 foot-pound, the work of raising 1 pound 10 feet high is 10 foot-pounds, which is the same as the work done in raising 10 pounds 1 foot high, and the same, again, as raising 2 pounds 5 feet high.

There are many kinds of work besides that of raising masses; but since, with the same resistance, the work of producing motion in any other direction is just the same as in a vertical direction, it is easy, in all cases in which we know the resistance and space through which the resistance is overcome, to find the equivalent in work done in raising a mass vertically. By thus securing a

common standard for measurement of work we are able to compare any species of work with any other. For instance, let us compare the work done by a man in sawing through a stick of wood, if his saw moves 10 meters against an average resistance of 12 kilograms, with that done by a bullet in penetrating a plank to a depth of 2 centimeters against an average resistance of 200 kilograms. Moving a saw 10 meters against 12 kilograms resistance is equivalent to raising a mass of 12 kilograms 10 meters high, or doing 120 kilogrammeters of work; a bullet moving 2 centimeters against a resistance of 200 kilograms does as much work as is required to raise a mass of 200 kilograms 2 centimeters high, or $200 \times 0.02 = 4$ kilogrammeters of work. $120 \div 4 = 30$ times as much work done by the sawyer as by the bullet.

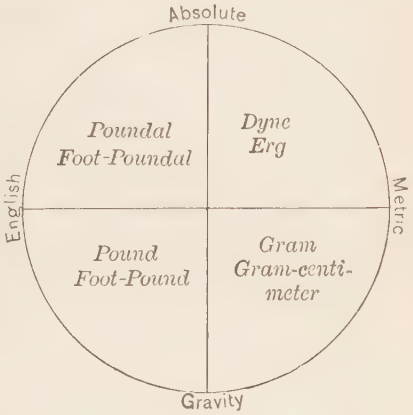


FIG. 52

79. Absolute units of work. If force be measured in dynes and distance in centimeters, the work done is expressed in a C.G.S. unit called an *erg*. *An erg is the work done or energy imparted by a force of one dyne working through a distance of one centimeter.*

The pupil will be assisted by the accompanying diagram (Fig. 52) in his first attempt to acquire and classify the units of force, energy, and work in the several systems. In purely scientific investigations absolute units are employed.

The following equivalents will be useful:

Gravitation Units		*Absolute Units*
1 gram-centimeter	=	g ergs
1 kilogrammeter	=	100,000 g ergs
1 foot-pound	=	0.13825×10^5 g ergs

80. Formulas for work or energy. *Force* and *distance* being essentials of work are necessarily the quantities employed in calculating work. A given force acting through a space of one foot does a certain quantity of work; it is evident that the same force acting through a space of two feet would do twice as much work. Hence the general formula

$$W = fs,$$

in which f represents the force employed, s the space through which the force acts, and W the work done.

In case a force encounters resistance, the magnitude of the force necessary to produce motion varies with the resistance. Often the work done upon a body is more conveniently determined by *multiplying the resistance by the distance through which it is overcome*. For example, a ball is shot vertically upward from a rifle in a vacuum; the work done upon the ball (by the explosive force of the gunpowder) may be calculated by multiplying the average force (difficult to ascertain) exerted upon it by the space through which the force acts (a little greater than the length of the barrel); or by multiplying the resistance to motion offered by gravity, i.e. its weight (easily ascertained), by the distance the ball ascends.

Let us calculate the energy stored in a bow by an archer whose hand, in bending the bow by pulling on the string, moves 6 inches ($\frac{1}{2}$ foot) against an average resistance of 20 pounds. Here $fs = 20 \times \frac{1}{2} = 10$ foot-pounds of work done upon the bow, or 10 foot-pounds of energy stored in the bow.

81. Formula for calculating kinetic energy. Suppose a body to have a mass m and a velocity v; it can do a definite quantity of work before it is brought to rest. If it be moving upward, a mutual work between it and the earth is performed in destroying each other's momentum. If its velocity be such that it will rise to a height s, then its kinetic energy is such that it will do mgs absolute units of work, or

$$E_k \text{ (kinetic energy)} = mgs. \qquad (1)$$

We may find, then, to what vertical height a body having a given velocity would rise if directed upward, and from formula (1) determine its kinetic energy. But a formula may be obtained which will give the same result with less trouble; thus, substituting g for a in the formula $v = at$, we have $v = gt$; whence

$$t = \frac{v}{g}, \text{ or } t^2 = \frac{v^2}{g^2}.$$

Again, $s = \frac{1}{2} gt^2$; substituting the value of t^2 in this equation, we have

$$s = \tfrac{1}{2} g \times \frac{v^2}{g^2} = \frac{v^2}{2g}.$$

Substituting for s in equation (1) its value, we have

$$E_k = \frac{mv^2}{2}, \tag{2}$$

a formula which will determine the kinetic energy of a body in absolute units when its mass and speed are known, since the energy is the same whatever be the direction of the motion.

Hence the kinetic energy of a body is half the product of its mass by the square of its speed.

If the result be desired in gravitation units, i.e. in gram-centimeters or foot-pounds, the number of absolute units must be divided by g, since g ergs (980) are equivalent to one gram-centimeter, or g foot-poundals (32.2) are equivalent to one foot-pound.

82. Energy distinguished from momentum. It is evident from formula (2) that *when the mass m of a body remains the same its energy is proportional to the square of its velocity; while its momentum*, as we have learned, *is proportional to its velocity.* In other words, the effect of increasing the velocity of a moving body would seem to be to increase its working power much more rapidly than its momentum.

Suppose a lead bullet to fall from a height of 50 centimeters upon a mass of moist clay; it will sink a definite distance. If, then, the same bullet be dropped from a height of 2 meters, it will strike the clay with *twice* the former speed. In the latter case it

penetrates to four times the depth that it did in the former, thus showing that its capability of doing work increased as the square of its speed.

Again, suppose a bullet of mass 30 grams to be shot with a speed of 100 meters per second from a gun of mass 3 kilograms; required the momentum and the energy of both pieces.

As $mv = m'v'$, we have $30 \times 10{,}000 = 3000\, v'$, from which v', or the gun's speed, is found to be 100 centimeters per second; and the momentum of *each* is 3×10^5 units.

From the energy formula, $E_k = \frac{1}{2}\, mv^2$, we calculate the energy of the gun to be 15×10^6 ergs; and that of the bullet to be 15×10^8 ergs, or one hundred times greater. From this it is seen that the kinetic energy in the case of equal action and reaction is by no means the same for the two parts of the system.

So it appears that *the energy of a moving body varies*, not as its velocity, but *as the square of its velocity*. Doubling the velocity increases the energy fourfold, trebling the velocity increases it ninefold, and so on; but the corresponding momentum is increased only twofold, threefold, etc. A bullet moving with a velocity of 400 feet per second will penetrate, not twice, but four times, as far into a plank as one having a velocity of 200 feet per second. A railway train having a velocity of 20 miles an hour will, if the steam be shut off, continue to run four times as far as it would if its velocity were 10 miles an hour. The reason is now apparent why light substances, even so light as air, exhibit great energy when their velocity is great.

Furthermore we have seen that momentum $= mv = ft$; and again, W (work done) or E_k (kinetic energy imparted) $= fs$.

The momentum, then, imparted to a body is the product of the force into the *time* it acts; energy imparted is the product of force into the *distance* through which it acts. It is evident, therefore, that *force may be measured by the momentum when time is considered, and by the energy which it imparts when the distance is considered.*

QUESTIONS AND PROBLEMS

1. Does the energy expended in raising the stones to their places in the Egyptian pyramids still reside in the stones?

2. What kind of energy is that contained in gunpowder?

3. Suppose that an average force of 25 lb. is exerted through a space of 10 in. in bending a bow. (*a*) What amount of energy will it give the bow? (*b*) What kind of energy will the bow, when bent, possess?

4. (*a*) What amount of kinetic energy is possessed by a mass of 20 lb. moving with a velocity of 300 ft. per second? (*b*) What amount of work can the body do?

5. A force of 500 lb. acts upon a body through a space of 20 ft. One fourth of the work is wasted in consequence of resistances. How much available energy is imparted to the body?

6. How much energy is stored in a body weighing 1000 lb. at a height of 200 ft. above the earth?

7. A horse draws a carriage on a level road at the uniform rate of 5 mi. an hour. (*a*) Does work accumulate? (*b*) What kind of energy does the carriage possess? (*c*) Suppose that the carriage were drawn up a hill, would energy accumulate? (*d*) What kind of energy would it possess when at rest on the top of the hill? (*e*) How would you calculate the quantity of energy it possesses when at rest on top of the hill? (*f*) Suppose that the carriage is in motion on top of the hill. What two formulas would you employ in calculating the total energy which it possesses?

8. How much work is done per hour if 80 kg. be raised 4 m. per minute?

9. (*a*) What energy must be imparted to a body of mass 50 g. that it may ascend 4 sec.? (*b*) How many times as much energy must be imparted to the same body that it may ascend 5 sec.? (*c*) Why?

10. Compare the momenta, in the two cases given in the last question, at the instants the body is thrown.

11. How much energy is stored in a body of mass 50 kg. at a height of 80 m. above the earth's surface?

12. How much kinetic energy would the same body have if it had a velocity of 100 m. per second?

13. Suppose it to fall in a vacuum, how much kinetic energy would it have at the end of the fourth second?

14. If it should fall through the air, what would become of a part of the energy?

15. A projectile of mass 25 kg. is thrown vertically upward with an initial velocity of 29.4 m. per second. How much energy has it?

16. What becomes of its energy during its ascent?

17. (*a*) Compare the momentum of a mass 50 kg. having a velocity of 2 m. per second with the momentum of a body of mass 50 g. having a velocity of 100 m. per second. (*b*) Compare their energies.

18. Which, momentum or energy, will enable one to determine the amount of resistance that a moving body may overcome?

19. Explain how a child who cannot lift 30 kg. can draw a carriage weighing 150 kg.

20. A United States 12-in. army gun, using a charge of 440 lb. of powder, throws a projectile of mass 1000 lb. with an initial velocity of 1975 ft. per second. (*a*) What quantity of energy is imparted to the projectile? (*b*) The maximum range of this gun when horizontal is 15 mi. If the velocity of the projectile were uniform, in how many seconds would it strike the ground? (*c*) If the projectile were directed vertically upward, during how many seconds would it rise, the resistance of the air being disregarded? (*d*) How high would it rise?

21. A street car having a mass of 3 T. was moving at the rate of 6 mi. per hour. The brakes were applied and the car was stopped in 4 sec. (*a*) What was the average force exerted by the brakes? (*b*) Suppose the car to have been stopped in the space of 40 ft., what was the average force applied?

83. Activity. The *activity* (sometimes called the *power*) of an agent is the *rate* at which it does or can do work, or it is the quantity of work it does *in a unit of time*, and is determined by the formula

$$A\ (\text{activity}) = \frac{W\ (\text{work})}{t\ (\text{time})}.$$

In estimating the total quantity of work done, the time consumed is not taken into consideration. The work done by a hod carrier in carrying 1000 bricks to the top of a building is the same whether he does it in a day or a week. But in estimating activity, as of a man, a horse, or a steam engine, in other words, *the rate* at which the agent is capable of doing work, it is evident that time is an important element. The work done by a horse in raising a barrel of flour 20 feet is about 4000 foot-pounds, but even a mouse could do the same quantity of work if time enough be allowed.

One of the units in which activity or rate of doing work is calculated is called (inappropriately) a horse-power. A horse-power is 550 foot-pounds per second, or 33,000 foot-pounds per minute.

The logical unit of activity is a unit of work in a unit of time, as one erg per second. The unit of activity, chiefly used in

measuring electrical activity, is the *watt*, or ten million ergs per second. A moderate estimate of man's activity is 100 watts.

Activity	*Equivalents*
1 erg per second	= 0.0000001 watt
1 horse-power	= 746 watts
1 foot-pound per minute	= 226,043 ergs per second
1 kilowatt	= 1000 watts

QUESTIONS AND PROBLEMS

1. For which is a truck horse valued, his energy or his activity?

2. Do we speak of the activity or the energy of a steam engine?

3. Which do we apply to levers and machines in general, power or force?

4. How much work can a 2 H.P. (horse-power) engine do in an hour?

5. (*a*) What quantity of work is required to raise 50 T. of coal from a mine 200 ft. deep? (*b*) An engine of how many H.P. would be required to do it in 2 hr.?

6. A car of mass 6000 kg. is drawn by a horse at a speed of 100 m. per minute. The index of the dynamometer to which the horse is attached stands at 40 kg. (*a*) At what rate is the horse working? (*b*) Express the rate in horse-power.

7. A dynamometer shows that a span of horses pull a plow with a constant force of 70 kg. What activity is required to work the plow if the horses travel at the rate of 3 km. per hour?

8. What is the activity of an engine which will raise 1,350,000 kg. 5 m. in an hour?

9. How long will it take a 3 H.P. engine to raise 10 T. 50 ft.?

10. How far will a 2-kilowatt engine raise 1000 kg. in 10 sec.?

11. A force of 10,000 dynes acting through a space of 100 m. per second furnishes an activity of how many watts?

12. The wind moves a vessel with a uniform velocity of 5 mi. an hour against a constant resistance of 2000 lb. What activity is furnished by the wind?

13. If a 2 H.P. engine can just throw 1056 lb. of water to the top of a steeple in 2 min., what is the height of the steeple?

14. A cannon ball of 10,000 g. is discharged with a velocity of 45,000 cm. per second. Find its kinetic energy.

15. In the last question, find the mean force exerted upon the ball by the powder, the length of the barrel being 200 cm.

16. Supply the following omissions with appropriate words such as "force," "work," "energy," "activity." When — acts through space — is

performed and – is imparted. The rate at which — is performed determines the — of the agent. The — of a bullet flying through vacant space. The — of a horse. The — of wind. The — of a bent bow. What — must a bullet of mass 1 oz. have that it may rise 4 sec.? What — is consumed by a steamer in crossing the ocean? What — is necessary that it may traverse 300 knots per day, and what must be the average — exerted to overcome the resistance at the required rate?

SECTION XI

MACHINES

84. It has been seen in the general discussion of energy, work, and activity that some device has usually been involved through the medium of which work has been done at the expenditure of energy. This agent is a *machine* of some kind, and a machine may, *in general*, be defined as any agent by means of which energy is either transferred or transformed. Perhaps the easiest concept of a machine is that of a simple lever by means of which a comparatively small force applied at the long arm may overcome a much greater one at the short arm. In the figure (Fig. 53) AB is a stiff rod pivoted at O, nearer the end B, so that $AO = 2\ OB$, say.

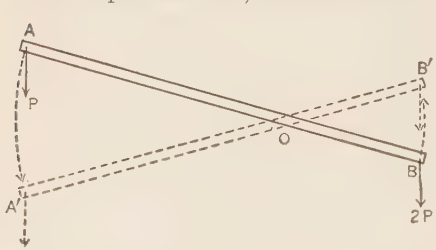


Fig. 53

In this case, if the weight of the rod be neglected, any force at A will balance one twice as great at B (see principle of moments, § 52). If now the point A be moved through a distance d to A' by a force P, the geometry of the figure shows that B will be raised a distance $d/2$. Now, since the work done in both cases is the product of the force by the distance, we have the work at A equal to $P \times d$; while that at B is equal to $2P \times d/2$, or $P \times d$, the same as at A. It is to be noted that we have neglected a very little friction at O, which means that in

a practical case the work expended at A must be slightly greater than that obtained at B; in other words, the machine is not quite *perfect*, and not quite as much work is gotten out of it as was put in, even in perhaps the most favorable case — that of a carefully arranged lever. The degree of perfection is technically called *efficiency*, and this may be defined as *the ratio of the useful work a machine does to the energy expended upon it*. It will be understood that in the example of the lever just studied the efficiency is very nearly 100 per cent, but in no case is it quite that, and in most cases the efficiency is far less.

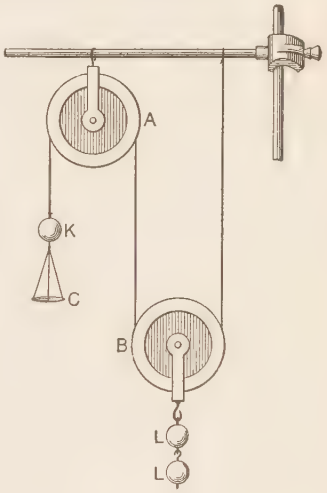


FIG. 54

As another example of a simple machine, suppose we have suspended, as in Fig. 54, a fixed pulley A and a movable pulley B. Let the scale pan C counterbalance the pulley B, so that there will be equilibrium. Suspend from B two masses L and L of equal weight, and on the side where the pan is, suspend a single one K equal to one of the former. The single mass supports the two; i.e. by the use of the machine a force of 1 is enabled to balance a force of 2. So far no work is done. Place a very small weight in the pan; the bodies L and L rise and work is done.

As the weight K plus a very small weight causes the motion, we shall regard this as the force (f); and as the weights L and L are the bodies moved (the pulleys and pan, being parts of the machine, may be disregarded), they may be regarded as the resistance (r) overcome, or the body on which work is done. Measure the respective distances through which f acts and r moves during the same time. It is found that r moves only one half as great a distance as that through which f acts; i.e. if r rise 2 feet, f

must act through 4 feet. Suppose that r is 2 pounds; then f is 1 + pounds. Now 2 (pounds) × 2 (feet) = 4 foot-pounds of work done on r. Again, 1 + (pounds) × 4 (feet) = a little more than 4 foot-pounds of work (or energy) expended.

It thus seems that although a machine will enable a small force to balance a large force, when work is performed the work applied to the machine is greater, rather than less, than that which the machine transmits to the resistance. The work applied is greater than the work transmitted by the amount of work wasted in consequence of friction and other resistances. So that *by the employment of a machine nothing is gained in work, but something is always lost.*

What, then, is the advantage gained in using this machine? Suppose that r is 400 pounds, and that the utmost force that a man can exert is a little more than 200 pounds. Then without the machine the services of two men would be required to move the resistance; whereas one man can move it with a machine, but he will be obliged to exert his 200 pounds twice as far as the resistance moves, — a matter of little consequence in comparison with the advantage of being able to do the work alone. The advantage gained in this instance seems to be one of *convenience.* Men, however, are accustomed to speak of it as "a gain of force" (or, more commonly and inaccurately, "of power"), inasmuch as a small force overcomes a large resistance.

Again, if it should be desirable to move a resistance with greater speed than it is possible or convenient for the force to act, it may be accomplished through the mediation of a machine, by applying to it a force proportionately greater than the resistance. This apparatus is one of many *contrivances called machines, through the mediation of which force can be applied more advantageously than when it is applied directly to the resistance.*

At present we deal with machines employed as means for transmitting and modifying motion and force. Later we shall consider machines whose function is to transform energy, such as the steam engine, dynamo, etc.

85. General law of machines. From the discussion above we derive the following formula for machines:

$$fs = rs' + w, \tag{1}$$

in which f represents the force applied, and s the distance through which f acts; r represents the resistance overcome, and s' the distance through which its point of application is moved; w represents the wasted work. A machine in which there is no wasted work is a *perfect machine.* Such a machine is purely ideal, as none exists. If in our calculations we regard a machine as perfect (though subsequently suitable allowance must be made for the wasted work), then our formula becomes

$$fs = rs', \tag{2}$$

whence $r : f :: s : s'$; i.e. *the force and resistance vary inversely as the distances which their respective points of application move.* In other words, the ratio of the resistance to the force is the reciprocal of the ratio of the distances which these points move; thus, if

$$r : f = 4, \text{ then } s' : s = \tfrac{1}{4}.$$

This law applies to machines of every description; hence it is called the *general* or *universal law of machines.*

Since fs, the work done upon a machine, is always greater than rs', the work transmitted by the machine, we infer that *no machine creates or increases energy.* No machine transmits more energy than it receives. A machine may enable us to gain intensity of force, but not energy. By taking s great enough f can be made as small as we please; in this case, *in proportion as force is gained, time, distance, or speed is lost.*

Formula (1) exhibits the relation between the work performed upon the machine and the entire work transmitted and transformed by the machine. But at any given instant while the machine is in operation it is evident that

$$fs = \tfrac{1}{2}\, mv^2 + rs' + w. \tag{3}$$

In formula (3) $\frac{1}{2}mv^2$ represents the kinetic energy of the moving parts of the machine.

86. Efficiency of machines. The *efficiency* of a machine is a fraction, usually a *per cent.* expressing the ratio of the energy given out by the machine and utilized to the total energy expended upon it. The limit of the efficiency of a machine is 100 per cent, which is the efficiency of an "ideal," or perfect, machine, in which no energy is lost. The object of improvements in machines is to bring their efficiency as near to unity as possible. For instance, if 50 foot-pounds of energy be expended on a machine, and friction convert 8 foot-pounds into heat, and 5 foot-pounds be lost in consequence of only a component of the working force being utilized, so that the machine is able to give out only 37 foot-pounds, its efficiency is 74 per cent ($= \frac{37}{50}$). If the friction can be reduced one half and an improvement be made in the machine which will render the entire working force effective, then there will be wasted only 4 foot-pounds of energy, the efficiency of the machine will be raised to 92 per cent ($= \frac{46}{50}$), and the quantity of work which it will accomplish will be increased in the ratio of 92 : 74.

87. The simple machines. Machines, however complicated or complex, are largely composed of a few simple machines. As usually given they are the *lever, wheel and axle, inclined plane, wedge, screw, pulley,* and *knee.*

While the general law of machines is always applicable, its application is not always convenient, since, for example, it necessitates putting the machine in motion in order to measure s and s' (the distances traversed respectively by the points of application of the force and resistance in the same time), an operation which would be very difficult and tedious in many cases. Hence a *special law* — one in which the relation between the ratio of gain and the ratio between certain dimensions of the machine is stated — is often more convenient in practice ; for example, in our discussion of the lever we discover that

$$r : f :: \text{force arm } (AO) : \text{resistance arm } (BO),$$

i.e. *the force and resistance vary inversely as the lengths of their respective arms.* Compare this special law with the general law. Place the fulcrum O at other points in the lever, thereby varying the length of the arms, and verify by numerous experiments the special law of levers.

88. Wheel and axle. As before stated, the lever, already explained, is the simplest of the machines, as well as the most efficient. The next in order is logically the wheel and axle, which is nothing more than a continuous lever.

The wheel and axle consists of two cylinders having a common axis, the larger of which is called the *wheel*, and the smaller the *axle*, as A and C (Fig. 55). Either may be moved by the hand or by a string wound around the circumference.

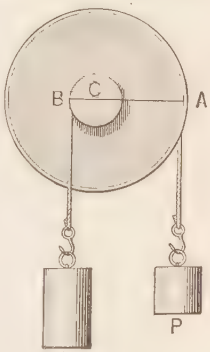


FIG. 55

The wheel is often replaced by a crank, as in the windlass, or by a spoke, as in the capstan, and is thus employed in hoisting apparatus, such as cranes, derricks, etc.

The wheel and axle viewed in section will be seen (Fig. 55) to be only a modification of the lever, which, unlike the latter, may be continuous in its operation. C' is the fulcrum, the radius $C'A$ the force arm, and the radius $C'B$ the resistance arm. The laws pertaining to this machine are virtually the same as those of the lever; for example, when the force P is applied to the wheel and the resistance Q is at the axle,

$$P : Q = \frac{1}{R\ (\text{radius of wheel})} : \frac{1}{R'\ (\text{radius of axle})}$$

$$= \frac{1}{C\ (\text{circumference of wheel})} : \frac{1}{C'\ (\text{circumference of axle})}.$$

89. Inclined plane. Any plane surface not horizontal or vertical, known as an *inclined plane*, may often be used as a mechanical advantage; for example, a plank resting with one end on a cart body and the other on the ground, a hillside, or a road grade. The

gradient is the quantity of rise per horizontal foot, or the ratio of the vertical rise to the horizontal distance.

When a body is pressed against a hard, smooth surface, the resistance offered by the surface is at right angles to the surface. A body, e.g. a sphere, may be supported on a horizontal surface, for the weight acting downward is counteracted by the upward

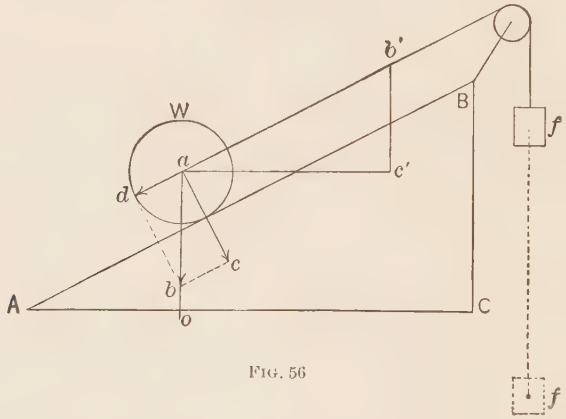


FIG. 56

reaction of the plane; but since on an inclined plane the reaction is not vertically upward, a body cannot rest on it without the aid of another force.

The mechanical advantage of this machine depends on the principle of the resolution of a force into its components. Let AB (Fig. 56) be an inclined plane whose gradient is $\frac{BC}{AC}$. Let a be the centroid of the weight W (technically called the *load*). The line of direction of the load is along the vertical ao, but the pressure exerted upon the plane is in the direction ac, and the reaction of the plane is in the direction ca. We may take any length along the vertical, as ab, to represent the load W. Draw bc parallel to AB, to meet ac. Complete the parallelogram $adbc$ with ab as its diagonal. The force ab is thereby resolved into two forces, ac representing the pressure upon the plane, and

ad representing an unbalanced force tending to move W along the plane BA. It is evident that to produce equilibrium, i.e. to support the body on the plane AB, a force equal to ad, but opposite in direction, must be employed. Now it may be proved geometrically that the triangles adb and BCA are similar; i.e.

$$da : ab :: CB : BA.$$

But da represents the force f necessary to produce equilibrium, while ab represents the load or resistance r; CB represents the height h, and BA the length l, of the inclined plane. Therefore

$$f : r :: h : l, \text{ or } f = \frac{h}{l} r.$$

Hence *a given force acting parallel with the direction of inclination of an inclined plane will support a weight as many times greater than itself as the length of the inclined plane is greater than its vertical height.*

COROLLARY. *With a given length of inclined plane the greater its vertical height, i.e. the steeper it is, the greater f must be.*

Suppose that a (Fig. 56), the point of application of r, be moved to b'. How high is the load raised? Through what distance does f act? Show that the general law of machines is applicable to this case.

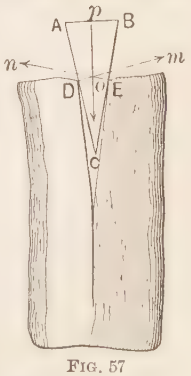


FIG. 57

90. The wedge. The wedge is a triangular prism, used commonly as a movable inclined plane for overcoming great resistances through short distances. It usually consists of two inclined planes as AcP and BcP (Fig. 57). The force applied acts in the direction po, and the resistance acts at right angles to the planes, or in the directions Dm and En. The force applied is of the nature of a percussion, as that of a sledge; besides, friction and other resistances form so considerable a factor in its use that no definite law of any practical value can be given other than that, with a given thickness, the longer the wedge the greater the gain in intensity of force.

91. The screw. The screw is another variety of the inclined plane, as may be shown by winding a triangular piece of paper around a cylinder, e.g. a lead pencil (Fig. 58). The hypotenuse will form a spiral about the cylinder resembling the threads of a screw.

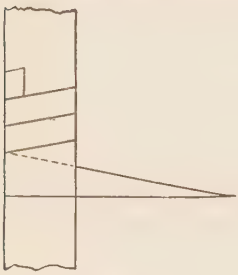


FIG. 58

In actual practice the screw consists of two parts, — (1) a convex grooved cylinder, or *screw* S (Fig. 59), which turns within (2) a hollow cylinder, or *nut* N. The concave surface of the latter is cut with a thread corresponding to the thread of the screw. The force is employed either to turn the screw within an immovable nut, or to turn the nut about a fixed screw.

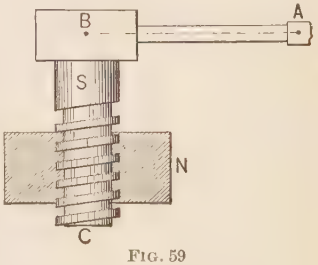


FIG. 59

In either case the force is usually applied to a lever or wheel fitted either to the screw or to the nut.

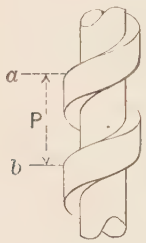


FIG. 60

During a single rotation of the screw or nut the load or resistance is moved a distance equal to the distance between the corresponding surfaces of two successive threads, usually termed the *pitch* of the screw, as ab (Fig. 60). Then, in conformity to the universal law of machines, *the force is to the resistance as the pitch is to the circumference of the circle described by the force.*

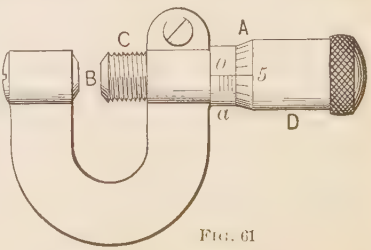


FIG. 61

Since a screw turning in a nut advances only its pitch distance at each revolution, a finely cut screw furnishes an instrument well adapted to measure very

minute distances. For example, if the screw C of the micrometer caliper (Fig. 61) have a pitch of one millimeter, and the thimble D be divided near its end A into one hundred parts, so as to register hundredths of a revolution, it is evident that any object, e.g. a hair, placed between its jaws at B can be measured to the hundredth of a millimeter.

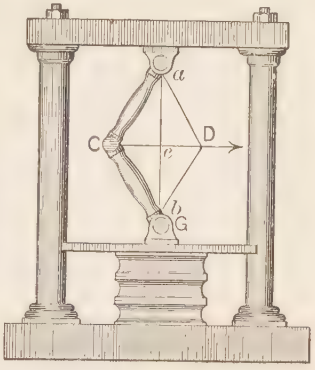


FIG. 62

92. The knee or toggle joint. This machine also is employed where great pressure has to be exerted through a small space, as in punching and shearing iron. Force applied at C (Fig. 62) in the direction CD will cause the jointed bars to be brought into line with each other and will tend to push apart the objects at the other extremities of the bars. The nearer these bars approach to a straight line, the greater pressure will a given force produce.

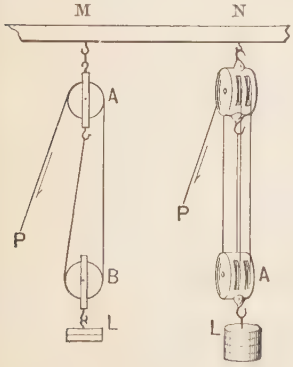


FIG. 63

COMBINATION OF MACHINES

93. Combination of pulleys. It has been shown that mechanical advantage is gained with the movable pulley. In Fig. 63, M, the cord passing around the movable pulley may be supposed to be divided into two parts, each supporting half the load; the tension in each part is half the load, and the supporting force P is half the load. In Fig. 63, N, the movable block A contains two pulleys, the cord supporting the load is divided into four parts, and P is one fourth of L; and generally with a combination of pulleys having a continuous cord

$$P = \frac{L}{n},$$

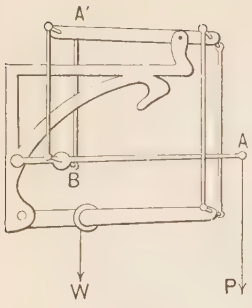


FIG. 64

i.e. *the force is equal to the load divided by the number of parts of the cord supporting the movable block.*

94. Combination of levers. In the arrangement shown in Fig. 64 there are three simple levers combined so as to form one *compound lever*. The triangular frame is rigidly fixed and the force P is applied at A: the resistance applied to this simple lever at B is identical with the force applied at A', and so on. Then

$$P = W \times \frac{\text{Continuous product of resistance arms}}{\text{Continuous product of force arms}}.$$

A combination of levers similar to this may be seen in scales used for weighing very heavy bodies, — the platform "scales," — in which a comparatively small weight counterbalances the heavy load.

95. Combinations of the wheel and axle. Fig. 65 represents a train of wheels in gear. A train of wheels being analogous to a compound lever, the mechanical advantage gained is obviously the ratio of the continued product of the radii of the wheels to the continued product of the radii of the axles.

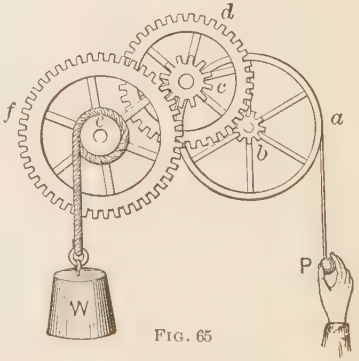


FIG. 65

96. Friction. Whenever two surfaces are in contact and relative motion occurs between them, there is more or less resistance developed, which opposes the motion. This resistance is called friction and plays a most important part in many phenomena.

Fig. 66 represents a body of any kind, say a brick, resting on a horizontal plane AB. Suppose a horizontal force P is applied to the body. Immediately the opposing friction F is called out,

which always equals P until the latter has reached a certain value, depending on the state of the surfaces in contact and the pressure between them; in other words, there is a maximum value for F in any given case, and until P becomes equal to this the body does not move. If P exceeds a little this maximum value of F, then the body starts to move in the direction in which P acts. In the case under consideration the pressure between the surfaces in contact is equal to the weight of the body, and the ratio of the maximum value of F to the pressure between the surfaces is called the *coefficient of friction*, or $\mu = \frac{F}{W}$.

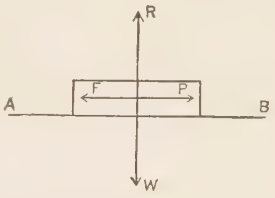


FIG. 66

97. Suppose now that the plane AB is inclined (Fig. 67); then W becomes resolved into two components R and P, the former being the mutual pressure between the surfaces and the latter being a force tending to draw the body down the incline. As before, the friction F is equal to P. If the inclination be increased till *sliding just begins*, F will have reached its maximum value, and $\frac{F}{R}$, the ratio of the friction to the pressure, will equal the coefficient of friction as before. From the equality of the angles α in the diagram it is seen that $\frac{P}{R}$ is the tangent of the angle of inclination of the plane AB. The angle of inclination of AB when sliding starts is called the angle of repose; therefore the coefficient of friction is equal to the tangent of the angle of repose, or, since $F = P$,

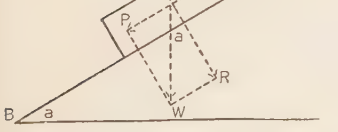


FIG. 67

$$\mu = \tan \alpha.$$

Evidently F approaches zero when the surfaces are made smoother and smoother, and so μ becomes less and less under these circumstances.

QUESTIONS AND PROBLEMS

1. (*a*) How is intensity of force gained by the use of a machine? (*b*) How is speed gained by the use of a machine?

2. (*a*) What is mechanical advantage? (*b*) Give a rule by which the mechanical advantage that may be gained by any machine may be calculated.

FIG. 68

3. Energy is applied to a machine at the rate of 250 ft.-lb. per minute and it transmits 200 ft.-lb. per minute. What is its efficiency?

4. (*a*) What advantage is gained by a nutcracker (Fig. 68)? (*b*) What is the ratio of gain?

5. (*a*) What advantage is gained by cutting far back on the blades of shears, near the fulcrum (Fig. 69)? Why? (*b*) Should shears for cutting metals be made with short handles and long blades, or the reverse? (*c*) What is the advantage of long blades?

6. Is work done when the moment of the force applied to a lever is equal to the moment of the resistance? Why?

7. If F (Fig. 70) be the fulcrum of the lever CA, and AB represent, on a scale of 1 cm. = 1 kg., a force applied at A, what force applied at C in the direction CD will produce equilibrium?

8. With a wheel and axle a force of 8 lb. sustains a weight of 56 lb. What is the ratio between the radii of the wheel and of its axle?

FIG. 69

9. A capstan turned by two horses is used to draw a boat; the horses are attached to the levers 12 ft. from the axis of the capstan, and the radius of the axle is 18 in. When each horse pulls with a force of 1000 lb., what force is exerted upon the boat?

10. A lever is 75 cm. long. Where must the fulcrum be placed in order that a force of 2 kg. at one end may balance a force of 4 kg. at the other end? What will be the pressure on the prop?

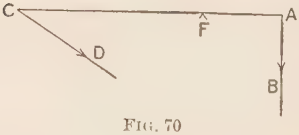


FIG. 70

11. Two weights of 5 kg. and 20 kg. respectively are suspended from the ends of a lever 70 cm. long. Where must the fulcrum be placed that they may balance?

12. (*a*) A skid 12 ft. long rests with one end on a cart at a height of 3 ft. from the ground. What force will roll a barrel of flour weighing 200 lb. over the skid into the cart? (*b*) What amount of work will be required?

13. (*a*) Draw a line to represent an inclined plane. Find the least force that will prevent a ball weighing 96 lb. from rolling down the plane. (*b*) Find the pressure which the ball will exert upon the plane.

14. An iron safe on trucks, weighing 2 T., is prevented from rolling down an inclined plane by a force of 250 lb. What is the ratio of the length of the plane to its height?

15. The gradient of an inclined plane is 1 ft. in 4 ft. To produce equilibrium on this plane, what relation must the force applied parallel with the plane bear to the load?

16. A plank 12 ft. long and weighing 24 lb. is supported by two props, one 3 ft. from one end and the other 1 ft. from the other end. What is the pressure on each prop?

17. What must be the diameter of a wheel in order that a force of 20 lb. applied at its circumference may be in equilibrium with a resistance of 600 lb. applied to its axle, which is 3 in. in diameter?

18. (*a*) Where is the fulcrum in a claw-hammer? (*b*) What is the ratio of the mechanical advantage gained by means of it?

19. In its technical meaning, a "perpetual-motion machine" is not a machine that will run indefinitely, but one which *can do work indefinitely without the expenditure of energy*. Show that such a machine is impossible.

20. When there are four movable pulleys in one block, what length of cord passes through the hands in raising a weight 6 in.?

21. What force must a man weighing 180 lb. use to support himself by means of the pulleys in Fig. 63, *N*?

22. The circumference of the circle described by the end of the lever of a screw is 9 ft., and there are three threads to the inch. What pressure will a force of 100 lb. exert?

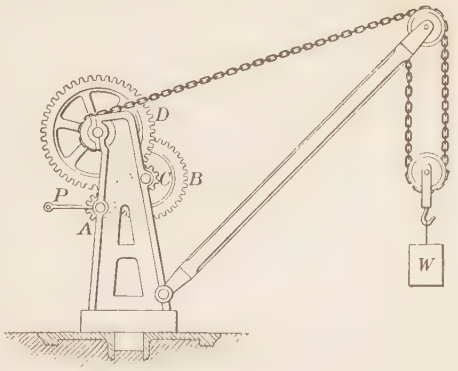


FIG. 71

23. Suppose the lengths of the arms of the several levers in Fig. 64 bear to each other the relations 5 : 1, 4 : 1, and 3 : 1. What force applied at *A* will support 180 lb. at *W*?

24. (*a*) Of how many and what simple machines does the *crane* (Fig. 71) consist? (*b*) How would you calculate the mechanical advantage gained by it?

25. Calculate the coefficient of friction when a body just begins to slide on a plane inclined to the horizontal at an angle of 45°.

SECTION XII

GRAVITATION, OR UNIVERSAL ATTRACTION BETWEEN MASSES

98. Gravitation is universal. An unsupported body falls to the earth. This is evidence of an action or stress between the earth and the body. It has been ascertained by careful observation that when a ball is suspended by a long string by the side of a mountain, the string is deflected from the vertical toward the mountain in consequence of an attraction between the mountain and the ball. Delicate experiments show that very small bodies tend to approach one another, and that the tendency of a body to fall to the earth is but a single instance of a tendency existing in all kinds and quantities of matter.

That there is an attraction between the sun and the earth, and the earth and the moon, is shown by their curvilinear motions. Tides and tidal currents on the earth are due to the attraction of the sun and the moon. This attraction which exists between all masses is called *gravitation*. When bodies under its influence tend to approach one another, they are said to *gravitate*. Since this attraction ever exists between all bodies, at all distances, it is called *universal gravitation*. The theory of universal gravitation was established by Newton. No concept in science rests on a surer foundation.

99. Weight. The attraction between the earth and a terrestrial body is called the *weight* of that body. The weight of a body, therefore, is the measure of the force of attraction between the body and the earth.

We do not know the cause of gravitation. Whether the seat of the attraction or energy is, as the language in common use indicates, in the bodies themselves, or whether it exists in some medium which we may suppose to surround all bodies and fill all intervening space, we do not know.[1]

[1] "It may turn out to be a property inherent in matter, and an exception to every known case; but it is more probable that it is not an inherent property in matter

100. Law of gravitation. Methods first given by Newton in the *Principia*, but too elaborate for our purpose, have established the fact that the magnitude of the attraction between any two bodies depends upon two things, — their *masses* and the *distance between their centroids*. "The law of universal gravitation" is as follows: *The attraction between every two particles of matter in the universe varies directly as the product of their masses, and inversely as the square of the distance between their centroids.*[1]

Representing the masses of two bodies by m and m', the distance between their centers of mass by d, and the attraction by f, this relation is expressed mathematically thus: $f \propto$ (varies as) $\frac{mm'}{d^2}$. For example, if the mass of either body be doubled, the product (mm') of the masses is doubled, and consequently the attraction is doubled. If the distance between their centers of mass be doubled, then $\left(\frac{1}{2^2} = \frac{1}{4}\right)$ the attraction becomes one fourth as great.

101. Galileo's experiment. Galileo let fall from the top of the leaning tower at Pisa iron balls of different masses, and found that they fell with equal acceleration and reached the ground at the same instant. This celebrated experiment established two important facts:

(1) *At any given place the acceleration due to gravitation is independent of the mass of the falling body:* in other words, for all bodies at the same place the acceleration due to the earth's attraction is the same.

Let f and f' be the intensities of two forces acting on two bodies whose masses are respectively m and m'. Then

$$f = ma, \text{ and } f' = m'a';$$

at all, but a property due to a strain in the medium in which all matter is immersed" (Lodge). The gravitation stress existing in this medium may be, in some respects, analogous to the stress which exists in a stretched rubber band, which would tend to bring together any bodies between which it might be stretched.

[1] The attraction between two gram masses whose centroids are one centimeter apart is about one fifteen-millionth of a dyne.

but, as proved by Galileo's experiment, in case m and m' are falling bodies,

$$a = a'.$$

Hence (dividing)

$$\frac{f}{f'} = \frac{m}{m'},$$

and, in general,

$$f \propto m;$$

i.e. (2) *the earth's attraction for different bodies at a given place near its surface varies directly as the masses of the bodies.*

Proposition (1) is seemingly contradicted by everyday experience, for if a coin and a piece of tissue paper be dropped from a height they fall with very different speeds and accelerations; but if they be placed in a long glass tube (Fig. 72), the air exhausted, and the tube turned end for end, it will be found that the coin and the paper fall in the vacuum with equal speeds. It is evident, then, that when there is a difference in the acceleration of falling bodies at the same place, it is not due to the force of gravitation but to something else, e.g. the resistance of the air.

FIG. 72

102. Variation of gravitation, or g, on the earth's surface. A spherical body of uniform density acts upon a particle outside it as if the entire mass were collected at its center. If the earth were a homogeneous sphere and at rest, then the value of g would be constant at its surface, since every point in it would be equidistant from the center. But the earth is a spheroid, its polar diameter being about 43 kilometers (nearly 27 miles) less than the equatorial diameter. Consequently the value of g is less in the equatorial than in the polar regions, i.e. a given body stretches a spring balance less as it is carried from either pole toward the equator. The loss of weight of any body, due to this increase of distance from the center of the earth in being transported from the poles to the equator, is estimated to be $\frac{1}{578}$ of its weight at the poles. But we have previously seen that the centrifugal force at the equator diminishes the weight

of a body $\frac{1}{289}$. Now in consequence of difference in distance from the center of mass of the earth and difference in velocity due to the earth's rotation, a body weighs at the equator $\frac{1}{568} + \frac{1}{289} = \frac{1}{192}$ less than at the poles.

103. Weight above the earth's surface. We infer from the law of gravitation that a body weighs more at the earth's surface than above it; in other words, bodies become lighter as they are raised above the earth's surface. But since the force diminishes as the square of the distance from the center (not from the surface) of the earth, and as the surface is about 4000 miles from the center, the diminution for a few miles or for any distance which we are able to raise bodies is scarcely perceptible; hence in all commercial transactions we may, without important error, buy and sell as if the weighing always took place at the same distance from the center of the earth, in which case mass is strictly proportional to weight.

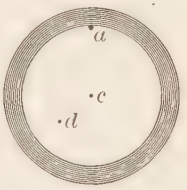


FIG. 73

104. Weight below the earth's surface. It may be demonstrated geometrically that if a particle be placed anywhere within a homogeneous hollow spherical shell of matter, as at a, c, or d (Fig. 73), it is in a state of equilibrium in regard to the attraction of the matter of the enveloping shell; i.e. a body thus placed within a hollow shell of the earth, of whatever thickness, would be under the influence of balanced forces and hence weightless. Hence if a body be taken below the surface of the earth, as from a to a' (Fig. 74), it is practically placed within a hollow spherical shell of the earth, and therefore is freed virtually from the gravitation influence of this shell. Its weight is now wholly determined by the gravitation of the smaller mass mn. So that as a body is carried below the surface of the earth it loses in weight as much as it would if it were being transferred to smaller and smaller earths. Obviously at the earth's center matter is under the

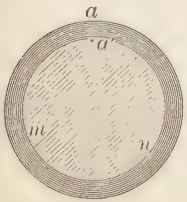


FIG. 74

influence of balanced forces and hence weightless. If the earth were a perfect sphere and homogeneous, the weight of a body would diminish uniformly as the distance from the surface increased.

QUESTIONS AND PROBLEMS

1. Why does a 100-lb. iron ball fall with no greater acceleration than a 1-lb. ball of the same material?

2. (*a*) Which falls with greater acceleration in the air, an iron ball or a wax ball? Why? (*b*) How would their accelerations compare in a vacuum? (*c*) Is the acceleration of a falling body independent of kind of matter?

3. If the earth's mass were doubled without any change of volume, how would it affect your weight?

4. On what principle may you determine that the mass of one body is ten times the mass of another body?

5. How many times must you increase the distance between the centers of two bodies that their attraction may become one fourth as great?

6. If a body on the surface of the earth be 4000 mi. from the centroid of the earth, and weigh at this place 100 lb., what would the same body weigh if it were taken 4000 mi. above the earth's surface?

7. The masses of the planets Mercury, Venus, Earth, and Mars are respectively very nearly as 7, 79, 100, and 12. Assuming that the distance between the centers of the first two is the same as the distance between the centers of the last two, how would the attraction between the first two compare with the attraction between the last two?

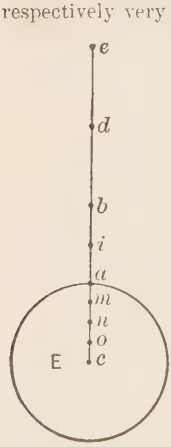


Fig. 75

8. What would be the answer to the last question if the distance between the centers of the first two were four times the distance between the centers of the last two?

9. If you hold a body on a spring balance in an elevator, what effect will be noticed as you start to ascend? What effect, as you start to descend? Explain.

10. Let *E* (Fig. 75) represent the earth as a perfect homogeneous sphere of a radius of 4000 mi. (*a*) If a body at *a* weigh 1 lb., what would it weigh at *m*, 1000 mi. below the surface? (*b*) What at *n*, *o*, and *c*, respectively 2000, 3000, and 4000 mi. below the surface? (*c*) What at *b*, *d*, *e*, and *i*, respectively 4000, 8000, 12,000, and 2000 mi. above the surface?

11. (*a*) What is a vertical line? (*b*) What angle does it make with the base of a spirit level when in position?

12. A body weighs 100 lb. at the earth's surface. At what two places would its weight on a spring balance be 50 lb.?

PROPERTIES OF MATTER

SECTION I

CONSTITUTION OF MATTER

105. The molecule. Physiology teaches us that, in order to smell any substance, we must take into our nostrils, as we breathe, small particles of that substance which are floating in the air. The air, for several meters around, is sometimes filled with fragrance from a rose. You cannot see anything in the air, but it is, nevertheless, filled with a very fine dust that floats away from the rose. At sea the odor of rosemary renders the shores of Spain distinguishable long before they are in sight. A grain of musk will scent a room for many years, by constantly sending forth into the air a dust of musk. Though the number of particles that escape must be countless, yet they are so small that the original grain does not lose perceptibly in weight.

These instances, and numerous others of common occurrence, give us only a feeble conception of the minuteness of particles of matter and of its extreme divisibility. Yet the smallest particle of dust of rosemary or musk, and the smallest particle of any substance which can be obtained by any mechanical means, is very large in comparison with bodies called *molecules*, which, of course, are too small to be seen, but of whose existence we have ample evidence. A single simple example of the proofs of their existence, though by no means the most conclusive, must suffice at this place. Matter, e.g. gold or water, is either continuous, as it appears to the eye, or it is discontinuous, that is granular, being composed of distinct particles called molecules, somewhat

as represented in Fig. 76. Matter is compressible and expansible. On the supposition that matter is continuous, these properties are unexplainable; but on the supposition that matter is molecular, these properties are easily explainable. A change of volume by contraction or expansion means simply a coming together or a separation of the molecules composing the body, as represented in Fig. 76.

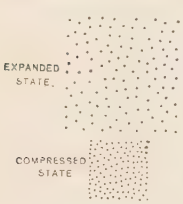


FIG. 76

106. Theory of the constitution of matter. For reasons which will appear as our knowledge of matter is extended, physicists have generally adopted the following theory of the constitution of matter: *Every body of matter is composed of exceedingly small disconnected particles called molecules. No two molecules of matter in the universe are in permanent contact with each other. Every molecule is in quivering motion, moving back and forth between its neighbors, hitting and rebounding from them. When we heat a body we simply cause the molecules to move more rapidly through their spaces, so they strike harder blows on their neighbors and usually push them away a very little; hence the body expands.*

107. Porosity. If the molecules of a body are never in contact except at the instants of collision, it follows that there are spaces between them. These spaces are called *pores.*

All matter is porous: thus water may be forced through the pores of cast iron; and gold, one of the densest of substances, absorbs liquid mercury.

Impenetrability [1] may be affirmed of molecules, but not necessarily of masses. The term *pore*, in physics, is restricted to the invisible spaces that separate molecules. The cavities that may be seen in a sponge are not pores, but holes; they are no more entitled to be called pores than are the cells of a honeycomb or the rooms of a house.

[1] The doctrine of impenetrability declares that "two bodies of matter cannot occupy the same space at the same time." In its strict scientific sense this doctrine is as axiomatic as the statement that "a body cannot be in two places at the same time."

By methods based on various lines of reasoning physicists ascertain approximately the probable size of the molecule. Lord Kelvin estimates that the diameter of the molecules of a gas cannot be less than one five-hundred-millionth of a centimeter. The minimum particle visible to the eye is a cube one four-thousandth of a millimeter on a side. Such a cube contains from sixty to one hundred million molecules.

The kinetic theory of gases teaches that in a cubic centimeter of any gas at atmospheric pressure and at a temperature of zero degrees there are about $3 \cdot 6 \times 10^{19}$ detached particles similar and equal to one another.

108. The atom. The theory given above assumes that the molecule is the limiting particle of possible physical division, i.e. the smallest particle of any substance which can preserve the properties of that substance; hence the molecule is sometimes termed the *physicist's unit*. The chemist finds it necessary to assume that the molecule is capable of a still further subdivision, a division which results in a complete change in the character of the substance operated on. Thus a molecule of sugar when subjected to chemical processes, which are virtually chemical divisions, yields carbon, hydrogen, and oxygen, substances entirely unlike sugar. These still smaller particles obtained by the division of the molecule are called *atoms*. The atomic theory assumes that the atom, as the word etymologically signifies, is indivisible, and it may be termed the *chemist's unit*. About seventy-five different kinds of atoms have been discovered. These constitute the so-called *elementary substances*. All other substances are compounds of certain of these elements of varying degrees of complexity. A molecule consists of a group of atoms bound together by chemical forces usually termed *chemical affinity*, and a mass consists of a group of molecules which may or may not be bound together by a physical or molecular force called *cohesion*.

109. The electron or corpuscle. Until quite recently it was supposed and always assumed that the atom was indivisible. Various electrical phenomena, however, indicate that an atom

contains a great many minute negatively charged particles in rapid motion, another portion of the atom making up an equal positive charge. These negative atomic constituent parts are called *electrons* or *corpuscles.*

It is supposed that all atoms are made up of different groupings and numbers of electrons. The subject is most intricate and not one to be further discussed in an elementary book.

SECTION II

THE STATES OF MATTER

110. Solids, liquids, and gases. In popular language there are said to be three states of matter, *the solid*, *the liquid*, and *the gaseous.*

Solids preserve a definite volume and shape when left to themselves; liquids tend to preserve a definite volume only, while their shape conforms to that of the containing vessel; gases tend to preserve neither a definite volume nor shape, but conform not only in shape but also in volume to the containing vessel, no matter how large this may be. We may have a vessel half full of liquid, but a mass of gas, no matter how small the quantity, always occupies the whole of a containing vessel. Gases tend to expand indefinitely and to assume an infinite volume with a correspondingly small density. Solids and liquids may have free surfaces; gases cannot retain permanently a free bounding surface independent of the containing vessel.

Which of the three states any portion of matter assumes depends upon its *temperature and pressure.* Just as at ordinary pressures of the atmosphere water is a solid (i.e. ice), a liquid, or a gas (i.e. steam), according to its temperature, so any substance may be made to assume any one of these forms unless a change of temperature causes a chemical change, i.e. causes it to break up into other substances. For example, wood cannot be melted because it breaks up into charcoal, steam, etc., before the melting point is reached. In order that matter may exist in a liquid (and

sometimes in a solid) state, a certain definite pressure is required. Ice vaporizes, but does not melt (i.e. liquefy) in a space from which the air (and consequently atmospheric pressure) has been removed. Solid carbon dioxide vaporizes, but does not melt unless the pressure is greater than the ordinary atmospheric pressure. Even charcoal could probably be liquefied, if sufficient pressure could be used along with a very high temperature.

As regards the temperature and pressure at which different substances assume the different states, there is great diversity. Oxygen and nitrogen gases liquefy and solidify only at extremely low temperatures. On the other hand, certain substances, as quartz and lime, are liquefied only by the most intense heat.

111. Fluids. The term implies the property of *flowing*. Since both liquids and gases possess this property in an eminent degree in consequence of great freedom of motion of their molecules around one another, they are both included under the common term *fluid*. Further on it will be seen that one of the chief distinctions between a solid and a fluid is that the former possesses rigidity, while the latter does not.

112. Vaporous state; critical state. Closer study makes evident that the above classification is purely arbitrary. The three states of matter sometimes merge into one another, so that there remains no distinct line of demarcation between them; for example, there is a state which may be regarded as intermediate between the liquid and the gaseous, called the *vaporous state*. If a substance in the gaseous form be compressed or cooled to such an extent that it will suffer but little further compression or cooling without passing into the liquid state, it possesses in this state certain peculiar properties and is known by the name of *vapor*. A vapor may be defined as a gas near its condensing point. When matter is at a temperature and under a pressure such that if heated a little more it becomes a vapor, or if allowed to cool a little more it becomes a liquid, it is said to be in the *critical state*. In this semi-liquid state the gaseous and liquid states meet and are indistinguishable. The highest temperature at which this occurs

is called the *critical temperature*, and the corresponding pressure the *critical pressure*. A vapor may be defined, also, as any gaseous substance at a temperature below its critical temperature.

For example, carbon dioxide at 31° and under a pressure of 73 atmospheres is in a critical state. Heated a little it certainly becomes gaseous; cooled a little it as certainly assumes the properties of a liquid, since it is far less compressible. But if the pressure be maintained, the transition from one to the other is hardly recognizable.

At a temperature *above* 31° no amount of pressure, however great, will liquefy the gas.

113. Ultragaseous or fourth state of matter. Air has been rarefied to the three-hundred-millionth of its normal density. But when gaseous matter is rarefied to even a millionth of the density of air at sea level, it exhibits extraordinary properties quite as different from the gaseous as this is from the liquid state, so that some are disposed to consider that there is an *ultragaseous* or *fourth state of matter*.

For the present views as to the structure of an atom, see § 109.

SECTION III

MOLECULAR FORCES

114. Molecular attractive forces. Many of the properties of matter are due to molecular forces, some of which now demand our attention. For convenience we call bodies of appreciable size *molar* (massive) bodies, or *masses*, in distinction from molecules (little masses). Action between molar bodies, usually at sensible distances, is called *molar force*: action between molecules, always at insensible distances, is called *molecular force*. According to the theory of the constitution of matter the molecules of every mass are in ceaseless motion, hitting and rebounding from one another. This tends to drive the molecules apart. In gaseous masses the molecules move without restraint; hence gaseous bodies always tend to expand.

In solids and liquids the molecules are held under the action of a very powerful attractive force called *cohesion*, which prevents their separation except under the action of considerable external force. It is the force which resists an effort tending to break, tear, or crush a body. The *tenacity* or *tensile strength* of solids and liquids, i.e. the resistance which they offer to being pulled apart, is due to this force. It is usually greater in solids than in liquids, and is entirely wanting in a true gas.

115. Strain; elasticity. *Strain* is a change of size or a change of shape. The former kind is called *compression* or *dilation*, and the resistance to it offered by the body is *incompressibility* or *elasticity of volume*. Any change-of-shape strain, such as flexion, torsion, and the like, is called *distortion*, and the resistance to it is called *rigidity* or *elasticity of figure*.

Frequently it is convenient to think of elasticity as qualitative rather than quantitative, as above. In this sense it is defined as that property in virtue of which a body tends to recover its size or shape after a strain. For example, in this sense air is perfectly elastic, though its *incompressibility* or quantitative elasticity is far less than that of most other substances. If the stress between the molecules in opposition to the distorting force continue constant, regardless of the time the strain is kept up, and restore the body to its normal condition immediately on the removal of the distorting force, without any permanent strain or "set," the body is said to be *perfectly elastic*. All fluids are perfectly elastic, and a few solids are approximately so, such as ivory, steel, and glass.

If a solid have little or no tendency to recover its size and shape after distortion, it is said to be *plastic* or *inelastic*. Such substances are putty, wet clay, and dough. A great number of substances are elastic when the distorting forces are small, but break or receive a "set" when these forces are too great. They are said to be elastic "within certain limits," called the *limits of elasticity*. If strained beyond those limits, they become more or less plastic. Hence the springs of a buggy sometimes become

set from bearing a too heavy load and lose permanently much of their elasticity ; i.e. they become to a degree plastic.

116. Viscosity. Support in a horizontal position, by one of its ends, a stick of sealing wax, and suspend from its free end a small mass, letting it remain in this condition several days, or perhaps weeks. At the end of the time the stick will be found permanently bent. Had an attempt been made to bend the stick quickly, it would have been found quite brittle.

It may seem like an abuse of the term to call sealing wax a fluid, yet the experiment shows it to be a fluid, or at least to possess fluidity, or freedom of motion of its molecules around one another, to a small degree. Resistance to deformation due to the friction of the molecules of a body in sliding over one another is called *viscosity.* Bodies that slowly suffer continuous and permanent deformation under the action of a continuous stress are said to be viscous. A lump of pitch in course of time loses its sharpness of outline and flows downhill of its own weight. It is very viscous. Cold molasses is quite viscous, but as its temperature is raised its viscosity diminishes, and it becomes more and more plastic or mobile. A perfectly rigid solid is one of infinite viscosity. A *perfect fluid* is a fluid which possesses no viscosity. Gases are viscous to some extent and are therefore *imperfect fluids.*

Bodies surrounded by air have on their surfaces an adherent film of air. When they move, this film rubs against the surrounding air, and thus movement is retarded by friction. To the viscosity of the air is due in part the retardation of the velocity of falling bodies. A penny and a piece of tissue paper fall with equal accelerations in a vacuum, but in the air the penny falls more rapidly because it presents less surface and therefore is retarded less, in proportion to its weight, by the friction of the air.

Falling water is retarded by the air; conversely, air is dragged down by falling water, as shown by the following experiment.

A (Fig. 77) is a long glass tube, funnel-shaped at one end, and having a little below this end a short branch tube *a*. Pour water

into the funnel and observe that it falls in a continuous stream until it reaches a, but below this point it is broken up by descending bubbles of air which it drags along with it after coming in contact with the air at a. If direct sunlight be allowed to strike the tube, the interior reflection from the water in this part of the tube is dazzling and beautiful.

FIG. 77

Connect with a by means of a rubber tube another short piece of glass tube b, and introduce the latter into a tumbler of water. The water descending in A exhausts the air in b, and the outside atmospheric pressure causes the water in the tumbler to ascend this tube b, and thus the tumbler may be emptied. If a closed vessel B be connected with a, it may be partially exhausted.

117. Hardness. *Hardness* is resistance to abrasion or scratching. To enable us to express degrees of hardness, the following table of reference is generally adopted:

MOHR'S SCALE OF HARDNESS

1. Talc	6. Orthoclase (feldspar)
2. Gypsum (or rock salt)	7. Quartz
3. Calcite	8. Topaz
4. Fluorspar	9. Corundum
5. Apatite	10. Diamond

By comparing a given substance with the substances in the table, its degree of hardness can be indicated approximately; thus, $H = 7$ means that the body is about as hard as quartz.

118. Malleability, ductility. Solids which possess that kind of fluidity which renders them susceptible of being rolled or hammered out into sheets are said to be *malleable*. Most metals are highly malleable. Gold may be hammered so thin as to be transparent, or to a thickness of one three-hundred-thousandth of

an inch. Most substances that are malleable are susceptible also of being drawn out into fine threads, e.g. wire of different metals. Such substances are said to be *ductile*. Platinum has been drawn into wire 0.000165 inch thick, or so fine as to be scarcely visible to the unaided eye.

SECTION IV

CAPILLARITY

119. Cohesion of liquids. Clean glass is wet by water. If a glass plate be dipped into water and then withdrawn, a layer of water clings to the glass. When the glass is withdrawn water is torn from water, and not glass from water. This shows that the attraction of the molecules of water for one another is weaker than the attraction between glass and water. Or if, to save words, we call the attraction between the solid and the liquid *adhesion*, then we may say that the *cohesion* between the molecules of the water is weaker than the *adhesion* between the glass and the water.

Clean glass is not wet by clean mercury, which shows that the adhesion between glass and mercury is not as great (about one third as great) as the cohesion in mercury. Generally speaking, a solid is wet by a liquid when the adhesion of the solid to the liquid is greater than the cohesion of the liquid, and is not wet when the cohesion is greater than the adhesion.

120. Surface tension. When a rubber band is strained or stretched it is said to be in a state of *tension*, and there exists between its molecules a contractile or resilient stress.

In liquids the molecules are within the limits of one another's attractions, which accounts for a greater or less viscosity or hindrance of flow and also for a certain phenomenon called *surface tension*. Every liquid behaves as if a thin film forming its external layer were in a state of tension, or were exerting a constant effort to contract.

It is not within the scope of this book to explain in full the dynamics of the molecular forces by which this result is brought

about; it must suffice to call the attention of the student to the peculiar condition, with reference to mutual attractions, of those molecules which compose the surface film. In the interior of a liquid each molecule is surrounded by other similar molecules, and the position which it assumes is that in which it is acted upon equally in all directions, and there is nothing to render the mutual attractions manifest. At a free surface, however, the molecules can be acted upon only by others lying internal to them. The result is a system of forces acting at right angles to the free surface of the liquid, and tending to reduce this surface to the least possible area. This tendency of a liquid surface to contraction means that it acts like an elastic membrane equally stretched in all directions, and by a constant tension. In the case of pure water at 20° C. this tension is about 81 dynes per linear centimeter.

Form a soap bubble at the orifice of the bowl of a tobacco pipe, and then, removing the mouth from the pipe, observe that the tension of the two surfaces (exterior and interior) of the bubble drives out the air from the interior, and that finally the bubble contracts to a flat sheet of minimum area.

As a consequence of surface tension, *every body of liquid tends to assume the spherical form*, since the sphere has less surface than any other form having equal volume. In large bodies the distorting forces due to gravity are generally sufficient to disguise the effect; but in small bodies, as in drops of water or mercury, it is apparent. Again, if the distorting effect of weight be eliminated in any way, as by immersing a quantity of oil in a mixture of water and alcohol of its own density, or by replacing the central portion of the body by a fluid much lighter than its own kind, as in the case of a soap bubble, the sphere is the resulting form.

121. Capillary phenomena. Surface tension is by no means peculiar to liquids. The surfaces of all bodies tend to contract. But since gases have no surfaces of their own, and the rigidity of solids prevents an alteration of shape, it is obvious why liquids

show the effects of surface tension most readily. But the surface tensions of solids and gases perform their part in determining certain phenomena. For example, if a glass rod be thrust vertically into water so as to leave a part projecting into the air, the surface of the water does not meet the rod at right angles, but is turned up so as to form a very small angle[1] with the surface of the glass, as ACB (Fig. 78). Here the three substances — water, glass, and air — are brought into contact, and there is a triplet of tensions in operation the resultant of which is a force which pulls the water up against the glass wall. On the other hand, if mercury, glass, and air be brought in contact, the relation between the triplet of forces becomes so changed as to cause the mercury to meet the glass at a very large angle, about 135°. It thus seems that when a solid, a liquid, and a gas are in contact, their boundary surfaces form *contact angles* with one another, determined by their relative surface tensions.

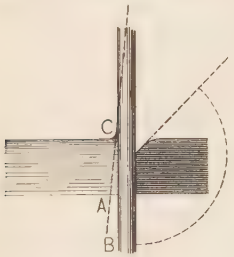


FIG. 78

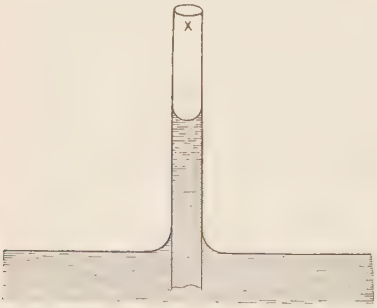


FIG. 79. Water

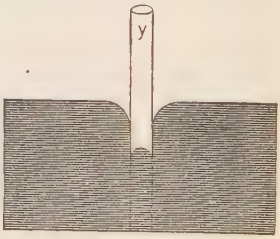


FIG. 80. Mercury

If a glass tube x (Fig. 79) of capillary (hairlike) bore be thrust into water, the water will rise in the bore considerably above the general level outside. If a similar tube y (Fig. 80) be thrust into mercury, the mercury within the bore will be

[1] If the glass be quite clean the angle is 0.

depressed below the surface outside. Phenomena of this kind are called *capillary phenomena*. The surfaces of the liquids inside the bores are curved, the surface of water being concave and that of mercury convex. The size of the bore of the tubes x and y is greatly exaggerated in order to show this more plainly. The concavity and convexity of these interior surfaces are a necessary consequence of the angles of contact with which these liquids meet glass. It remains only to explain the elevation and depression of the column of liquid in the tube. This may be done in part by analogy. Let AB (Fig. 81) represent a clothesline suspended slackly between two posts. From this line hang by strings small stones a, b, c, etc. If the hempen line become wet, as in a rain, it contracts and straightens, as shown by the dotted line AB; in other words, the contractile force which is exerted obliquely (e.g. nn, Fig. 81) is resolvable into two forces, one of which is horizontal and the other vertically upward; the latter tends to elevate the stones. In a similar manner the curved surfaces of water and mercury tend to contract and become flat. In the case of the water surface (which is concave) the contractile force tends to elevate the pendent liquid; but in the case of the mercury surface (which is convex) the tendency is to produce depression. On the nature of the curvature depends the direction in which the contractile force acts on the pendent liquid. Now it is evident that water will be drawn up by this contractile force until the weight of the column balances this force; and mercury will be depressed until the force is balanced by the pressure of the mercury outside the tube. Capillary phenomena are, therefore, phenomena of surface tension.

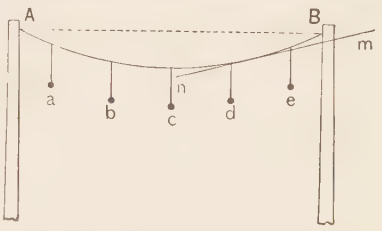


FIG. 81

The phenomena of capillary action are well shown by placing various liquids in U-shaped glass tubes having one arm reduced

to a capillary size, as A and B in Fig. 82. Mercury poured into A assumes convex surfaces in both arms, but does not rise as high in the small arm as it stands in the large arm. Pour water into B, and all the phenomena are reversed. Fig. 83 shows the forms that the surfaces of water and mercury take when contained in the same glass tube.

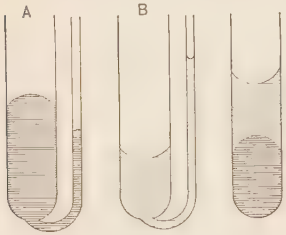


FIG. 82 FIG. 83

The following laws of capillary action may be verified by experiment:

(1) *Liquids rise in tubes when they wet them, and are depressed when they do not.*

(2) *The elevation or depression varies inversely as the diameter of the bore.*

(3) *The elevation and depression vary with the nature of the liquids employed, and with the substance of the tube.*

(4) *The elevation or depression varies inversely as the temperature.*

SECTION V

DIFFUSION OF FLUIDS

122. Other molecular phenomena. Besides the phenomena we have just studied, there are a great many others depending in part on molecular attraction, but much more on molecular motions, and which now we must consider more in detail.

The molecules of all bodies are constantly in a state of motion. The higher the temperature the greater are their velocities. In a solid "a molecule, though in continual motion, never gets beyond a certain very small distance from its original position in the body. In fluids there is no restriction to the excursions of a molecule. True the molecule travels only a very small distance before it encounters another molecule, but after this encounter there is nothing which determines the molecule rather to return

towards the place whence it came than to push its way into new regions. Hence in fluids the path of a molecule is not confined to a limited region, but may penetrate to any part of the space occupied by the fluid" (Maxwell).

123. Diffusion of liquids. Partially fill a glass jar (Fig. 84) with water. Then introduce beneath the water, by means of a long tunnel, a concentrated solution of sulphate of copper. The lighter liquid rests upon the heavier, and the line of separation between the two is at first distinctly marked. But in the course of days or weeks this line will gradually become obliterated; the heavier blue liquid will gradually rise, and the lighter, colorless liquid will descend, till they become thoroughly mixed.

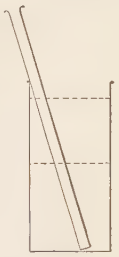


FIG. 84

Take about 1 cubic centimeter of bisulphide of carbon, color it by dropping into it a small particle of iodine, and pour this colored solution into a test tube nearly filled with water. The colored liquid, being heavier than the water, sinks directly to the bottom and shows no tendency to mix with the water. But in the course of time you discover that the colored liquid diminishes in quantity and finally disappears. The peculiar odor of this substance, which pervades the air in the vicinity, shows that a considerable portion has evaporated; but it must have worked its way gradually through the water above it.

If, in the last two experiments, you examine the liquid with a microscope during the operation, you will not be able to trace any currents; hence the motion of the liquids is not in mass, but by molecules, — a true intermolecular motion.

An intermingling of the molecules of two liquids caused by their own motions is called *diffusion of liquids.*

124. Diffusion of gases. Fill a test tube with oxygen gas and thrust into it a lighted splinter; the splinter burns much more rapidly than in the air. Fill another tube with hydrogen gas and keep the tube inverted (this gas being about 14.4 times as light as air, there will be no danger of its falling out). Thrust in

a lighted splinter; the gas takes fire and burns with a pale flame at the mouth of the tube. Next fill one tube with oxygen and the other with hydrogen gas, and place the mouth of the latter over the mouth of the former, as in Fig. 85. In about a minute apply a lighted splinter to the mouth of each tube (let the mouth of each tube be freely open to prevent accident); a slight explosion takes place in each instance. It is apparent that although the oxygen gas is many times as heavy as the hydrogen, some of it has risen into the upper tube, while some of the lighter hydrogen has descended into the lower tube, and the two gases have become diffused.

Fig. 85

There are liquids between which diffusion does not take place, but it does take place between any two gases whenever they are placed in contact.

In consequence of this universal tendency to diffusion, gases will not remain separated, — i.e. a lighter resting upon a heavier, as oil rests upon water. This is of immense importance in the economy of nature. The largest portion of our atmosphere consists of a mixture of oxygen and nitrogen gases. There are always present also small quantities of other gases, such as carbon dioxide, ammonia gas, and various others, which are generated by the decomposition of organic matter. These gases, obedient to gravity alone, would arrange themselves according to their weight, — carbonic-acid gas at the bottom, or next the earth, followed respectively by oxygen, nitrogen, ammonia, and other gases. Neither animal nor vegetable life could exist in this state of things. But in consequence of the diffusibility of these gases, they are found intimately mixed and in about the same relative proportions, whether in the valley or on the highest mountain peak.

125. Osmose. Certain liquids when separated from each other by membranous septa diffuse through these septa, and the diffusion may be more rapid than when no septa intervene. Such membranes as a bladder, cow's pericardium, and parchment paper are

especially suited to this purpose. Diffusion through septa is called *osmose*. There is a wide range of relative diffusibilities. At one end of the scale are such substances as solutions of urea, common salt, and such others as crystallize, and are called *crystalloids*. These diffuse rapidly. At the other extreme are such substances as starch, gum, albumin, gelatin and gluelike matter, called *colloids*. These diffuse very slowly. If a mixture composed of a crystalloid and a colloid be placed in a bladder, for instance, and the bladder be suspended in water, the crystalloid will diffuse rapidly through the septum while the colloid will diffuse very slowly. In this way a separation of the two substances may be effected. Separation by this process is called *dyalysis*.

FIG. 86

126. Osmose of gases. Seal the open end of a thin, unglazed earthen cup, such as is used in a Bunsen battery, with plaster of Paris, through which extends a glass tube. Place the exposed end of the tube in a cup of colored water. Lower a glass jar filled with hydrogen or coal gas over the porous cup, as in Fig. 86. Instantly air is forced down through the tube and escapes in bubbles from the colored liquid. The gas in the larger vessel forces its way through the pores of the cup, diffuses through the air contained in it, and causes an unusual pressure on the colored liquid, as is evident by the air that is forced out through it. In a minute remove the glass jar. The hydrogen now escapes through the sides of the cup, and mixes with the air on the outside; a partial vacuum is formed in the cup, and water rises in the tube. In each case air passes through the sides of the porous cup, but the influx and efflux of the hydrogen is much more rapid than that of the air.

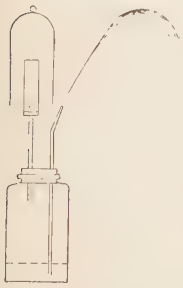


FIG. 87

An interesting modification of this apparatus is the *diffusion fountain* (Fig. 87). By passing the glass tube of the porous cup

through the cork of a tightly stoppered vessel, and having another glass tube pass through another perforation in the same cork, water is forced out in a jet several feet in height, when the hydrogen jar is held over the porous cup.

Children well understand that toy balloons, which are made of collodion and filled with coal gas, collapse in a few hours after they are inflated. This is caused by the escape of the gas by osmose. Nature furnishes an illustration of osmose of gases in respiration. In the lungs the blood is separated from the air by the thin, membranous walls of the veins. Carbon dioxide escapes from the blood through these septa, and oxygen gas enters the blood through the same septa.

The phenomena of diffusion both in liquids and gases furnish strong and tangible evidence that these bodies consist of molecules in a state of continual motion.

DYNAMICS OF FLUIDS

SECTION I

TRANSMISSION OF PRESSURE

127. Law of hydrostatic and pneumatic transmission of pressure. That branch of science which treats of liquids in a state of equilibrium or rest is called *hydrostatics;* that branch which treats of liquids in motion is called *hydrokinetics;* and that branch which treats of the dynamics of air and other gases is called *pneumatics.* With the exception of phenomena occasioned by difference in compressibility and expansibility, liquids and gases are subject to the same laws and may be treated together, in so far as they are alike under the common term *fluid.*

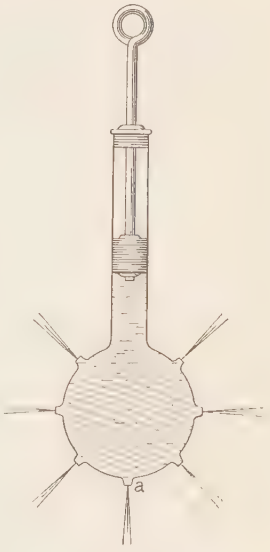


FIG. 88

Fill the globe and cylinder (Fig. 88) with water and thrust the piston into the cylinder. Jets of water will be thrown not only from that aperture *a* in the globe toward which the piston moves and the pressure is exerted, but from all the apertures.

If a finger be placed loosely over the end of a water faucet, spray will be thrown in all directions. It thus appears not only that external pressure is exerted upon that portion of the liquid that lies in the path of the force, but that it is transmitted equally to all parts and in all directions.

When pressure is exerted upon a solid, on account of its rigidity it is incapable of transmitting the pressure in other than the direction in which it is pressed. With fluids it is widely different. On account of the mobility of their molecules they are incapable of resisting a change of shape when acted upon by a force which is not equally applied over the whole surface of the body of fluid; hence any force applied to a fluid body must be transmitted by the fluid in every direction. Consequently every portion of the interior walls of the containing vessel with which the fluid is in contact is subjected to pressure.

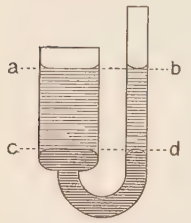


FIG. 89

Fig. 89 represents a U-tube with arms of quite unequal bore. We will suppose, for illustration, that the diameters are respectively 40 millimeters and 10 millimeters; then the ratio of the areas of the transverse sections of the bores will be $40^2 : 10^2 = 16$; that is, when the tube contains a liquid the area of the free surface of the liquid in the large arm will be sixteen times as great as of that in the small arm. Pour mercury into the tube until it stands about 1 centimeter above the bottom of the large arm, as *cd*. The mercury stands at the same level in both arms. Pour water upon the mercury in the large arm until this arm lacks only about 1 centimeter of being full. The pressure of the water causes the mercury to rise in the small arm and to be depressed in the large arm. Pour water very slowly into the small arm from a beaker having a narrow lip, until the surfaces of the water in the two arms are at the same level as *ab*. It is evident that the quantity of water in the large arm is sixteen times as great as that in the small arm.

This phenomenon appears paradoxical (*apparently* contrary to the natural course of things), until we master the important hydrostatic principle involved. We may best understand this by imagining the body of liquid in the large arm to be divided into cylindrical columns of liquid of the same size as that in the small arm. There will evidently be sixteen such columns. Then whatever

pressure is exerted on the mercury by the water in the small arm is transmitted by the mercury to each of the sixteen columns, so that each column receives an upward pressure or a supporting force equal to the weight of the water in the small arm.

The pressure exerted by a fluid upon the vessel containing it is normal to the walls of the vessel. Fluid pressure is expressed by stating the force exerted on a *unit area*, as 2 pounds per square inch, 5 grams per square centimeter, etc. The total pressure on any surface is the product of the pressure per unit area multiplied by the number of units of area.

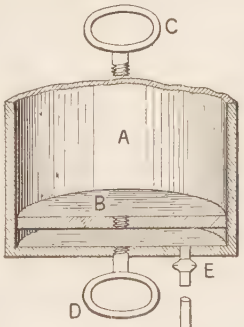


FIG. 90

Fig. 90 represents a section of a very useful demonstration apparatus. *A* is a hollow cylinder closed at one end. *B* is a nicely fitting piston which may be pushed into or drawn out of the cylinder by the handle *C* when screwed into the piston. *D* is another handle permanently connected with the closed end of the cylinder. *E* is a nipple opening into the space below the piston. To this may be attached a thick-walled rubber tube *F*. *G* is a stopcock or a funnel, either of which may be inserted at will into the free end of the tube.

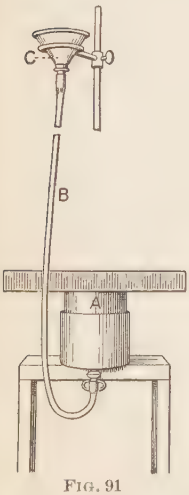


FIG. 91

Support the apparatus with the open end upward, force the piston in, place on it a block of wood *A* (Fig. 91), and on the block a heavy weight. Attach one end of the rubber tube *B* (12 feet long) to the apparatus and insert

a funnel C in the other end of the tube. Raise the latter end as high as practicable, and pour water into the tube. Explain how the few ounces of water standing in the tube can exert a pressure of many pounds on the piston, and cause it to rise together with the burden that is on it.

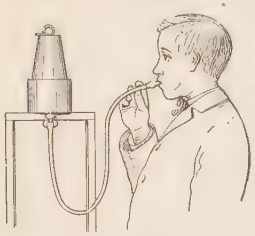


FIG. 92

Remove the water from the apparatus, place on the piston a 16-pound weight, and blow (Fig. 92) from the lungs into the apparatus. Notwithstanding that the actual pushing force exerted through the tube by the lungs probably does not exceed a few ounces, the slight increase of pressure caused thereby, when exerted upon the 26 square inches (about) of surface of the piston, causes it to rise together with its burden.

A pressure exerted on a fluid inclosed in a vessel is transmitted undiminished to every part of that vessel; and the total pressure exerted on the interior of the vessel is equal to the area multiplied by the pressure per unit of area.

128. The hydraulic press. Closely allied to the experimental apparatus just described is the *hydraulic press*. It contains two pistons, t and s (Fig. 93). The area of the lower surface of t is, say, one hundred times that of the lower surface of s. As the piston s is raised and depressed, water is pumped up from the cistern A, is forced into the cylinder x, and exerts an upward pressure against the piston t one hundred times greater than the downward pressure exerted upon s. Thus, if a pressure of one hundred pounds is applied at s, the cotton bales will be subjected to a pressure of five tons.

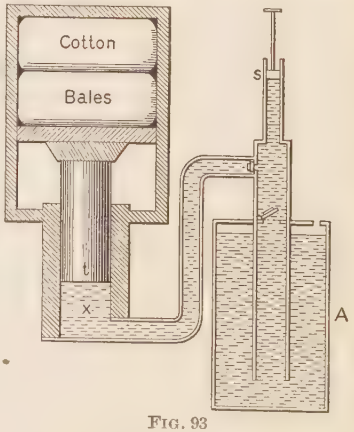


FIG. 93

The pressure that may be exerted by these presses is enormous. By means of them the hand of a child can break a strong iron bar. But observe that although the pressure exerted is very great, the upward movement of the piston t is very slow. In order that the piston t may rise 1 centimeter, the piston s must descend 100 centimeters. The disadvantage arising from slowness of operation is insignificant, however, when we consider the great advantage accruing from the fact that one man can produce as great a pressure with the press as a hundred men can exert without it.

The press is used for compressing cotton, hay, etc., into bales, and for extracting oil from seeds. The modern engineer finds it a most efficient machine whenever great resistances are to be moved.

129. Pressure of fluid due to its weight. Fluids exert pressure due to their weight. Imagine a vessel filled with shot; you will understand that the upper layer of shot will press upon the layer next beneath with a force equal to its weight, the second upon the third with a force equal to the sum of the weights of the first two, and so on. You will also readily conclude that the pressure exerted upon the successive layers will be exactly *proportional to their depths*, unless in consequence of the great pressure to which the lowest layers are subjected there should be a crowding together of the shot so as to make them more compact. In this case there would be a slight variation from the rule as stated. For a like reason the downward pressure in a body of liquid increases *as its depth*, except in so far as the pressure is modified in consequence of the compressibility of liquids. Liquids are, however, so slightly compressible that any variation in consequence of the compression is usually neglected, and the principle is stated in general that *pressure at any point in a liquid varies as its depth.*

Since the shot possess a certain degree of mobility or freedom of motion around one another, their weight will cause to some extent a lateral pressure against one another and against the walls of the containing vessel. In consequence of the *extreme* mobility of the molecules of fluids the downward pressure due to gravitation

at any point in a fluid gives rise to an *equal* pressure at that point in *all* directions. Hence the so-called *Pascal's principle: At any point in a fluid at rest the pressure is equal in all directions.*

Thus, let *a*, *b*, *c*, etc. (Fig. 94), represent imaginary surfaces, and the arrowheads the direction of pressure exerted at points in these surfaces at equal depths in a liquid. The pressures exerted at these several points are equal.

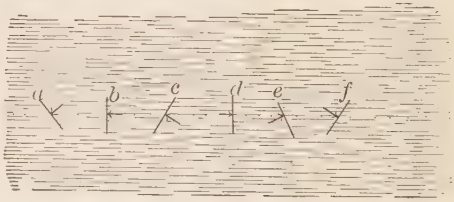


FIG. 94

The truth of this principle is obvious, for if there be any inequality of pressure at any point, the unbalanced force will cause particles at that point to move, which is contrary to the supposition that the fluid is *at rest*. Conversely, when there is motion in a body of fluid it is evidence of an inequality of pressure.

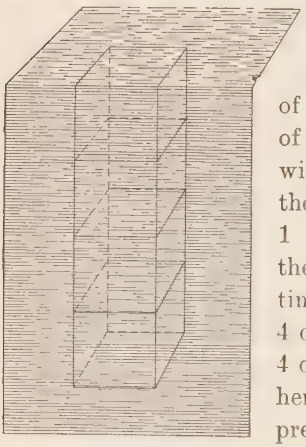


FIG. 95

130. Methods of calculating liquid pressure. Conceive of a square prism of water (Fig. 95) in the midst of a body of water, its upper surface coinciding with the free surface of the liquid. Let the prism be 4 centimeters deep and 1 centimeter square at the end; then the area of one of its ends is 1 square centimeter and the volume of the prism is 4 cubic centimeters. Now the weight of 4 cubic centimeters of water is 4 grams; hence this prism must exert a downward pressure of 4 grams upon an area of 1 square centimeter. But at the same depth the pressure in all directions is the same; hence, generally, the pressure at any depth in water may be taken approximately as one gram per square centimeter for each centimeter of depth

($\doteq$ 1000 kilograms per square meter for each meter of depth; or, since the weight of water is about 62.3 pounds per cubic foot, the pressure is 62.3 pounds per square foot for each foot of depth). In any other liquid, to determine the pressure at any depth the water pressure at the given depth must be multiplied by the specific gravity (§ 153) of the liquid.

FIG. 96

The conclusions arrived at may be summarized as follows: *In a mass of liquid at rest the pressure is the same at all points in any horizontal plane, and is equal to the weight of a column of the liquid one square centimeter in section extending vertically from the horizontal plane to the upper surface of the liquid.*

A and *B* (Fig. 96) are two bottomless vessels which can be alternately screwed to a supporting ring *C* (Fig. 97). The ring is itself fastened by means of a clamp to the rim of a wooden water pail. A circular disk of metal *D* is supported by a rod connected with one arm of the balance beam *E*. When the weight *F* is applied to the other arm of the beam, the disk *D* is drawn up against the ring so as to supply a bottom for the vessel above. Take first the vessel *A*, screw it to the ring, and apply the weight to the beam, as in Fig. 99. Pour water slowly into the vessel and move the index *a* up the rod, keeping it just at the surface of the water, until the downward pressure of the water upon the bottom tilts the beam, pushes the bottom down from the ring, and allows some of the water to fall into the pail. Remove vessel *A* and attach *B* to the ring, as in Fig. 98. Pour water into vessel *B* as before; when the surface of the water reaches the index *a* the bottom is again forced off. That is, *the pressures upon the*

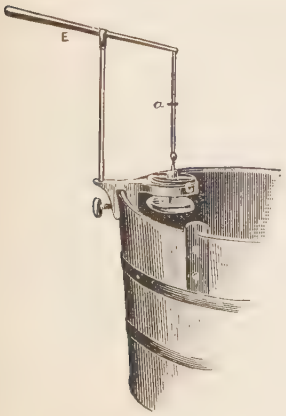


FIG. 97

bottoms of all vessels (of whatever capacity or shape) are the same, provided the bottoms be of the same area and the depth and density of the liquid be the same.

131. Rules for calculating liquid pressure. *The total pressure due to gravity on any portion of the horizontal bottom of a vessel containing a liquid is equal to the weight of a column of the same liquid whose base is the area of that portion of the bottom pressed upon, and whose height is the depth of the water in the vessel.* Thus, suppose that we have three vessels having bottoms of the same size: one of them has flaring sides, like a wash basin; another has cylindrical sides; and the third has conical sides, like a coffeepot. If the three vessels be filled with water to the same depth, the total pressure upon the bottom of each will be equal to the weight of the water in the vessel of cylindrical shape. Suppose that the area of the bottom of each is 108 square inches, and that the depth of water is 16 inches, then the cubical contents of the water in the cylindrical vessel is 1728 cubic inches, or 1 cubic foot. The weight of 1 cubic foot of water is 62.3 pounds. Hence the total pressure upon the bottom of each vessel is 62.3 pounds.

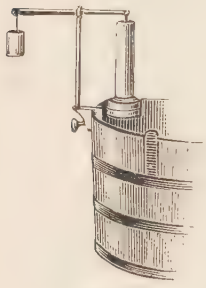


FIG. 98

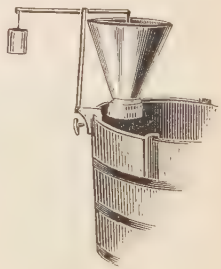


FIG. 99

Evidently, the lateral pressure at any point of the side of a vessel depends upon the depth of that point; and, as depth at different points of a side varies, *to find the total pressure upon any portion of a side of a vessel, find the weight of a column of liquid whose base is the area of that portion of the side, and whose height is the average depth of that portion.*

132. The surface of a liquid at rest is level. By jolting a vessel the surface of a liquid in it may be made to assume some such

form as that seen in Fig. 100. Can it retain this form? Take two molecules of the liquid at the points *a* and *b* in the same horizontal plane. The total downward pressures upon *a* and *b* are as their respective depths *ca* and *db*. But since (assuming the mass of liquid to be at rest) the pressure at a given depth is equal in all directions, *ca* and *db* represent the lateral pressures at the points *a* and *b* respectively. But *db* is greater than *ca*; hence the molecules *a* and *b*, and those lying in a straight line between them, are acted upon by two unequal forces in opposite directions. Hence the liquid cannot be at rest in the position assumed, and there will therefore be a movement of molecules in the direction of the greater force (toward *a*) till there is equilibrium of forces, which will occur only when the points *a* and *b* are equally distant from the surface; or, in other words, *there will be no rest till all points in the surface are on the same level.*

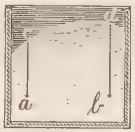


FIG. 100

This fact is commonly expressed thus: "Water always seeks its lowest level." In accordance with this principle water flows down an inclined plane and will not remain heaped up. An illustration of the application of this principle, on a large scale,

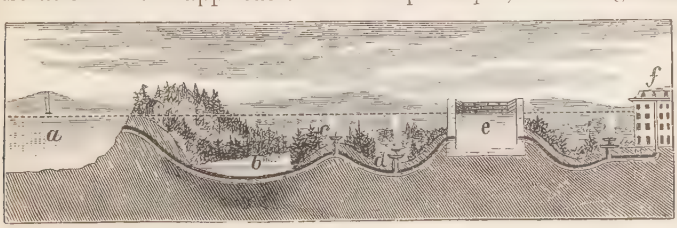


FIG. 101

is found in the method of supplying cities with water. Fig. 101 represents a modern aqueduct through which water is conveyed from an elevated pond or river *a*, beneath a river *b*, over a hill *c*, through a valley *d*, to a reservoir *e*, from which it is distributed by service pipes to the dwellings in a city. The aqueduct is tapped at different points, and fountains would rise to the level

of the water in the pond were it not for the resistance of the air and the check which the ascending stream receives from the falling drops. Where should the pipes be made stronger, on a hill or in a valley? Where will water issue from faucets with greater speed, in a chamber or in a basement? How high may water be drawn from the pipe in the house f?

133. Artesian wells. In most places the crust of the earth is composed of distinct layers of earth and rock of various kinds. These layers frequently assume concave shapes, so as to resemble cups placed one within another. Fig. 102 represents a vertical

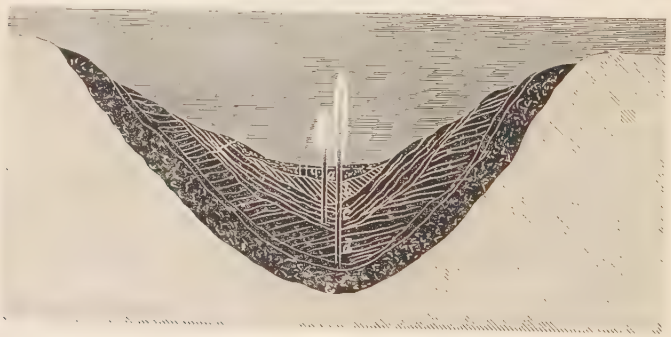


FIG. 102

section exposing a few of the surface layers of the earth's crust: a is a stratum of loose sand or gravel; b, a clay bed; c, a stratum of slate; d, a stratum of limestone; the whole resting on a bed of granite e. If you hollow out a lump of clay and pour water into the cavity, you will find that the water will percolate through the clay very slowly. Water that falls in rain passes readily through the gravel a till it reaches the clay bed b, where it collects. Hence a *well* sunk to the clay bed will fill with water as high as the water stands above the clay. Water also works its way from elevated places down between the strata of rocks. If a hole be bored through the slate c, water will rise above the surface of the ground in a fountain, seeking the level of its source on the hill; and if bored still lower, through the stratum d, a still higher

fountain may result. Such borings are called *artesian wells*. Water frequently forces its way through fissures in the rocky strata to the surface, as at t, and gives rise to *springs*.

QUESTIONS AND PROBLEMS

1. The areas of the bottoms of vessels A, B, and C (Fig. 103) are equal. The vessels have the same depth, and are filled with water. Which vessel contains the most water? On the bottom of which vessel is the pressure equal to the weight of the water which it contains? How does the pressure upon the bottom of vessel B compare with the weight of the water in it?

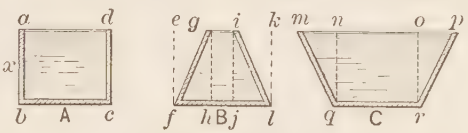


FIG. 103

2. A cubic foot of water weighs about 62.3 lb., or 1000 oz. Suppose that the area of the bottom of each vessel is 100 sq. in. and the depth is 10 in. What is the pressure on the bottom of each?

3. Vessel A is a cubical vessel. What is the total pressure against one of its vertical sides?

4. Suppose that vessel A were tightly covered, and that a tube 10 ft. long were passed through a perforation in the cover so that the end should just touch the upper surface of the water in the vessel; then suppose the tube to be filled with water. What additional pressure would each wall of the cube sustain?

5. Suppose that the area of the end of the large piston of a hydraulic press is 100 sq. in. What should be the area of the end of the small piston that a force of 100 lb. applied to it may produce a pressure of 2 tons?

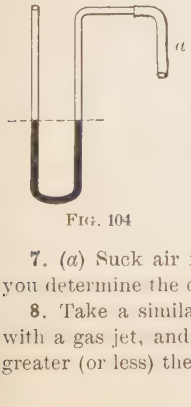


FIG. 104

6. Take a glass U-tube (Fig. 104) about 40 in. high, having a stout rubber tube a attached, and containing mercury with the surfaces at the same level in both arms. (*a*) Blow into the tube; the surfaces of mercury will at once assume different levels. How will you determine the pressure which you exert upon the mercury through the air in the tube (the *specific gravity* of mercury being 13.6)?

7. (*a*) Suck air from a. What happens to the mercury? (*b*) How will you determine the diminution of pressure which you produce by suction?

8. Take a similar tube containing water instead of mercury, connect it with a gas jet, and turn on the gas. How would you determine how much greater (or less) the gas pressure is than that of the atmosphere?

9. How great is the hydrostatic pressure in fresh water at the depth of 50 ft.?

10. Why does not a person who dives to the bottom of a pond feel the weight of the column of water above him?

11. Why does not the weight of the greater quantity of liquid in a coffee-pot when filled cause the liquid to rise higher in the spout than the surface of the liquid in the pot?

12. (*a*) A house is supplied with water by a system of pipes from a distant reservoir, as is customary in cities. What data would you require in order to compute the pressure at any point in the pipe? (*b*) How much greater is the pressure at a point in the pipe in the cellar than at another point in the attic? (*c*) Is the pressure in the pipe the same when water is running from a faucet in the house as when the water is at rest? Why?

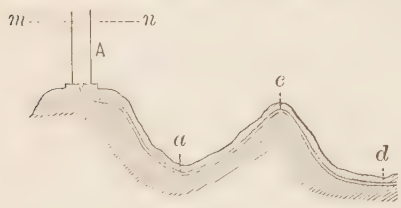


FIG. 105

13. *A* (Fig. 105) represents a standpipe for furnishing the neighboring district with water by the action of gravity. The standpipe is supplied with water from a lake in the vicinity by means of a pumping engine. Vertical distances are represented on a scale of $\frac{1}{4}$ in. = 50 ft. If the standpipe be filled to the level *mn* and the water be at rest in the main pipe leading from it, what pressure is the pipe sustaining at points *a*, *c*, and *d* respectively?

SECTION II

ATMOSPHERIC PRESSURE

134. Introduction. We live at the bottom of a rare and elastic aerial ocean, called the *atmosphere*, extending an undetermined distance into space. Every molecule in the gaseous ocean is drawn towards the earth's center by gravitation, and the atmosphere is thus bound to the earth by this force, just as is the liquid ocean. Evidently the pressure in the atmosphere due to its weight increases with the depth; or, since in our position we are more accustomed to speak of *height* in the atmosphere, decreases with the height. The pressure does not diminish regularly with the height, as in an ocean of incompressible fluid. Air is very

compressible and therefore varies in density. The lower strata of air sustaining the weight of air above are relatively much compressed, more dense, and elastic. The density and elasticity of the air diminish more rapidly than the height above sea level increases. Owing to this fact, the greater part of the mass of the atmosphere is within three and a half miles of sea level. Above this height the air is much rarefied and vanishes, as it were, very gradually into empty space.

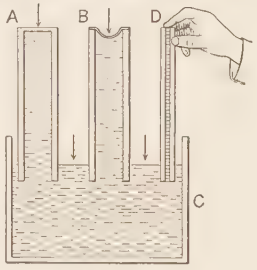


FIG. 106

Fill two glass jars (Fig. 106) with water, *A* having a glass bottom, and *B* a bottom provided by tying a piece of sheet rubber tightly over the rim. Invert both in a larger vessel of water *C*. The water in *A* does not feel the downward pressure of the air directly above it, the pressure being sustained by the rigid glass bottom; but it indirectly feels the pressure of the air on the surface of the water in the open vessel, and it is this pressure that sustains the water in the jar. But the rubber bottom of the jar *B* yields somewhat to the downward pressure of the air, and is forced inward.

FIG. 107

Fill a glass tube *D* with water, keeping the lower end in a vessel of water, and the upper end tightly closed with a finger. Why does not the water in the tube fall? Remove your finger from the closed end. Why does the water fall?

Fill (or partly fill) a tumbler with water, cover the top closely with a card or writing paper, hold the paper in place with the palm of the hand, and quickly invert the tumbler (Fig. 107). If the experiment be carefully done, the water will not fall out? Why?

135. Measurement of atmospheric pressure. Fig. 108 represents a U-shaped glass tube (closed at one end) about 86 centimeters in height, and with a bore of about 1 square centimeter section. The closed arm having been filled with mercury, the tube is placed with its open end upward, as in the figure. The mercury in the closed arm sinks about 5 centimeters to A, and rises 5 centimeters in the open arm to C; but the surface A is 76 centimeters higher than the surface C. This can be accounted for only by the atmospheric pressure. The column of mercury BA, containing 76 cubic centimeters, is an exact counterpoise for a column of air of the same diameter extending from C to the upper limit of the atmospheric ocean, — an unknown height.

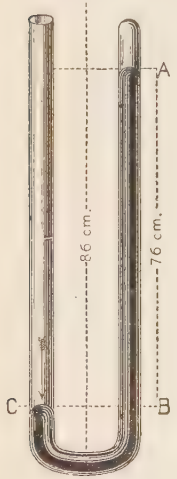


FIG. 108

The weight of the 76 cubic centimeters of mercury in the column BA is about 1033 grams. Hence the weight of a column of air of 1 square centimeter section, extending from sea level to the upper limit of the atmosphere, is about 1033 grams. But in fluids gravity causes equal pressure in all directions. Hence, *at the level of the sea all bodies are pressed upon in all directions by the atmosphere with a force of about 1033 grams per square centimeter.*

If this be reduced to the English system, it will be found to be equivalent to about 14.7 pounds per square inch, and a pressure of 15 pounds per square inch is quite generally adopted by engineers as a unit of gaseous pressure, called an *atmosphere*. Physicists, however, generally measure pressure in terms of *centimeters* or *millimeters* of mercury at 0° C.; that is, the height in *millimeters* of mercury that the pressure to be measured sustains in the tube.

136. The barometer. The height of the column of mercury supported by atmospheric pressure is quite independent of the area of the surface of the mercury pressed upon; hence the

apparatus is more conveniently constructed in the form represented in Fig. 109.

A straight tube about 86 centimeters long is closed at one end and filled with mercury. The tube is inverted, with its open end tightly covered with a finger, and this end is inserted into a vessel of mercury. When the finger is withdrawn the mercury sinks until there is equilibrium between the downward pressure of the mercurial column *AB* and the pressure of the atmosphere. The empty space at the top of the tube is called a *Torricellian*[1] *vacuum*. An apparatus designed to measure atmospheric pressure is called a *barometer* (pressure measurer). Beside the tube and near its top is a scale graduated in inches or centimeters, indicating the height of the mercurial column. For ordinary purposes this scale needs to have a range of only six or eight centimeters, so as to include the maximum fluctuations of the column.

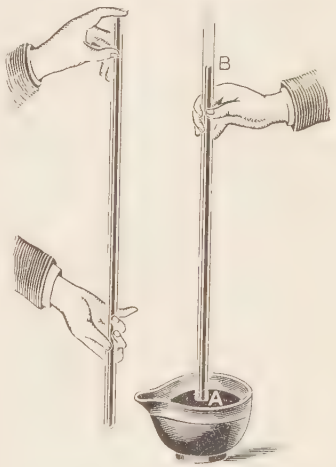


FIG. 109

137. The Fortin barometer. We will suppose the scale of a barometer to be fixed so as to indicate correctly the height of the surface of mercury in the tube above that in the cistern at a time, for instance, when this distance is 30 inches. A point on the surface of the mercury in the cistern in this case is called technically the *zero point*. Now should the mercury fall in the tube to 29 and the mercury in the cistern remain at zero, then the scale reading would indicate correctly the barometric height; but the mercury does not remain at zero, but rises a little (less as the diameter of the cistern is greater), and consequently the scale reading is too great. When the mercury is higher in the tube

[1] The first barometer was constructed by Torricelli, a Florentine, in 1643.

than 30 all the readings will be too small. Evidently, then, the mercury in the cistern must be brought to zero at every observation in order to eliminate this error. This is easily accomplished with the Fortin barometer. The bottom of the cistern of this barometer is pliable leather resting on a thumbscrew A (Fig. 110). Projecting from the tube inside of the cistern is a little pointer B of colored glass or ivory. The lower end of this pointer, called the *fiducial point*, corresponds to the zero point. The level of the mercury in the cistern must be set to this point by raising or lowering the cistern base by the adjusting screw before taking a reading. A sliding piece C (Fig. 111), furnished with a vernier,[1] can be slid along the tube so as to enable one to read with great accuracy the top of the column.

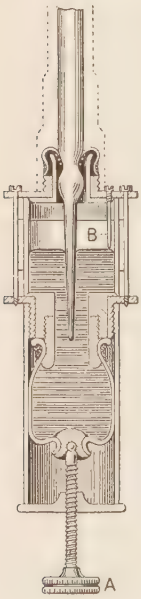


FIG. 110

In refined scientific researches it is necessary to make suitable allowances for expansion and contraction of the mercury and of the scale attending changes in temperature; hence a very sensitive thermometer is attached to the barometer. Allowances for capillary depression must also be made.

FIG. 111

138. The aneroid barometer. The *aneroid* (without moisture) barometer employs no liquid. It contains a cylindrical box D (Fig. 112), having a very flexible top. The air is partially exhausted from within the box. The varying atmospheric pressure causes this top to rise and sink much like the chest of a man in breathing. Slight movements of this kind are communicated to the index needle A by means of multiplying apparatus, such that a small movement of one part causes a large movement of another part. The dial is graduated

[1] For construction and methods of using verniers the student is referred to one of the many books on practical physics.

to correspond with a mercurial barometer. The observer turns the button C and brings the brass needle B over the black needle A, and at his next observation any departure of the latter from the former will show precisely the change which has occurred between the observations.

The aneroid can be made more sensitive, i.e. so as to show smaller changes of atmospheric pressure, than the mercurial barometer. Owing to this as well as to its convenient size and portability, the aneroid has become quite popular. Unfortunately, however, it does not preserve its accuracy for a great length of time, and hence must be adjusted from time to time to a standard mercurial barometer.

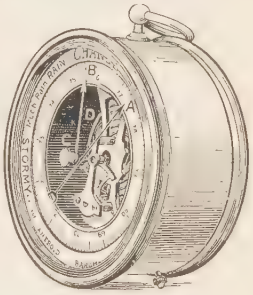


FIG. 112

139. Standard pressure. Many physical operations require a standard pressure for reference. The standard generally adopted is the pressure exerted by a column of pure mercury at 0° C. and 76 centimeters (29.922 inches) high, which is about the average height of the barometric column at sea level in latitude 45°. As we have seen, the pressure corresponding to this height is 1033 grams per square centimeter or 14.7 pounds per square inch.

140. Barometric measurement of heights. Since atmospheric pressure varies with the height above sea level, it is evident that changes in elevation may be determined from changes of pressure as indicated by the barometer. In other words, the height of a mountain may be ascertained from barometric readings made on the summit and at sea level. Such determinations are more reliable for moderate elevations, as there are elements of greater or less uncertainty in measuring great heights. For moderate heights the barometric column falls at the very nearly uniform rate of one inch for every 900 feet of ascent.

If a mercurial barometer stand at 760 millimeters on the floor, the same barometer on the top of a table 1 meter high should

stand at a height of 759.91 millimeters, a change scarcely perceptible. The aneroid is, however, sometimes made so sensitive that the change of pressure experienced in this short distance is rendered quite perceptible.

The shading in Fig. 113 is intended to indicate roughly the variation in the density of the air at different elevations above sea level. The figures in the left margin show the height in miles; those in the first column on the right, the corresponding average height of the mercurial column in inches; and those in the extreme right, the density of the air compared with its density at sea level.

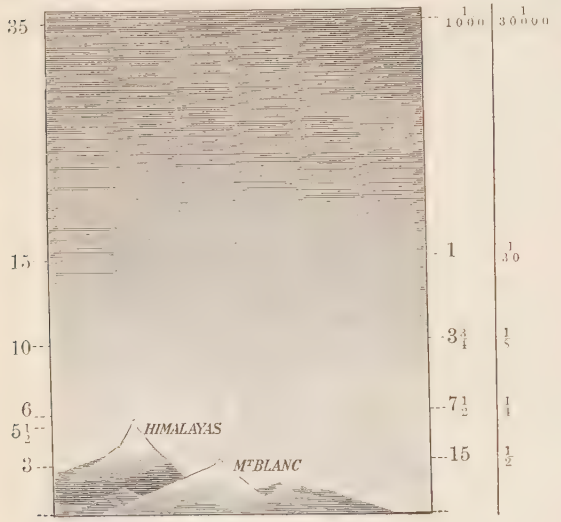


FIG. 113

It is calculated that if an opening could be made in the earth 35 miles below sea level, the density of the air at the bottom would be 1000 times that at sea level, so that water would float in it.

If the aërial ocean were of uniform density, and of the same density that it is at sea level, its depth would be a little less than 5 miles. Only a few peaks of the Himalayas would rise above it.

141. The barometer in meteorology. The barometer is sometimes called a "weather glass," chiefly because its scale frequently bears the words *fair*, *rainy*, *storm*, etc. These words are very objectionable, since they are totally wrong from a meteorological

point of view. To form a good forecast of the weather a barometer, a thermometer, and a hygrometer must be consulted, and one must be familiar with the laws which govern the relations between atmospheric pressure, temperature, moisture, etc. In forming a judgment of forthcoming weather, the point at which the mercury stands should not be so much regarded as the fact whether it is rising or falling. The following general rules may not be amiss, though even these have many exceptions:

(1) A steady barometer at about its mean height, with a seasonable temperature and dry air, indicates a continuation of fine weather.

(2) A *rise* from this point indicates decidedly fine and dry weather; a *fall* indicates rain or higher wind.

(3) A gradual rise or fall of the mercury indicates a less immediate change than a more rapid motion.

Fluctuations in barometric pressure are of hourly occurrence. Some of the many conditions which influence the atmospheric pressure are the following: (1) *Temperature.* A rise of temperature tends to diminish the air pressure. In general, the barometer falls as the thermometer rises. Heat expands the air, causing a lateral flow away from the heated areas. (2) *Humidity.* Moist air is lighter than dry air having an equal pressure. (3) *Currents in the atmospheric ocean.* On weather charts, lines called *isobars* are drawn through places having the same pressure. Frequently the isobar indicating the lowest pressure incloses an area more or less circular. This is called a "center of depression." The surrounding air tends to flow into it from all sides. The greater the difference of pressure between the two places, i.e. the steeper the barometric gradient, the more rapid the flow. The direction of the wind, however, is so modified by the rotation of the earth that the flow is not directly toward the center, but spirally toward it, the motion in the northern hemisphere being opposite to the direction in which the hands of a watch move.

SECTION III

RELATION BETWEEN THE DENSITY, VOLUME, AND PRESSURE OF A BODY OF GAS

142. Boyle's (or Mariotte's) law. Fig. 114 represents a bent glass tube the short arm of which is closed; the long arm, which should be at least 34 inches (85 centimeters) long, is open at the top. Pour mercury into the tube till the surfaces in the two arms stand at zero. Now the surface in the long arm supports the pressure of an atmosphere; therefore the pressure of the air inclosed in the short arm, which exactly balances this, must be about 15 pounds to the square inch. Next pour mercury into the long arm till the surface in the short arm reaches 5, or till the volume of air inclosed is reduced one half, when it will be found that the height of the column AC is just equal to the height of the barometric column at the time the experiment is performed. It now appears that the pressure of the air in AB balances the atmospheric pressure *plus* a column of mercury AC which is equal to another atmosphere; therefore the pressure of the air in AB equals two atmospheres. But the air has been compressed into half the space it formerly occupied, and is consequently twice as dense. If the length and strength of the tube would admit of a column of mercury above the surface in the short arm equal to twice AC, the air would be compressed into one third its original bulk; and, inasmuch as it would balance a pressure of three atmospheres, its pressure would be increased threefold.

FIG. 114

This experiment may be conducted in a more scientific manner, as follows:

Let the mercury be at the same level AB (Fig. 115) in both arms of the tube. The body of air to be experimented with is in the short arm between A and C. The dimensions of this body can vary only in height; hence its height H may represent its volume. Measure H, i.e. the distance between A and C, and regard the number of centimeters as representing the volume V. Its pressure P evidently is the same as that of the atmosphere at the time of experimenting. Consult a barometer and ascertain the height of the barometric column; let this height represent P. Pour a little mercury into the tube; the mercury rises to A_1 and B_1. Measure from A_1 to C; this number represents the volume V_1 of the body of gas now. Measure the vertical distance between A_1 and B_1; this number represents the increase in pressure, which, added to P, will give its present pressure P_1.

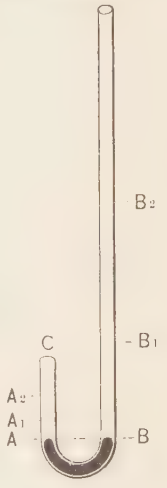


FIG. 115

Now pour more mercury into the long arm, so that it will rise to some such points as A_2 and B_2. Determine as before the new volume V_2 and new pressure P_2. So continue to add mercury a third and a fourth time, and get new values for the volume V_3 and V_4, and for the pressure P_3 and P_4. Arrange the results as follows:

$V =$	$P =$	$V \times P =$
$V_1 =$	$P_1 =$	$V_1 \times P_1 =$
$V_2 =$	$P_2 =$	$V_2 \times P_2 =$
etc.	etc.	etc.

It will be found that the series of products in the last column are approximately equal (due allowance being made for errors in measurement, etc.); consequently P varies inversely as V. Hence the law:

The volume of a given mass of gas at a constant temperature varies inversely as its pressure.

For many years after the announcement of this law, first by Boyle and a little later by Mariotte, it was believed to be rigorously correct for all gases; but more recently, more precise experiments have shown that it is approximately but not rigidly true for any gas, that the departure from the law differs with different gases, and that *each gas possesses a special law of compressibility.* There is a limit beyond which this law does not hold. This limit is soonest reached with those gases, like carbon dioxide, chlorine, etc., that are most readily liquefied by pressure. A gas is nearer perfect, or conforms more nearly to Boyle's law, in proportion as it is at a greater distance, as regards both temperature and pressure, from its liquefying point. When a gas is near the critical state its density increases more rapidly than its elasticity.

143. Manometer, or pressure gauge. The manometer, an instrument for measuring the pressure of a gas or vapor contained in a closed vessel under considerable pressure, illustrates the application of Boyle's law to a practical purpose.

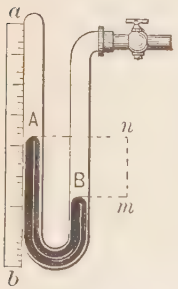


FIG. 116

This instrument consists, as in Fig. 116, of a bent tube AB closed at one end a, and containing within the space Aa a quantity of air, which is cut off from external communication by a column of mercury. The apparatus is so constructed that when the pressure on B is equal to that of an atmosphere, the mercury stands at the same height in both branches. But if the pressure increase, the mercury is forced into the left branch, so that the air in that branch is compressed and its elasticity proportionately increased. The pressure of the gas exerted at B is then equal to the pressure of the compressed air, together with that of a column of mercury mn equal to the difference of level of the liquid in the two branches. This pressure is expressed in *atmospheres* on the scale ab.

144. Elasticity of gases. *The elasticity of all fluids is perfect.* By this is meant that the force exerted in expansion is, except

within certain limits rarely reached, equal to the force used in compression; and that, however much a fluid is compressed, it will always completely regain its former bulk when the pressure is removed. Hence the barometer, which measures the compressing force of the atmosphere, also measures at the same time the elasticity of the air. A so-called "vacuum gauge" (Fig. 117) is simply a short mercury barometer, — short because it is seldom necessary to make measurements except in tolerably high vacua, where the mercurial column is correspondingly low. For instance, this apparatus placed under the receiver of an air pump from which air is exhausted will measure the elasticity of the air in the receiver. This known, the degree of exhaustion is readily determined.

Fig. 117

Fig. 118

Take a glass tube (Fig. 118) having a bulb blown at one end. Nearly fill this tube with water, so that when it is inverted there will be only a bubble of air in the bulb. Insert the open end in a glass of water, place under a receiver, and exhaust. Nearly all the water will leave the bulb and tube. Why? What will happen when air is admitted to the receiver?

145. Suction. Liquids are said to be raised by the "force of suction," which seems to imply that a lifting force acting from above *pulls* the liquid up. Does this explain suction?

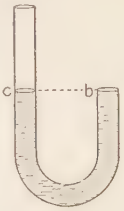


Fig. 119

Fill a glass U-tube, having unequal arms (Fig. 119), with water to the level *cb*. Close the end *b* with a finger and try to suck the liquid out of the tube. You find it impossible. Remove the finger from *b*, and you can suck the liquid out with ease. Why?

SECTION IV

INSTRUMENTS USED FOR RAREFYING AIR

146. The air pump. The air pump is used to rarefy air in a closed vessel. Fig. 120 will serve to illustrate its operation. R is a glass *receiver* within which the air is to be rarefied; B is a hollow cylinder of brass, called the *pump barrel;* the plug P, called a *piston*, is fitted to the interior of the barrel, and can be moved up and down by the handle H; s and t are valves. A valve acts on the principle of a door intended to open or close a passage. If you walk against an ordinary door on one side, it opens and allows you to pass; but if you walk against it on the other side, it closes the passage and stops your progress. Suppose the piston to be in the act of descending; the compression of the air in B closes the valve t and opens the valve s, and the inclosed air escapes. After the piston reaches the bottom of the barrel it begins its ascent. This would cause a vacuum between the bottom of the barrel and the ascending piston (since the unbalanced pressure of the outside air immediately closes the valve s), but the pressure of the air in the receiver R opens the valve t and the air flows through, filling this space. As the air in R expands it becomes rarefied and exerts less pressure. The external pressure of the air on R, being no longer balanced by the pressure of the air within, presses the receiver firmly upon the plate L. Each repetition of a double stroke of the piston removes a portion of the air remaining

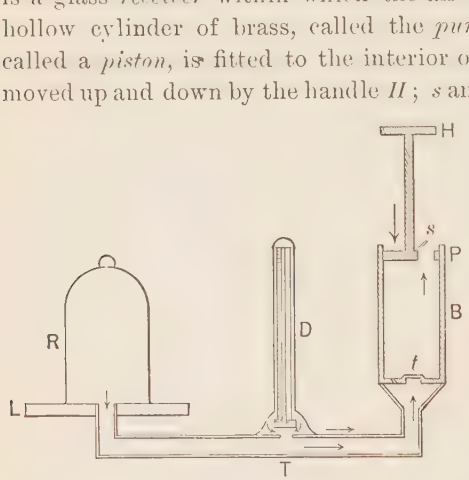


FIG. 120

in R. The air is removed from R by its own expansion. D is a vacuum gauge (§ 144). The receiver will always be filled with air, although it may be exceedingly rarefied, and the operation of exhaustion is practically ended when the pressure of the air in R becomes too feeble to lift the valve t, unless the apparatus be so constructed that the valves are opened and closed by mechanical action. It is obvious that if s and t opened downward instead of upward, then as the piston is raised and depressed, air would be compressed in R. A *condenser* is merely a pump with its valves reversed, and is used to condense air.

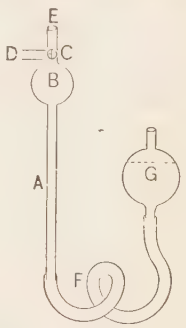


FIG. 121

147. The mercury air pump. In recent years the so-called mercury air pump has largely displaced the pump described above, since it is capable of producing a much greater rarefaction. In brief, it makes use of the Torricellian vacuum, such as is formed in the top of a barometer tube. On account of its simplicity, the Geissler pump, the first of the kind invented, is chosen for illustration. A (Fig. 121) is a glass tube more than thirty-four inches long, having a globelike enlargement B of about a liter capacity. Above this globe leads a tube containing a three-way stopcock and a branch tube D. By means of this stopcock B may be placed in communication either through D with the atmosphere, as shown in M (Fig. 122), or through E with the receiver to be exhausted, as shown in N. Connected with the lower end of A by means of a thick rubber tube F is a vessel G containing mercury. The pump is operated as follows: C is turned as in M, and G is raised so that mercury will flow from it into B and fill it, the air escaping through D. Then the stopcock is placed as in N, and G is lowered so as to allow the mercury to flow back into it. A Torricellian vacuum would be formed in B were it not in communication through E

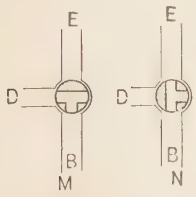


FIG. 122

with the receiver. As it is, the air in this space expands and fills *B*, and is thus to this extent rarefied. By a sufficient number of repetitions of this process a very high vacuum is obtainable. There are many modifications of this pump, in some of which the stopcock is dispensed with, and consequently the trouble of operating it is avoided.

With the common pump a vacuum of a millimeter of mercury is considered exceedingly good; but with a mercury pump it is easy to obtain a vacuum of 0.00076 of a millimeter, which represents about one millionth the normal pressure of the atmosphere. Rood in 1881 succeeded in obtaining vacua as high as a three-hundred-millionth of an atmosphere.

SECTION V

SIPHONS AND PUMPS FOR LIQUIDS

148. The siphon. A siphon is an instrument used for transferring a liquid from one vessel to another over an elevation through the agency of atmospheric pressure. It consists of a tube of any material (rubber is often most convenient) bent into a shape somewhat like the inverted letter U. To set it in operation, fill the tube with a liquid, stop each end with a finger or cork, place it in the position represented in Fig. 123, remove the stoppers and the liquid will flow out at the orifice *o*. The upward pressure of the atmosphere against the liquid in the tube is the same at both ends; hence these two forces are in equilibrium. But the downward pressure of the column of liquid *ab* is greater than the downward pressure of the column *dc*; hence equilibrium is destroyed and the movement is in the direction of the greater, i.e. the unbalanced, force. The unbalanced force which causes the flow is equal to the downward pressure of the column *eb*.

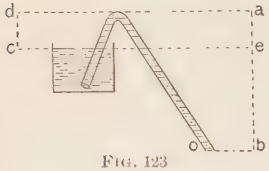


FIG. 123

If one end of the tube filled with liquid be immersed in a liquid in some vessel, as in *A* (Fig. 124), and the other end be brought below the surface of the liquid in the vessel and the stoppers be removed, the liquid in the vessel will flow out

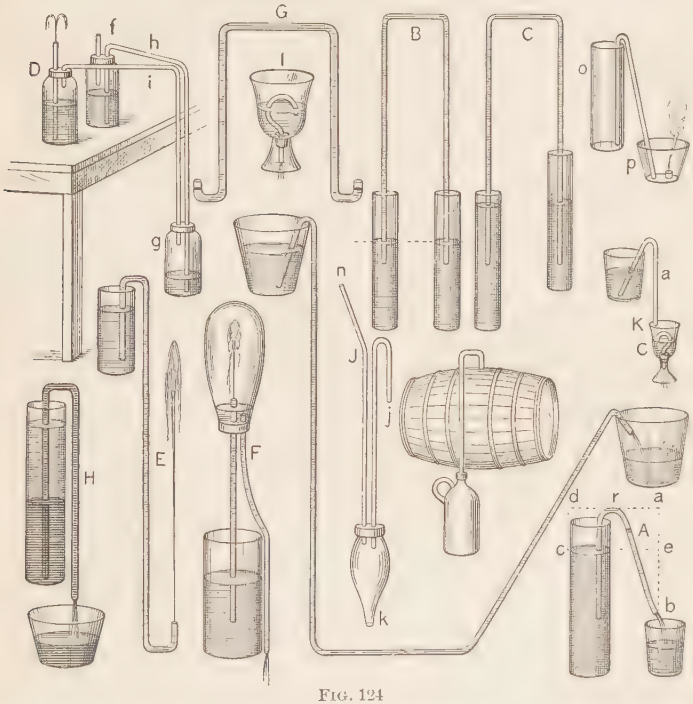


FIG. 124

through the tube as long as the distance *cb* remains greater than zero.

If one of the vessels be raised a little, as in *C*, the liquid will flow from the raised vessel till the surfaces in the two vessels are on the same level. The remaining diagrams in this cut represent some of the great variety of uses to which the siphon may be put. *D*, *E*, and *F* are different forms of siphon fountains. In *D* the siphon tube is filled by blowing in the tube *f*. Explain the

remainder of the operation. A siphon of the form *G* is always ready for use. It is only necessary to dip one end into the liquid to be transferred. Why does the liquid not flow out of this tube in its present condition? *H* illustrates the method by which a heavy liquid may be removed from beneath a lighter one. By means of a siphon a liquid may be removed from a vessel in a clear state, without disturbing sediment at the bottom. *I* is a *Tantalus cup*. A liquid will not flow from this cup till the top of the bend of the tube is covered. It will then continue to flow as long as the end of the tube is in the liquid. The siphon *J* may be filled with a liquid that is not safe or pleasant to handle, by placing the end *j* in the liquid, stopping the end *k*, and sucking the air out at the end *l* till the lower end is filled with the liquid.

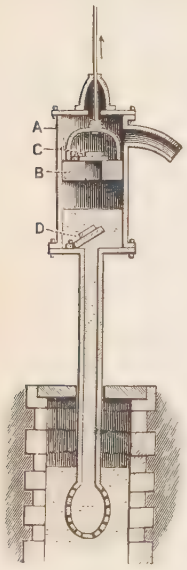


FIG. 125

Gases heavier than air may be siphoned like liquids. Vessel *o* contains carbonic acid gas. As the gas is siphoned into the vessel *p* it extinguishes a candle flame. Gases lighter than air are siphoned by inverting both the vessels and the siphon.

The siphon cannot elevate liquids, it can merely transfer them to places of lower level. Obviously the height of the bend to which different liquids can be raised varies with their respective densities. Thus the height to which mercury can be raised is the barometric height, while water may be raised 13.6 times as high.

149. Lifting or suction pump. The common *lifting pump* is constructed like the barrel of an air pump. Fig. 125 represents the piston *B* in the act of rising. As the air is rarefied below it, water rises in consequence of atmospheric pressure on the water in the well, and opens the lower valve *D*. Atmospheric pressure closes the upper valve *C* in the piston. When the piston is pressed down (Fig. 126), the lower

valve closes, the upper valve opens, and the water between the bottom of the barrel and the piston passes through the upper valve above the piston. When the piston is raised again (Fig. 127), the water above it is raised and discharged from the spout.

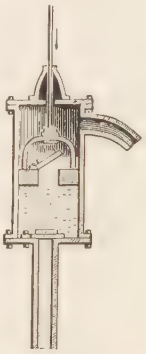


Fig. 126

The liquid is sometimes said to be raised in a lifting pump by the "force of suction." There is no such *force*.

Calling the density of mercury 13.6 grams per cubic centimeter and disregarding the vapor pressure of mercury (0.02 millimeters at 20° C.), a water barometer would be $76 \times 13.6 = 1033.6$ centimeters high when the mercury barometer stands at 76 centimeters, provided there were no pressure exerted by the vapor of water. But at 20° C. the pressure of water vapor is 1.74 centimeters of mercury $= 1.74 \times 13.6\ (= 23.7)$ centimeters of water. Therefore the height of the water barometer would be $1033.6 - 23.7 = 1009.9$ centimeters, and this is the limit of the height to which water can be raised by the pressure of air in a suction pump under these conditions.

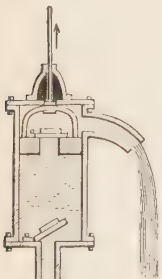


Fig. 127

At 50° the limit would be $1033.6 - (9.2 \times 13.6) = 908.4$ centimeters. At 100° the height would be $1033.6 - (76 \times 13.6) = 0$ centimeters, the pressure of water vapor being equal to the pressure of the atmosphere.

150. Suction and force pump combined. In this pump the ordinary piston with valve and leather washers is replaced by a solid cylinder of metal *B* (Fig. 128), called the *plunger*. This passes through a stuffing box *D*, in which it fits air-tight. Valves opening upward and outward are placed at *A* and *C* respectively. When the plunger is raised *A* opens and *C* closes, and water is raised into the barrel by atmospheric pressure. When the plunger descends *A* closes and *C* opens, and

the water is forced up through the pipe E to a height dependent on the pressure produced through the plunger. An air dome F is usually connected with these pumps to regulate the pressure so as to give through the delivery pipe a very steady stream. This dome contains air. When the plunger descends it forces water violently into E and thus tends to produce a severe strain in the pipe. But the water enters the dome and compresses the elastic air within, thus greatly reducing the shock. As soon as the down stroke of the piston ceases, the valve C closes and the compressed air in the dome forces the water out through E in a continuous stream.

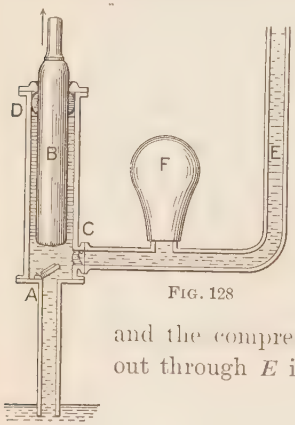


FIG. 128

SECTION VI

BUOYANCY OF FLUIDS

151. Archimedes' principle. Suppose *dcba* (Fig. 129) to be a cubical block of marble immersed in a liquid. It is obvious that the downward pressure upon the surface *da* is equal to the weight of the column of liquid *edao*. The upward pressure on the surface *cb* is equal to the weight of a column of liquid *ecbo*. The difference between the upward pressure against *cb* and the downward pressure on *da* is the weight of a column of liquid *ecbo* less the weight of a column of liquid *edao*, which is a column of liquid *dcba* (*ecbo* − *edao* = *dcba*). But a column of liquid *dcba* has precisely the volume of the solid submerged. Therefore *a body is buoyed up by a fluid in consequence of the unequal pressures upon its top and bottom at different depths, and the amount of the buoyancy is the weight of a volume of that fluid equal to the volume of the immersed solid.*

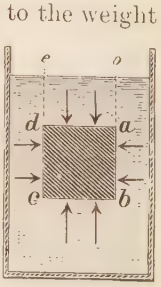


FIG. 129

This principle, known as "the principle of Archimedes" from the name of its discoverer, may be thus stated: *a body immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced.* The difference between the weight of a body and the buoyant force of a fluid in which it is submerged may be called the *effective weight* of the body in that fluid.

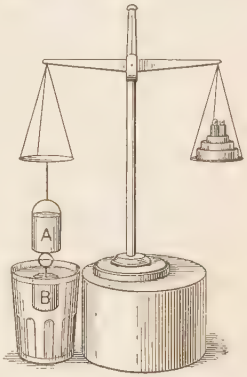


FIG. 130

152. Experimental proof of Archimedes' principle. Suspend from one arm of a balance beam a cylindrical bucket *A* (Fig. 130), and from the bucket a solid cylinder whose volume is exactly equal to the capacity of the bucket; in other words, the latter would just fill the former. Counterpoise the bucket and cylinder with weights.

Place beneath the cylinder a tumbler of water, and raise it until the cylinder is completely submerged. The buoyant force of the water destroys the equilibrium. Pour water into the bucket; when it becomes just even full the equilibrium is restored.

Now it is evident that the cylinder immersed in the water displaces its own volume of water, or just as much water as fills the bucket. But the bucket full of water is just sufficient to restore the weight lost by the submersion of the cylinder.

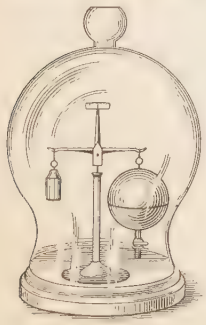


FIG. 131

A floating body, as a cork on water, has no effective weight. *It sinks until it displaces a weight of the fluid equal to its own weight, or until it reaches a depth where the upward pressure of the fluid is equal to its own weight.*

Place under the receiver of an air pump a device (Fig. 131) consisting of a scalebeam, a small weight, and a hollow brass sphere, and exhaust the air. In the air the weight

and sphere balance each other; but when the air is removed the sphere sinks, showing that in reality it is heavier than the weight. In the air each is buoyed up by the weight of the air it displaces; but as the sphere displaces more air it is buoyed up more. Consequently, when the buoyant force is withdrawn from both, their equilibrium is destroyed.

Ordinary weighing conducted in the air consists, therefore, in a comparison of effective weights in that fluid. The *absolute weight* is evidently the effective weight plus the weight of the excess of air displaced by the body over that displaced by the weights, or it is the weight of the body in a vacuum.

The density of the atmosphere is greatest at the surface of the earth. A body free to move cannot displace more than its own weight of a fluid; therefore a balloon, which is a large bag filled with a gas about fourteen times lighter than air at the sea level, will rise till the weight of the balloon, together with that of its car and cargo, equals the weight of the air displaced.

SECTION VII

DENSITY AND SPECIFIC GRAVITY

153. Definitions. The *density* of a substance at any temperature is *the mass of a unit volume* of the substance at that temperature. Thus the density of water at 4° C. is 1 gram per cubic centimeter, and the density of cast iron at the same temperature is about 7.12 grams per cubic centimeter. The mean density of a body is found by dividing its mass by its volume. Thus, if the mass of a body be 30 grams and its volume be 6 cubic centimeters, its mean density is $(30 \div 6 =)$ 5 grams per cubic centimeter.

Now as the weight of a body is proportional to its mass, the ratio of the weight of any body to that of an equal volume of water is known as the *specific gravity* of the body. This is evidently the number expressing the average density of the substance in grams per cubic centimeter. We speak of the *density*

of a substance; of the *specific gravity* of a body. The former is a concrete quantity, the latter an abstract number.

For example, the *density* of mercury is 13.6 grams per cubic centimeter in the metric system, or $(13.6 \times 62.3 =)$ 850 pounds per cubic foot (about) in the English system. The *specific gravity* of mercury, however, is 13.6 in any system.

It is to be noted, therefore, that in the metric system the number which expresses the numerical value of the density of a substance and the number which expresses the specific gravity of a body of the substance are identical, while this is not the case in the English system.

154. Experimental methods of finding specific gravity. The special method to be employed depends on whether the substance is in the solid, liquid, or gaseous state.

(1) The principle of Archimedes is usually employed to find the specific gravity of solid bodies. The method is as follows:

From a hook beneath one of the pans of a balance suspend by a fine thread the body whose specific gravity is to be found, and weigh it. Let the weight be W. Then immerse the body in a beaker of water at a temperature not far from 4° C., and while yet hanging from the balance pan, weigh again. Let this weight be W_1. The loss of weight in the water, or $W - W_1$, is evidently the weight of an equal volume of water; therefore, from the definition of specific gravity,

$$\text{Specific gravity} = \frac{W}{W - W_1}.$$

If the body chosen floats on water, a piece of some heavy substance, e.g. lead, must be attached in order that the body may be made to sink. The extra piece is called a *sinker*.

By the above method find the weight of water displaced by the combination, and also that displaced by the sinker alone. Subtract the latter from the former. This gives the weight of water displaced by the light body alone, and, as before, its specific gravity is the ratio of its weight to the weight of an equal volume of water.

(2) *Liquids. The specific-gravity bottle.* This in one form consists of a thin glass flask of capacity usually not more than 100 cubic centimeters with a ground stopper pierced with a capillary hole. The clean flask is weighed empty (Fig. 132) by balancing it with weights a. When filled with the liquid to be tested, additional weights b have to be placed on the pan. If when filled with water, weights c are required to balance it, the specific gravity of the liquid is evidently $\frac{b}{c}$, since this is the ratio of the weights of equal volumes of the liquid and of water.

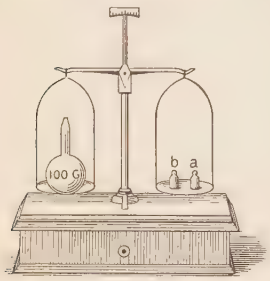


FIG. 132

If very exact results are desired, specific gravities must always be referred to water at a definite temperature, say at 4°, and in this case all temperatures must be observed during the experiment and proper reductions made.

Another method for finding the specific gravity of liquids depends on the principle of Archimedes. Let a small body be chosen that is not acted on chemically by either the liquid or by water, say a lump of glass of mass 8 or 10 grams. Find its loss of weight w in the given liquid, and also its loss of weight w' in water. Then, as before, the specific gravity of the liquid is $\frac{w}{w'}$.

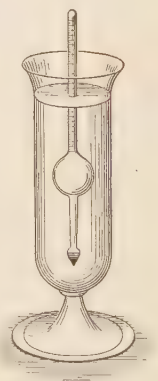


FIG. 133

155. Hydrometers. Referring again to the principle of Archimedes, it is clear that the amount a given body sinks in different liquids may be used to compare the specific gravities of the liquids. An instrument especially constructed for this purpose is shown in Fig. 133 and is called an *hydrometer*. It is usually of glass weighted at the bottom to make it float upright and has a stem of uniform size.

The latter may be graduated to read directly the specific gravity of the liquid in which it is immersed, or it may have scales made for various special purposes. If for the former use, the graduation unity is placed near the top if the instrument is intended for liquids heavier than water, or at the bottom if for liquids lighter than water.

For the many other forms of hydrometers the reader is referred to the author's laboratory manual.

(3) *Gases.* The specific gravity of a gas is found by the application of the same principles employed in determining that of a liquid, but the operation is attended with peculiar difficulties. Air is extracted from a light, capacious glass or copper globe by means of a good air pump, and the empty vessel is weighed. By means of a stopcock it is then put in communication with a reservoir containing the gas to be tested, which must be perfectly pure and dry. Gas is allowed to enter the globe slowly until the pressure is the same as the atmospheric pressure outside, and then it is allowed to stand until the gas acquires the temperature of the air outside. The vessel thus filled with gas is again weighed; its gain in weight is the weight of the gas (W) which fills the globe. The globe is again exhausted and filled, at the same temperature and pressure, with a gas which is employed as a standard. The weight of the gas now filling the globe is found. Then the weight of a given gas divided by the weight of an equal volume of the standard gas at the same temperature and pressure is the specific gravity of the given gas.

For many purposes it is most convenient to employ hydrogen — the lightest gas — as a standard for gases. Then, assuming the specific gravity of hydrogen to be 1, that of air is 14.47, of oxygen, 16, etc. A cubic centimeter of hydrogen at 0° C. and at the barometric pressure of 760 millimeters has a mass of 0.00008957 gram, and a cubic centimeter of dry pure air under the same conditions has a mass of 0.0012932 gram.

QUESTIONS AND PROBLEMS

1. By placing the neck of a bottle in your mouth can you suck liquid out of the bottle? Explain.

2. (*a*) What is the mass of a liter of hydrogen under a pressure of 760 mm. and at 0° C.? (*b*) What is the mass of a liter of dry air under the same conditions?

3. Into what space must you compress 30 cu. ft. of air that its pressure may be made five times as great?

4. If when the barometer stands at 760 mm. a cubic meter of air be forced into a vessel whose capacity is 1000 cc., what pressure will be exerted upon its interior walls?

5. How high can ether be raised by a lifting pump?

6. Why do ironclad vessels float in water?

7. A block of ice weighing 500 g. floats on water. (*a*) What volume of water does it displace? (*b*) What volume of ice is out of water?

8. Will ice float or sink in alcohol?

9. The effective weight of a stone in water is 56 g.; its weight in air is 112 g. (*a*) What is the volume of the stone? (*b*) What is its density?

10. How many cubic centimeters of dry air at 760 mm. and 0° C. weigh as much as 1 cc. of water at 4° C.?

11. If 4 cu. ft. of a body have a mass of 180 lb., what is the specific gravity of the body?

12. How much will 1 kg. of copper weigh in water?

13. A solid body weighs 10 lb. in air and 6 lb. in water. (*a*) What is the weight of an equal volume of water? (*b*) What is its specific gravity? (*c*) What is the volume of the body?

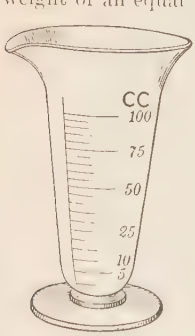


FIG. 134

14. A piece of cork floating on water displaces 2 lb. of water. What is the weight of the cork?

15. In which is it easier for a person to float, in fresh water or in sea water? Why?

16. Fig. 134 represents a beaker graduated in cubic centimeters. Suppose that when water stands in the graduate at 50 cc. a pebble is dropped into the water, causing it to rise to 75 cc. (*a*) What is the volume of the stone? (*b*) How much less does the stone weigh in water than in air? (*c*) What is the weight of an equal volume of water?

17. If a piece of cork be floated on water in a graduate and displace (i.e. cause the water to rise) 7 cc., what is the weight of the cork?

18. You wish to measure out 50 g. of sulphuric acid. To what number on a beaker graduated in cubic centimeters will that correspond?

19. State how you would measure out 80 g. of nitric acid in a measuring beaker.

20. A mass of lead weighs 1 kg. in air. What will it weigh in a vacuum?

21. A mass whose weight in air is 30 g. weighs in water 26 g., and in another liquid 27 g. What is the specific gravity of the other liquid?

22. A silver spoon weighing 150 g. is supported by a string in water. What part of the weight is sustained by the string and what part is supported by the water?

23. Find the specific gravity of wax from the following data: weight of a given mass of wax in air is 80 g.; wax and sinker displace 102.88 cc. of water; sinker alone displaces 14 cc.

24. An empty glass globe weighs 100 g.; full of air it weighs 102.4 g.; full of chlorine gas it weighs 105.928 g. What is the specific gravity of chlorine gas?

25. What mass of alcohol can be put into a vessel whose capacity is 1 l.?

26. A solid floats at a certain depth in a liquid when the vessel which contains it is in the air. If the vessel be placed in a vacuum, will the solid sink, rise, or remain stationary?

27. When the volume of a body of gas diminishes, is it due to contraction or compression, i.e. to internal or external forces?

28. What is the height of the barometer column when the atmospheric pressure is 10 g. per square centimeter?

29. A barometer in a diving bell stands at 196 cm. when a barometer at sea level stands at 76 cm. What is the depth of the surface of water inside the bell? below the air-exposed surface of the water above?

30. A person can lift just 200 kg. of copper in water. How much can he lift in the air?

ENERGY OF MASS VIBRATION—SOUND WAVES

SECTION I

ORIGIN OF SOUND WAVES; TRANSMISSION OF SOUND WAVES

156. Listen to a sounding church bell. It produces a sensation; it is heard. The ear is the organ through which the sensation of hearing is produced. The bell is at such a distance that it cannot act directly on the ear; yet something must act on the ear, and it must be the bell which causes that something to act.

Strike a bell or a glass bell jar, and touch the edge with a small ivory ball suspended by a thread; you not only *hear* the sound, but, at the same time, you *see* a tremulous motion of the ball, caused by a motion of the bell. Touch the bell gently with a finger and you feel a tremulous motion. Press the hand against the bell; you stop its vibratory motion, and at that instant the sound ceases. Strike the prongs of a tuning fork and press the stem against a table; you hear a sound. Thrust the ends of the prongs just beneath the surface of water; the water is thrown off in a fine spray on either side of the vibrating fork. Watch the strings of a piano, guitar, or violin, or the tongue of a jew's-harp, when sounding. You can *see* that they are in motion, and that the motion is vibratory in character. *Sound, therefore, originates in mass vibration.*

How can a bell sounding at a distance affect the ear? If the bell while sounding possess no peculiar property except motion, then it has nothing to communicate to the ear but motion. But motion can be communicated by one body to another at a distance only through some medium.

Lay a thick tuft of cotton wool on the plate of an air pump, and on this, face downward, place a loud-ticking watch, and cover with the receiver. Notice that the receiver, interposed between the watch and your ear, greatly diminishes the sound, or interferes with the passage of *something* to the ear. Take a few strokes of the pump and listen; the sound is more feeble, and continues to grow less and less distinct as the exhaustion progresses, until either no sound can be heard when the ear is placed close to the receiver, or an extremely faint one, as if coming from a great distance. The removal of air from a portion of the space between the watch and the ear destroys the sound. Let in the air again, and the sound is restored.

The vibrations of a sonorous body cannot affect the organ of hearing without a material medium of communication between them.

157. The propagation of vibratory motion through an elastic medium. Fig. 135 represents a brass wire wound into the form of a spiral spring long enough to reach across the room. Attach one end to the bottom of a cigar box, and fasten the box to the wall. Hold the other end of the spiral firmly in one hand, and with the other hand insert a knife blade between the turns of the wire, and quickly rake it for a short distance along the spiral toward the box, thereby crowding closer together for a little distance (*B*) the turns of wire in front of the hand, and leaving the turns behind pulled wider apart (*A*) for about an equal distance. The crowded part of the spiral may be called a *condensation*, and the stretched part a *rarefaction*. The condensation, followed by the rarefaction, runs with great velocity through the spiral, strikes the box, producing a sharp thump; is reflected from the box to the hand, and from the hand again to the box, producing a second thump; and by skillful manipulation three or four reflections may be produced in rapid succession. If a piece of twine be tied to some

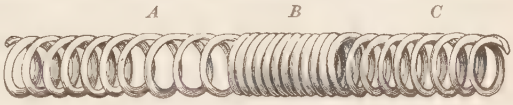


FIG. 135

turn of the wire, it will be seen, as each wave passes it, to receive a slight jerking movement forward and backward in the direction of the length of the spiral.

How is energy transmitted through the spring so as to deliver the blow on the box? Certainly not by a bodily movement of the spiral as a whole, as might be the case if it were a rigid rod. The movement of the twine shows that the only motion which the coil undergoes is a vibratory movement of its turns. Here, as in the case of water waves, energy is transmitted through a medium by the successive vibration of its parts.

The effect of applying force with the hand to the spiral spring is to produce in a certain section B of the spiral a crowding together of the turns of wire, and at A a separation; but the elasticity of the spiral instantly causes B to expand, the effect of which is to produce a crowding together of the turns of wire in front of it, in the section C, and thus a forward movement of the condensation is made. At the same time the expansion of B causes a filling up of the rarefaction at A, so that this section is restored to its normal state. This is not all: the folds in the section B do not stop in their swing when they have recovered their original position, but, like a pendulum, swing beyond the position of rest, thus producing a rarefaction at B, where immediately before there was a condensation. Thus a forward movement of the rarefaction is made, and thus a pulse or wave is transmitted with uniform velocity through a spiral spring or any *elastic* medium.

Elasticity is essential in a medium that it may transmit waves; and the greater the elasticity, the greater the facility and rapidity with which this is done.

158. Air as a medium of wave motion. Air is a fluid and therefore has only volume elasticity. The only waves it can propagate are waves composed of compressions and rarefactions. In a previous chapter we have seen how a wave is the result of a transmission of harmonic motion or harmonic vibrations through a series of particles. A sound wave results when consecutive portions

of the sound medium vibrate harmonically and successively and in the same direction as that in which the sound wave moves. There are two important distinctions between these waves and waves of water, or waves sent along a cord when one end is shaken: the former consist of condensations and rarefactions; the latter of elevations and depressions. In the former the vibration of the particles is in the same line with the path of the wave, and hence they are called *longitudinal* vibrations; in the latter the vibrations take place at some angle to the path of the wave, and are therefore called *transverse* vibrations.

Boys often amuse themselves by inflating paper bags and bursting them with a quick blow, producing with each a single loud report. First the air is suddenly and greatly condensed by the blow, and the bag is burst; the air now, as suddenly and with equal force, expands, and by its expansion condenses the air for a certain distance all around it, leaving a rarefaction where just before had been a condensation. If many bags were burst at the same spot in rapid succession, the result would be that alternating shells of condensation and rarefaction would be thrown off, all having a common center, enlarging as they advance, as do the ripples formed by stones dropped into water; except that, in this case, the waves are not like rings, but like hollow globes, — not circular, but spherical. In this manner sound waves produced by the vibration of a sounding body travel through the air.

As a wave advances, each individual air particle concerned in its transmission performs a short excursion to and fro in the direction of a straight line radiating from the center of the shells or hollow globes. *A sound wave travels its own length in the time that a particle occupies in going through one complete vibration.*

Take a strip of black cardboard 4.5 inches × 1 inch. Cut a slit about one sixteenth of an inch wide lengthwise and centrally through the strip nearly from end to end. Place the slit across the page just below Fig. 136, and draw the book along underneath in the direction of the arrow. Imagine that the short dark dashes seen through the slit represent a series of air particles,

and the slit itself represents the direction in which a series of sound waves is traveling. It will be seen that each air particle moves a little to and fro in the line in which the sound waves travel, and comes back to its starting point; but the condensations and rarefactions, represented by a group (half a wave length) of dashes becoming alternately closer together or farther apart, are transmitted through the whole series of air particles.

FIG. 136

159. Nature of sound and sound waves. *Sound is a sensation caused usually by air waves beating upon the ear.*

As commonly used, the term *sound* is ambiguous, being applied to both a sensation and the physical cause of the sensation. In a scientific treatise ambiguity and consequent confusion are disastrous. No apology, therefore, is required for restricting the term to its legitimate signification. With sound sensation we have little to do, as this is a physiological rather than a physical phenomenon. No more appropriate name than sound

wave can be applied to the physical agent with which we are to deal; it suggests at once the reality, and is not suggestive of some vague, mysterious *thing* shot through space.

Sound waves are waves in any medium (usually air) *that are capable of producing the sensation of sound.* A body vibrating in an elastic medium, e.g. in air, does not necessarily produce sound waves; in other words, not all waves are sound waves. For example, the energy of the vibrations may be insufficient, or the vibrating body may be so small (or the medium so rare) that it cuts through the medium without condensing it sufficiently to produce audible effects.

160. Solids and liquids as sound media. If a watch be placed face upward on a long board (or table) near to one of its ends, and covered with loose folds of cloth until its ticking cannot be heard through the air in any direction at a distance equal to the length of the board, the ear placed in contact with the farther end of the board will hear the ticking of the watch very distinctly.

Place one end of a long pole on a cigar box, and apply the stem of a vibrating tuning fork to the other end; the sound vibrations will be transmitted through the pole to the box, and a sound will be given out by the box as though it, and not the tuning fork, were the origin of the sound.

SECTION II

SPEED OF SOUND WAVES

161. Speed of sound waves dependent on elasticity and density of medium. It may be demonstrated that *in harmonic motion the velocity with which a particle of an elastic medium vibrates, and therefore the speed of wave propagation in the medium, is directly proportional to the square root of the elasticity of the medium, and inversely proportional to the square root of its density.* The relation of these quantities is shown in the formula

$$V \propto \sqrt{\frac{e}{d}}.$$

If the elasticity and density of the medium vary alike, it is evident that the speed of the sound wave is unaffected. Hence the speed of a sound wave is unaffected by changes in air pressure as measured by a barometer, or by elevation above sea level. Temperature, however, affects only the density of air. Elevation of temperature of the air diminishes its density, and therefore tends to increase the speed of the sound wave. Moisture in the air renders it less dense (pressure remaining constant), and thereby tends to increase the speed. The velocity of a sound wave is greatest in the direction of the wind. Speed of sound waves is very nearly independent of pitch and intensity.

The greater density of solids and liquids, as compared with gases, tends, of course, to diminish the speed of sound waves; but their greater incompressibility more than compensates for the decrease of speed occasioned by the increase of density. As a general rule, solids are more incompressible than liquids; hence sound waves generally travel faster in the former than in the latter. For example, sound waves travel in water about four times as fast as in air, and in iron and glass sixteen times as fast.

The speed of sound waves in free air at 0° C. is 332.4 meters (nearly 1091 feet) per second. The increase of speed per degree centigrade is 0.608 meters (23.9 inches). The speed in other gases $= \frac{v}{\sqrt{d}}$, in which v is the speed in air and d the density of the given gas referred to air. For example, in hydrogen, whose density is $\frac{1}{14.4}$ that of air, the speed at 0° C. is about 4163 feet per second. The speed of sound waves in any medium may be calculated from the formula given above and by experimental methods to be given further on.

SECTION III

ENERGY OF SOUND WAVES. LOUDNESS

162. Energy of sound waves. Gently tap the prongs of a tuning fork and dip them into water, — the water is scarcely moved by them; increase the energy of the blow, — the vibrations become wider and the water spray is thrown to a greater distance. The same thing occurs when the fork vibrates in the air; though we do not see the air particles as they are batted by the moving fork, yet we feel the effects as a sound sensation, and we judge of their energy by the intensity of the sensation which they produce.

Fix your attention upon a particle of air as a sound wave passes it. A harmonic motion is impressed upon it. At a certain point of its excursion its velocity is at its maximum. Now since the energy of a moving particle varies as the square of its velocity, the *intensity* of the impact which it is capable of producing upon the tympanum of the ear is *proportional to the square of this maximum velocity.*

It is also clear that if the amplitude of vibration of a particle be doubled while its period remains constant, its velocity is doubled, and therefore its energy is increased fourfold. Hence, (1) *the energy of a sound wave is proportional to the square of the amplitude of the vibration of particles.* An amplitude of less than 10^{-7} centimeters is sufficient to cause hearing.

Loudness of sound refers to the intensity of a sensation. We have no standard of measurement for a sensation, so we are compelled to measure the energy of the sound wave, knowing at the same time that *loudness is not proportional to this energy.*

163. Effect of the density of the medium. In the experiment with the watch under the receiver of the air pump, the sound grew feebler as the air became rarer. Aëronauts are obliged to exert themselves more to make their conversation heard when they reach great heights than when in the denser lower air. In diving bells, where the pressure and therefore the density of the air is

great, persons are obliged to speak in undertones. In a rare medium a vibrating body during a single vibration either sets in motion fewer particles, as in the case of the partially exhausted receiver, or, as in the case of hydrogen gas, it sets in motion particles of less mass than in a dense medium; consequently it parts with its energy more slowly, and the sound is therefore weaker.

(2) *The energy of gaseous sound waves is proportional to the density of the medium in which they are produced.*

164. Effect of distance. It is a matter of everyday observation that the loudness of a sound diminishes very rapidly as the distance from the source of the waves to the ear increases. As a sound wave advances in an ever-widening sphere, a given quantity of energy becomes distributed over an ever-increasing surface; and as a greater number of particles partake of the motion, the individual particles receive proportionately less energy; hence it follows — as a consequence of the geometrical truth that "the surface of a sphere varies as the square of its radius" — that (3) *the energy of a sound wave varies inversely as the square of the distance from the source.* This is known as the *law of inverse squares.* For example, if two persons, A and B, be respectively 500 and 1000 rods from a ringing bell, the waves that reach A will have four times the energy that the same waves have when they reach B.

165. Speaking tubes. Long tin tubes, called *speaking tubes,* passing through many parts of a building, enable persons at the distant extremities to carry on conversation in a low tone of voice, while persons in the various rooms through which the tubes pass hear little or nothing. The reason is that the sound waves which enter the tube are prevented from expanding, and consequently the energy of the sound waves is little affected by distance, except as it is wasted by friction of the air against the sides of the tube, and by internal friction due to the viscosity of the air.

166. Facility of sound-wave propagation depends on the homogeneousness of the transmitting medium. Observations and experiments of Humboldt, Tyndall, and Henry have established the following

facts: rain, hail, snow, and fog offer little or no obstruction to the passage of a sound wave. The air associated with a fog is, as a general rule, highly homogeneous and favorable to the transmission of sound. An atmosphere optically opaque may be acoustically transparent; hence the great value of fog horns. On the other hand, streams of air differently heated, or saturated in different degrees with aqueous vapor, though invisible to the eye, form acoustic clouds which may greatly interfere with the propagation of sound waves.

167. Transmission of sound waves affected by winds. It not infrequently happens that sound waves are audible two or three times as far to the leeward as to the windward. Sound waves are borne along with the wind, but, of course, are impeded by it when the direction of wave propagation is opposite to that of the wind.

SECTION IV

CHANGES IN DIRECTION OF SOUND WAVES

168. Reflection. So long as sound waves travel in a uniform medium they are propagated in the form of concentric spheres; but when they meet with an obstacle they follow the general law of elastic impact; that is, they return upon themselves, forming new concentric waves, called *reflected waves*, which seem to emanate from a second center on the other side of the reflecting surface. This phenomenon is called the *reflection of sound waves*. A (Fig. 137) represents a vibrating particle or a sonorous center from which emanates a series of waves. PQ represents a flat surface turned toward the waves. Take, for example, the incident wave $MCDN$ emitted from the center A; the corresponding reflected wave is represented by the arc CKD of a circle whose center a is as far beyond the surface PQ as A is in front of it.

Join any point C of the reflecting surface to the sonorous center; the line AC then represents one of an infinite number

of directions in which energy is transmitted by a sound wave. Such a line may conveniently be called a *sound ray*. Let fall the line HC normal to the surface at the point of incidence C. The angle ACH is called the *angle of incidence*. The ray AC after reflection takes the direction CB, which is a prolongation of aC. The angle BCH is called the *angle of reflection*. An observer at B receives sound waves not only directly from A in the line AB, but also from C in the line CB. Hence he hears two sounds, — one directly from point A, and the other after reflection from point C. The latter travels from A to C and from C

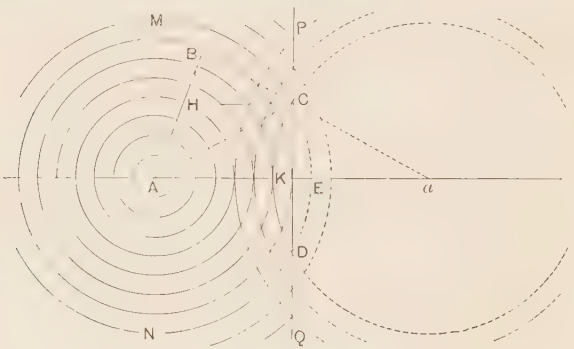


FIG. 137

to B, a longer distance than AB, and is therefore heard later than the former. If the interval of time between their arrivals at B be greater than about a fifth of a second, the ear is able to separate the two sensations, and the latter appears as an *echo*. If the interval of time be too short, then only a single and perhaps somewhat blurred and indistinct sound is heard. The latter phenomenon is usually called *resonance*. Such an effect is experienced frequently by a person listening to his own voice in a large hall.

If the obstacle PQ present a concave surface, the wave front after reflection will be less convex, and may become plane or even concave according to the degree of the concavity of the reflector and the position of the sounding body.

169. Sound waves reflected by concave mirrors. Arrange an experiment, as shown in Fig. 138, in which G and H are two concave surfaces, A a watch, and B a small funnel connected to a rubber tube CD. The extremity D being placed in the ear, the ticking of the watch can be heard very distinctly, as though it were somewhere near the mirror H. Though the mirrors be 12 feet apart, the sound will be louder at B than at an intermediate point E.

How is this explained? Every air particle in a certain radial line, as Ac, receives and transmits motion in the direction of this line; the last particle strikes the mirror at c, and, being perfectly elastic, bounds off in the direction cc', communicating its motion to the particles in this line. At c' a similar reflection gives motion to the air particles in the line $c'B$. In consequence of these two reflections, all divergent sound rays, as Ad, Ae, etc., that meet the mirror G are there rendered parallel, and are afterwards rendered convergent at the mirror H. The practical result of the concentration of this energy is that a sound of comparatively great intensity is heard at B. The points A and B are called the *foci* of the mirrors. The front of the wave as it leaves A is convex, in passing from G to H it is plane, and from H to B concave. If you fill a large circular tin basin with water and strike one edge with a knuckle, circular waves with concave fronts will close in on the center, heaping up the water for a moment at that point.

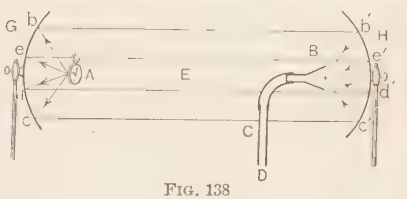


FIG. 138

Long "whispering galleries" have been constructed on this principle. Persons stationed at the foci of the concave ends of the long gallery can carry on a conversation in a whisper, which persons between cannot hear. The external ear is a wave condenser. The hand held concave behind the ear, by its increased surface, adds to the efficiency of the ear.

170. Refraction. If the ear be placed at the small end of a tunnel C (Fig. 139), the ticking of a watch A about 4 meters

distant may be heard. If now a collodion balloon B filled with carbonic acid gas be introduced between the ear and the watch, and very near the latter, the sound becomes louder.

The cause is obvious; for let the curved lines a, b, c, etc., represent sections of sound waves with convex fronts, and let B be a spherical body of carbonic acid gas which is denser than air; then it is clear that owing to the slower progress of the waves in the denser gas they would become flattened on entering this gas, and the waves of convex fronts may be changed to waves of plane fronts. Again, points at the extremities of the waves, having less distance to travel in the denser gas than points near the center, would emerge first and get in advance, and thus the wave fronts which are plane or nearly so while wholly in the dense gas become concave on leaving it. By these changes in the form of the wave fronts, sound energy which was originally becoming diffused through wider and wider space, and therefore becoming less intense as it progressed, is so changed in direction in passing into and out of a medium of greater density that the energy is finally concentrated at a distant point, as at C, and thereby intensified.

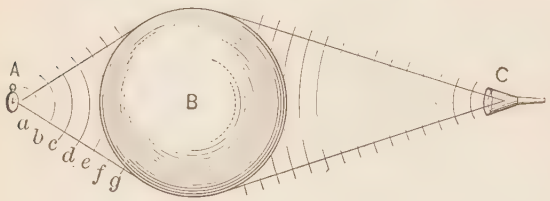


FIG. 139

Any change in direction of sound, caused by passing from one medium into another of different density, is called *refraction*.

171. Diffraction. When sound waves encounter an obstacle, a series of secondary waves are formed with the edges of the obstacle as centers. These waves appear to flow around behind the object, so that the obstacle is able to produce only a partial sound shadow. "Sounds heard around a corner" are thus accounted for. This bending, as it were, of waves around an obstacle is called *diffraction*.

SECTION V

REËNFORCEMENT OF SOUND WAVES

172. Consonance. Set a tuning fork in vibration; you can scarcely hear the sound unless the fork is held near the ear. Press the stem against a table; the sound rings out loud, but the waves seem to proceed from the table.

When only the fork vibrates, the prongs, presenting little surface, cut their way through the air, producing very slight condensations and consequently waves of little energy. When the fork rests upon the table the vibrations are communicated to the table; the table with its larger surface throws a larger mass of air into vibration, and thus greatly intensifies the sound. This is called *consonance*. The strings of the piano, guitar, and violin owe as much of their loudness of sound to their elastic sounding boards as the fork does to the table.

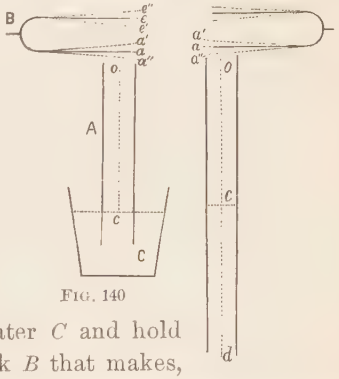


FIG. 140

173. Resonance. Take a glass tube *A* (Fig. 140) about 16 inches long and 2 inches in diameter; thrust one end into a vessel of water *C* and hold over the other end a vibrating fork *B* that makes, say, 256 vibrations in a second. Gradually lower the tube into the water, and when it reaches a certain depth, i.e. when the column of air *oc* attains a certain length, the sound becomes quite loud; as the tube is lowered below this point the sound rapidly dies away.

Columns of air, as well as sounding boards, serve to *reënforce* sound waves. Under such circumstances, unlike sounding boards, they can respond loudly to only one tone, or to a few tones of widely different pitch. They are then called resonators.

How is resonance effected? When the prong a moves from one extremity of its arc a' to the other a'', it sends a condensation down the tube; this condensation, striking the surface of the water, is reflected by it up the tube. Now suppose that the front of this reflected condensation should just reach the prong at the instant it is starting on its retreat from a'' to a'; then the reflected condensation will conspire with the condensation formed by the prong in its retreat to make a greater condensation in the air outside the tube. Again, the retreat of the prong from a'' to a' produces in its rear a rarefaction, which also runs down the tube, is reflected, and will reach the prong at the instant it is about to return from a' to a'', and to cause a rarefaction in its rear; these two rarefactions moving in the same direction conspire to produce an intensified rarefaction. The original sound waves thus combine with the reflected to produce resonance; but this can happen only when the like parts of each wave coincide appropriately, for if the tube were somewhat longer or shorter than it is, it is plain that condensations and rarefactions would meet in the tube and tend to destroy each other.

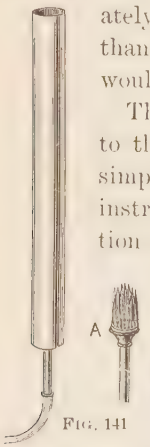


Fig. 141

The loudness of sound of all wind instruments is due to the resonance of the air contained within them. A simple vibratory movement at the mouth or orifice of the instrument, scarcely audible in itself (such as the vibration of a reed in reed pipes, or a pulsatory movement of the air, produced by the passage of a thin sheet of air over a sharp wooden or metallic edge, as in organ pipes, flutes, and flageolets, or, more simply still, by the friction of a gentle stream of breath from the lips sent obliquely across the open end of a closed tube or pen case), is sufficient to set the large body of inclosed air in the instrument into vibration, and the sound thus reënforced becomes audible at long distances.

Attach a rose gas burner A (Fig. 141) to a metal gas tube about 1 meter in length, and connect this with a gas nipple by means of a rubber tube. Light the gas at the rose burner and

you will hear a low, rustling noise. Support a long tube in a vertical position, and gradually raise the burner into the tube; when it reaches a certain point not far up, the body of air in the tube will catch up the vibrations and give out deafening sound waves that will shake the walls and furniture in the room.

174. Measurement of wave length and of speed. Experiments like that described in § 173 enable us readily to measure the length of the wave produced by a fork whose vibration number is known, and also to measure the velocity of sound waves. It is evident that if a condensation generated by the prong of the fork in its forward movement from a' to a'' (Fig. 140) meet with no obstacle, its front, meantime, will traverse the distance *od*, or twice the distance *oc*; hence the length of the condensation is the distance *od*. But a condensation is only one half of a wave, and the passage of the prong from a' to a'' is only one half of a vibration; consequently the distance *od* is one half of a wave length, and the distance *oc* is one fourth of a wave length. The measured distance of *oc* in this case is about 13 inches; hence the length of wave produced by a fork making 256 vibrations in a second is (13 inches $\times$ 4 =) 52 inches, or $4\frac{1}{3}$ feet. And since a wave from this fork travels $4\frac{1}{3}$ feet in $\frac{1}{256}$ of a second, it will travel in an entire second ($4\frac{1}{3}$ feet $\times$ 256 =) 1109 feet. The distance *oc* varies with the temperature of the air.

It is evident that the three quantities expressed in the formula

$$\text{velocity} = \text{number of vibrations} \times \text{wave length},$$

or

$$v = n\lambda,$$

bear such a relation to one another that if any two be known, the remaining quantity can be computed. *It will further be observed that with a given velocity the wave length varies inversely as the number of vibrations;* i.e. the greater the number of vibrations per second, the shorter the wave length.

175. Interference of sound waves. Hold a vibrating fork over a resonance jar, as in Fig. 142. Roll it over slowly in the fingers. At certain points a quarter of a revolution apart, when the fork is in an oblique position with reference to the edge of the jar as

represented in the figure, the reënforcement from the tube almost entirely disappears, but it reappears at the intermediate points.

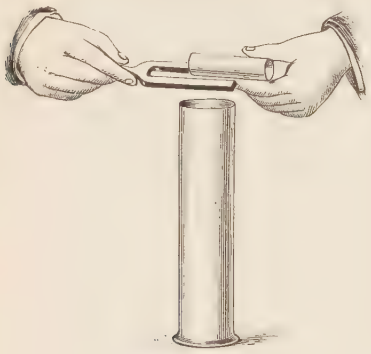


FIG. 142

That is, there are four intervals in the space around the fork where the two series of waves generated by the two tines interfere to produce mutual destruction. These are called technically *the cones of silence*. Return to the position where there is no resonance, and inclose in a loose roll of paper the prong farthest from the tube, without touching the fork, so as to prevent the sound waves produced by that prong from passing into the tube; the resonance resulting from the vibrations of the other prong immediately appears.

Close the tubes of a bicyclist's whistle alternately (Fig. 143), and try each separately. The sound is clear and smooth. Sound both tubes at the same time, and you obtain the usual rough and discordant sound.

FIG. 143

The two tubes of unequal length give out waves of slightly different length, so that at certain short intervals both waves will interfere in the same phase (i.e. condensation with condensation) and produce intensified sounds which are heard at long distances, while at other intervals they interfere in opposite phases (i.e. condensation with rarefaction), and the result of their mutual destruction is to cause the otherwise smooth sound to become broken or rattling.

Two sound waves may combine to produce a sound louder or weaker than either alone would produce, or may even cause silence. This phenomenon is called *interference*.

176. Forced and sympathetic vibrations. Suspend from a frame several pendulums, *A*, *B*, *C*, etc. (Fig. 144). *A* and *D* are each 3 feet long, *C* is a little longer, and *B* and *E* are shorter. Set *A* in vibration; the slight impulses will be communicated through the frame to *D*, causing it to vibrate. The vibration period of *D* being the same as that of *A*, all the impulses tend to accumulate motion in *D*, so that it soon vibrates through arcs as large as those of *A*. On the other hand, *C*, *B*, and *E*, having different rates of vibration from that of *A*, will at first acquire a slight motion, but soon their vibrations will be in opposition to those of *A*, and then the impulses received from *A* will tend to destroy the slight motion previously acquired.

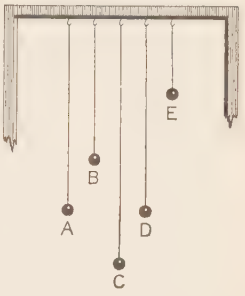


FIG. 144

Press down gently one of the keys of a piano so as to raise the damper without making any sound, and then sing loudly into the instrument the corresponding note. The string corresponding to this note will be thrown into vibrations that can be heard for several seconds after the voice ceases. If another note be sung, this string will respond only feebly.

Raise the dampers from all the strings of the piano by pressing the foot on the right-hand pedal, and sing strongly some note into the piano. Although all the strings are free to vibrate, only those will respond loudly that correspond to the note you sing, i.e. those that are capable of making the same number of vibrations per second as are produced by your voice.

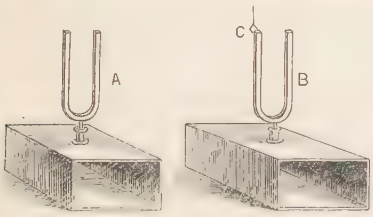


FIG. 145

Take two forks *A* and *B* (Fig. 145) tuned exactly in unison and mounted on resonance boxes, and place them from three to ten meters apart. Fasten, by a bit of sealing wax, a thread to a small ivory or glass ball *C*, and suspend so as to touch a side of

one of the prongs of the fork *B*. Set the fork *A* in vibration by drawing a resined bass-viol bow strongly across the ends of its prongs. In about ten seconds stop the vibrations of *A* with the fingers, and you will see and hear the little ball rattling against the prong of the fork *B*; remove it, and place the ear near the fork *B*, and you will hear a distinct sound, showing that the fork *B* has been thrown into a state of vibration by the fork *A*.

So the pulses that traverse the air between the forks, so gentle that only the sensitive organ of the ear can perceive them, become effective enough to move the rigid steel when the energy of their blows, dealt at the rate of perhaps 512 in a second, accumulates. The large number of impulses makes up for the feebleness of the individual ones.

These experiments show that a vibrating body tends to make other bodies near it vibrate, even if their periods of vibrations be different. Vibrations of this kind, such, for example, as those of *B*, *C*, and *E* (Fig. 144), and those generated in the sounding boards of pianos, violins, etc., are called *forced vibrations*. But if the period of the incident waves of air be the same as that of the body which they cause to vibrate, the amplitude and intensity of the vibrations become very great, like that of the pendulum *D*, and those of the piano strings which gave forth the loud sounds. Such are called *sympathetic vibrations*.

SECTION VI

MUSICAL SOUND. PITCH

177. Distinction between noise and musical sound. If the body that strikes the air deal it but a single blow, like the discharge of a firecracker, the ear receives but a single shock and the result is called a *noise*. If several shocks be slowly received by the ear in succession, the ear distinguishes them as so many separate noises. If, however, the body that strikes the air be in vibration, and deal it a great number of little blows in a second,

or if a large number of firecrackers be discharged one after another very rapidly, so that the ear is unable to distinguish the individual shocks, the effect produced is that of one continuous sound, which may be pleasing to the ear; and, if so, it is called a *musical sound*. But continuity of sound does not necessarily render it musical. The sound produced by a hundred children beating various articles in a room with clubs might not be lacking in continuity, but it would be an intolerable noise. There would be wanting those elements that please the ear, namely regularity both in periodicity and intensity of the shocks which it receives. The distinction between music and noise is, generally speaking, a distinction between the agreeable and the disagreeable, between *regularity* and confusion.

178. Pitch. In the contemplation of sounds of various kinds it will doubtless have been noted that any particular one is characterized by other features than intensity or loudness.

There are at least two other elements in which sounds may differ from one another. They are known as *pitch* and *quality*. The former depends on the vibration frequency of the sounding body, which in turn determines the wave length in the propagating medium; while the quality of a sound depends on the complexity of the motion of the sounding body, which determines the particular shape assumed by the wave.

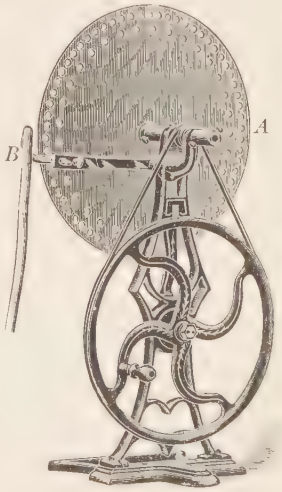


FIG. 146

Cause the circular sheet-iron disk *A* (Fig. 146) to rotate, and hold a corner of a visiting card so that at each hole an audible tap shall be made. Notice that when the separate taps or noises cease to be distinguishable, the sound becomes musical; also, that the pitch of the musical sound

depends upon the rapidity of the rotation, i.e. upon the frequency of the taps.

Hold the orifice of a tube *B* so as to blow through the holes as they pass. When rotating slowly, separate puffs, from which it hardly seems possible to construct a musical sound, are heard. When, however, the ear is no longer able to detect the separate puffs, the sound becomes quite musical and the pitch rises and falls with the speed.

Pitch sensation then depends upon the number of sound waves striking the ear per second. If the source of the sound waves and the receiving ear be both stationary, the pitch depends upon the frequency of vibration, or wave length ; i.e. *the greater the number of vibrations per second, or the shorter the wave length, the higher the pitch.*

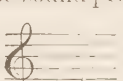
Since pitch depends upon the number of sound waves striking the ear per second, a sound must rise in pitch if we rapidly approach the source of the sound waves, or if the source rapidly approach us. In the former case more sound waves will then strike the ear per second than would under any other circumstances. In the latter case the wave lengths in the air are actually shorter, resulting in a sound of higher pitch. The pitch of the whistle is higher on the rapid approach of a locomotive, and lower as the engine travels away, than it would be if the whistle and the ear were relatively at rest.

179. Determination of pitch. The siren. While the apparatus illustrated in Fig. 146 might be used to determine roughly the pitch of a sound, an improvement on it by Caignard de la Tour, and later by Helmholtz, called the siren, is capable of far more accurate results.

By boring the holes obliquely through the revolving disk and directing the air perpendicularly against the sides of the openings, the disk may be revolved by means of the pressure of the impinging air.

The complete instrument has an attachment called a *counter*, which shows the number of revolutions the wheel makes in a given time.

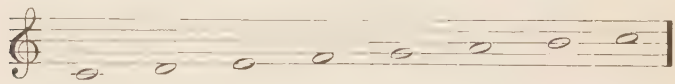
Suppose that it is required to ascertain the number of vibrations per second necessary to produce a given pitch. Take some instrument that gives the required pitch, e.g. a tuning fork, and set it in vibration ; also rotate the siren, causing the pitch of its sound gradually to rise until it corresponds with the pitch of the fork ; then, sustaining that pitch, set the counter in operation, and at the end of a given time read off the number of revolutions made by the wheel ; this number multiplied by the number of holes in the wheel gives the number of sound waves produced by the wheel during the given time, and the number of vibrations made by the fork in the same time ; and this number divided by the number of seconds employed gives the number of vibrations that must be made in a second by any instrument in order to produce a sound of the given pitch. With the siren one may easily determine the number of vibrations made by the wing of a fly or other insect.

180. The musical scale. Suppose a body, e.g. a tuning fork, to make 261 vibrations per second ; the sound produced is recognized by our musical sense as the note  which corresponds to the so-called *middle* C (c', or French ut_3) of a piano tuned to the national standard pitch.

The pitch of a sound produced by twice as many vibrations as that of another sound is called the *octave* of the latter. Between two such sounds the voice rises or falls in a manner very pleasing to the ear, by a definite number of steps called *musical intervals*. This gives rise to the so-called *diatonic scale*, or *gamut*. Long before any one had attempted to find the frequency of vibration of a sounding body, men had used a succession of sounds, differing in pitch, determined only by their musical sense and not by arbitrary agreement. The number of vibrations which shall constitute a given note is purely arbitrary, and differs slightly in different countries ; but the ratios between the vibration numbers of the several notes of the gamut and the vibration

number of the first or fundamental note of the gamut are the same among all enlightened nations.

The successive tones of the major scale of C are related to one another with respect to vibration frequency as follows:



	c′	d′	e′	f′	g′	a′	b′	c″
	ut_3	re_3	mi_3	fa_3	sol_3	la_3	si_3	ut_4
No. of vibrations	261	293.62	326.25	348	391.5	435	489.37	522
Ratios	256 :	288 :	320 :	341.3 :	384 :	426 :	480 :	512
or	1 :	$\frac{9}{8}$:	$\frac{5}{4}$:	$\frac{4}{3}$:	$\frac{3}{2}$:	$\frac{5}{3}$:	$\frac{15}{8}$:	2

In most scientific acoustic instruments the frequency corresponding to middle C is 256 per second.

The ear is wholly incapable of determining the number of vibrations corresponding to a given tone, but it is capable of determining with wondrous precision the *ratio* of the vibration numbers of two notes; hence all music must depend upon the recognition of such ratios, and for this reason the vibration ratios given above are of the utmost importance. An octave below c′ is c; two octaves below, c_1, and so on. In a similar manner the octaves below any other tone are indicated.

If we refer now to the lowest line in the preceding scheme, it is seen that the successive intervals in the series are not equal; thus,

c′	d′	e′	f′	g′	a′	b′	c″
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$

gives three different interval values, — $\frac{9}{8}$, $\frac{10}{9}$, and $\frac{16}{15}$. The first of these is called a major tone, the second a minor tone, and the last a major semitone or limma.[1] Moreover, the interval c : e, or $\frac{5}{4}$, is a major third, while the interval e : g, or $\frac{6}{5}$, is called a minor

[1] Strictly the term *semitone* should be used only in referring to an even-tempered scale.

third. Again, the interval between the major third and the minor third is $\frac{5}{4} \div \frac{6}{5} = \frac{25}{24}$ and is called a minor semitone.

It will be noticed that in the above scale the semitones come between the third and the fourth tones, and between the seventh and the eighth. Such a musical scale is known as the major scale in the key of C. The minor scale is constructed with a as the keynote a minor third below C, and has the semitones between the second and the third tones, and between the fifth and the sixth.

These two scales are adequate for any music composed in the particular key upon which they are based. Since, however, in rendering either of the above scales one may start with any particular note, and not necessarily from either c′ or a, it is clear that in this case various changes must be made in order that the succession of intervals shall remain the same. If, for example, g′ be made the keynote, only one of the necessary semitones, namely that between b′ and c″, is found to be properly placed. Furthermore, a semitone occurs between e″ and f″, where it is not desired. If, however, f″ be raised a half tone, then it will be found that the sequence of tones and semitones will occur exactly as in the key of c′. The process of raising a tone by a semitone is called sharping, and that of lowering it by a similar amount is called flatting. The vibration number of the new tone is found by multiplying or dividing that of the natural note by $\frac{25}{24}$, the interval of a minor semitone.

Since every change of key requires the sharping or flatting of some of the notes, it is evident that on all musical instruments means must be provided for sounding all the extra notes required. This is very easy in all such instruments, as the violin, in which any arbitrary pitch may be produced by properly fingering the string; but on a piano every note sounded requires a separate key, so that to play in natural, pure intervals in all the keys in which modern music is composed would require no less than seventy-two keys to the single octave.

181. Temperament. So large a number being impossible both in construction and in execution, a compromise is effected on all

instruments, like the piano, having fixed keys. The system of *temperament*, as it is called, now generally adopted, was developed by Bach. This system ignores the distinctions between major and minor tones and semitones, and deems as equivalent the sharp of one tone and the flat of the next above. The number of keys in an octave is thereby reduced to those representing the seven original tones, namely the white keys, and the five black ones used to sound the flats and sharps. "Music founded on the tempered scale must be considered as imperfect music, and far below our musical sensibility and aspirations. That it is endured and even thought beautiful only shows that our ears have been systematically falsified from infancy" (Blaserna).

182. Limits of the musical scale and of audibility. The lowest note of a $7\frac{1}{3}$ octave piano makes about 27 vibrations per second; the highest about 4224 vibrations per second; but these extreme notes have little musical value, and the lowest notes are chiefly used for their harmonics only. The range of the human voice lies between about 60 and 1300 vibrations per second,[1] or more than $4\frac{1}{2}$ octaves; an ordinary singer has about the compass of 2 octaves.

The ear is capable of hearing vibrations far exceeding in number the requirements of music. It can appreciate sounds arising from 32 to 38,000 vibrations[2] per second, i.e. a range of about 11 octaves, and a corresponding range of wave length between seventy feet and three or four tenths of an inch. These numbers vary considerably, however, with the person. Exceptional ears can hear as many as 50,000 vibrations. Some ears can hear a bat's cry or the creaking of a cricket; others cannot. Singing mice are sometimes placed on exhibition. Of those who go to hear them, some can hear nothing, others a little, and others again can hear much. In the ability to hear sharp sounds no animal is superior to the cat, which finds her prey in the dark by its squealing. High tones are heard with difficulty in the presence of low ones. A lower tone tends to drown a higher one.

[1] Pietro Blaserna, in his "Theory of Sound."

[2] Preyer places the lowest limit for some ears at sixteen vibrations per second.

QUESTIONS AND PROBLEMS

1. Find the vibration number for each note of the scale of which c″ is the first note.

2. What is the vibration number of c an octave below c′?

3. Find the wave length corresponding to each note of the scale of which c′ is the first, when the temperature of the air is 16° C.

4. Find the length of a resonance tube (disregarding its diameter) closed at one end, which will respond to c″ when the temperature is 16° C.

5. (*a*) The interval between e′ and c″ is called a *minor sixth*. What is the vibration ratio for this interval? (*b*) What is the note a minor sixth above a′?

6. Make out a series of fractions which shall express the vibration ratios of each tone in the diatonic scale c″, d″, etc., as compared with c′; i.e. continue the series of ratios given in § 180 through another octave.

7. What is the vibration number of a′ in a scale in which c′ (the keynote) = 256 vibrations?

8. (*a*) What is the effect of striking a bell with different degrees of force? (*b*) What change in the vibrations is produced? (*c*) What property of sound remains the same?

9. (*a*) Strike a key of a piano and hold it down. What is the only change you observe in the sound produced, while it remains audible? (*b*) What is the cause of this change?

SECTION VII

COMPOSITION OF SONOROUS VIBRATIONS AND THEIR RESULTANT WAVE FORMS

183. Coexistence and superposition of waves. Interference. When two or more currents of waves traverse the same medium at the same time and in the same direction, so that one set of waves is, as it were, superposed upon another, there are imparted to every particle of the medium simultaneously all the vibratory motions peculiar to the several waves. When two or more systems of waves act on a particle at the same time, they are said to *interfere*. The resultant motion of any particle at a given instant would be found on the principle of the composition of motions. This will be best understood by means of graphical representations. In

Fig. 147, *A*, are represented by dotted lines the wave lines of two coexisting currents of waves having the same wave length and phase, but the amplitude of one greater than that of the other. For example, the amplitudes of the vibrations for the particle *a* are respectively *ac* and *ae*. Their algebraic sum is *ad*. In like manner, the displacement of any particle of the medium traversed by the several wave currents at any instant is determined. The heavy line represents the form of the joint wave resulting from the combination of the two. It will be seen that the only change is one of amplitude or intensity.

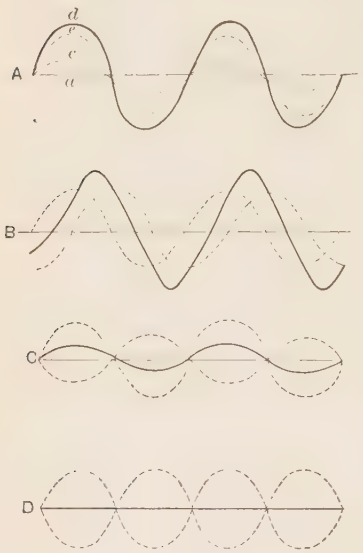


Fig. 147

In *B* are two wave currents whose waves are of the same length and amplitude but with a difference of phase of $\frac{1}{4}$ of a period, i.e. one is a quarter of a wave length behind the other. The result is a wave of the same length but of differing phase and amplitude.

In *C* are two sets of waves of like length, of different amplitudes, and of opposite phases, or one is half a wave length behind the other. The result is a set of waves of the same length but of diminished intensity. In *D* the conditions are the same as in *C* except that the components have the same amplitude. The result is that the two components destroy each other, and the particles of the medium are undisturbed, as indicated by the straight line.

In Fig. 148, *A*, are given two wave currents whose wave lengths are as $1 : \frac{1}{2}$ and whose phases in the beginning agree. The resultant of this combination with still another of $\frac{1}{4}$ the wave length of the longest is shown in *B*. In *C* is the same

combination as in *A*, but the phases differ by $\frac{1}{4}$ of a period of the shorter wave.

Fig. 149 represents wave lines drawn by a *vibrograph*. The second line represents a sound two octaves above that which the

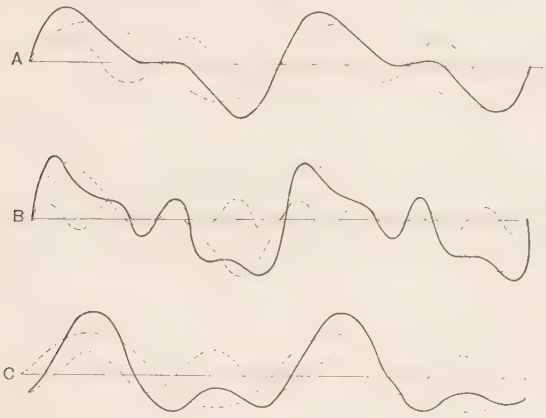


Fig. 148

first line represents, and the third line shows the result that is produced by causing the fork to have two sets of simultaneous vibrations.

One may see a typical representation of superposed waves, or currents of small waves and wavelets creeping over the backs

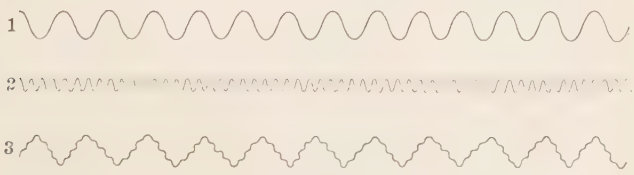


Fig. 149

of larger ones and carving their surfaces into ragged and ever-changing outlines, in watching the billows of the sea, especially when the surface is swept by a breeze of varying intensity.

In the diagrams given above only transverse vibrations are represented, but the results there depicted apply equally well to longitudinal vibrations and waves of condensations and rarefactions. In Fig. 150 the heavy line AB is a *typical* representation of the resultant of two currents of aërial sound waves an octave

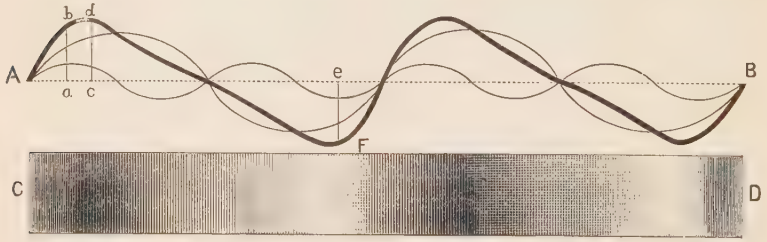


FIG. 150

apart, while the rectangular diagram CD is intended to represent a portion of a transverse section of a body of air traversed by the joint wave corresponding to the heavy wave line above. The depth of shading in different parts indicates the degree of condensation or rarefaction at those parts.

SECTION VIII

VIBRATION OF STRINGS

184. Sonometer. This instrument consists of two or more piano wires of different thicknesses stretched lengthwise over a resonance box. One end of each wire is attached to the shorter arm of a bent lever A or B (Fig 151), and the tension of the wire is regulated both by the lengths of the longer arms employed and by the magnitude of the weights suspended therefrom. The length of the vibrating portion of the strings is regulated by the sliding bridge C.

Remove the bridge C, pluck one of the strings with the fingers at the middle point, causing it to vibrate as a whole, and note the pitch of the sound. Place the bridge under the same wire, and

move it gradually toward one end of the sonometer, thereby shortening the vibrating portion; the pitch rises as the vibrating portion is shortened. Vary the position of C until a pitch is obtained an octave above the pitch given at first when the entire wire was vibrating. It will be found that the length of the wire which gives the higher note is just half the original length; i.e. *by halving the wire its vibration number is doubled.* At two thirds its original length it gives a note at an interval of a fifth

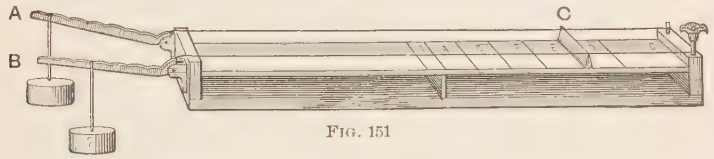


FIG. 151

above that given by its original length; and generally *the reciprocals of the fractions representing the relative vibration numbers of the several notes of a scale represent the relative lengths of the wires that produce these notes.*

Now increase the tension of the wire; the pitch rises. Increase the tension until the pitch has risen an octave; it will be found that the tension has been increased fourfold.

Next try two wires whose lengths and tensions are the same, but whose diameters are, say, as 1 : 2, and whose masses are consequently as 1 : 4; the pitch given by the wire of greater mass is an octave lower than the pitch given by the other wire.

These conclusions may be summarized thus: *The vibration numbers of strings of the same material vary inversely as their lengths and the square roots of their masses per unit length, and directly as the square roots of their tensions.*

185. Stationary waves. Hold one end of a rubber tube about two meters long, while the other is fixed, and send along it a regular succession of equal pulses from the vibrating hand; it will be easy, by varying the tension a little, to obtain a succession of loops (Fig. 152) separated by points that are nearly or quite at rest. Unlike the earlier experiments, the waves here do not appear to travel along the tube; yet in reality they do traverse

it. The deception is caused by stationary points being produced by the interference of the advancing and retreating waves.

This interference of direct and reflected waves gives rise to an important class of phenomena called *stationary waves*. The points of least motion, as *a*, *b*, and *e* (Fig. 152), are called *nodes* (from fancied resemblance to knots); the points of greatest amplitude, as *d* and *c*, are called *antinodes*; and the portions between the nodes are called *loops* or *ventral segments*.

FIG. 152

In a similar manner a string may be made to vibrate in three, four, etc., parts, as shown in *C*, *D*, and *E* (Fig. 153). The pitch of the tone produced by a string when it vibrates as a whole, as in *A*, is called the *fundamental* pitch of the string. The vibration frequency when the string divides into halves, as in *B*, is twice as great, and consequently the pitch of the tone produced is an octave above that of the fundamental. Generally the vibration frequency varies as the number of loops into which the string divides.

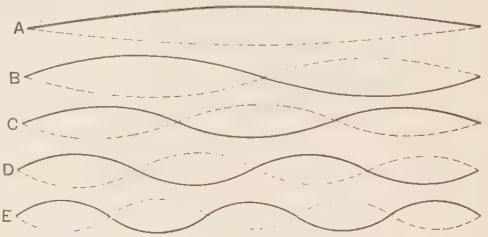


FIG. 153

Tones produced by a string or other body that vibrates in parts are called *overtones* or *upper partial tones*. If the overtones harmonize with the fundamental of the vibrating body, they are called *harmonics*.

186. Complex vibrations. Press down the c'-key gently so that it will not sound, and while holding it down strike the c-key strongly. In a few seconds release the key, so that its damper will stop the vibrations of the string that was struck, and you will hear a sound which you will recognize by its pitch as coming

from the c′-wire. Place your finger lightly on the c′-wire, and you will find that it is indeed vibrating. Press down the right pedal with the foot so as to lift the dampers from all the wires, strike the c-key, and touch with the finger the c′-wire; it vibrates. Touch the wires next to c′, namely b and d′; they have only a forced vibration. Touch g′; it vibrates.

Now it is evident that the vibrations of the c′- and g′-wires are sympathetic. But a c-wire vibrating as a whole cannot cause sympathetic vibrations in a c′-wire; but if it vibrates in halves, it may. Hence we conclude that when the c-wire was struck it vibrated not only as a whole, giving a sound of its own pitch, but also in halves; and the result of this latter set of vibrations was that an additional sound resulted from this wire, just an octave higher than the first-mentioned sound.

Again, the g′-wire makes 391.5 vibrations in a second, or three times as many (130.5) as are made by the c-wire; hence the latter wire, in addition to its vibrations as a whole and in halves, must have vibrated in thirds, inasmuch as it caused the g′-wire to vibrate. It thus appears that a string may vibrate at the same time not only as a whole, but also in halves, thirds, etc., and the result is that *a sensation is produced that is compounded of the sensations of several sounds of different pitch*. A sound so simple that it cannot be resolved is called a *simple tone*.

187. Tones and notes. A sound composed of two or more tones is called a *note*.

Not only do stringed instruments produce *notes*, but no ordinary musical instrument is capable of producing a *simple tone*, i.e. a sound generated by vibrations of a single period. In other words, *when any note of any musical instrument is sounded there is produced, in addition to the primary tone, a number of other tones in a progressive series*.[1] The primary or lowest tone of a note is called the *fundamental tone*.

[1] The so-called *Fourier theorem* translated from the language of kinematics into that of acoustics asserts that "every regular musical sound is resolvable into a definite number of simple tones whose relative pitch follows the law of the partial-tone series."

Strings when struck produce many overtones, according to the *place* where they are struck, the *nature of the stroke*, and the *density* and *elasticity* of the string.

188. Beats. If the two lowest adjacent notes of a large organ be sounded together, a peculiar wavy or throbbing sound is heard, caused by an alternate rising and sinking in loudness. Each recurrence of the maximum intensity is called a *beat*.

Let the continuous curved line AC (Fig. 154) represent a series of waves caused by sounding the lower note, and the dotted line a series of waves proceeding from the upper one. Now both waves may start together at A; but as the waves from the lower

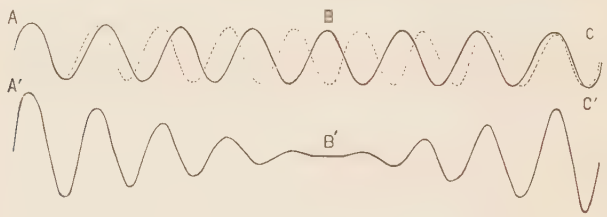


FIG. 154

note are given less frequently, so are they correspondingly longer; and at certain intervals, as at B, condensations will correspond with rarefactions, producing by their interference momentary silence, too short, however, to be perceived; but the sound as perceived by the ear is correctly represented in its varying loudness by the curved line $A'B'C'$.

It will be apparent from the study of Fig. 154 that exactly one beat will occur in each interval of time during which the acuter of two simple tones performs *one* more vibration than the graver tone.

Hence the number of beats per second due to two simple tones is equal to the difference of their respective vibration numbers.

189. Origin of harmony and discord. The harmonics in any tone are produced successively by two, three, etc., times the number of vibrations made by its fundamental. Hence, if any two notes

an octave apart — for instance, c' and c'' — be sounded simultaneously, there will result for

$$\left.\begin{array}{l} c',\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ \text{etc.} \\ c',\ \ \ 2,\ \ \ 4,\ \ \ 6,\ \text{etc.} \end{array}\right\} \text{times the number of vibrations made}$$

by the fundamental of c'; so that the fundamental of c'', and its overtones (with the exception of the highest overtones, which are too feeble to affect the general result), are in perfect unison with certain overtones of c'. Not only is there perfect agreement among the overtones of two notes an octave apart when sounded together, as when male and female voices unite in singing the same part of a melody, but the richness and vivacity of the sound are much increased thereby.

Discord produced by two sounds is explained by the fact that the sounds produce *beats*, which do not coalesce because the interval between them is too long.

As the frequency of the beats increases, a point is finally reached where they cease to be recognized as distinct sounds and where they blend into a more or less pure tone. Beats may thus coalesce to produce beat tones that are musical.

It must not, however, be inferred that dissonance disappears immediately upon the intermittences becoming too rapid for individual recognition. If two tones form a narrower interval than a minor third, the combined sound is harsh and unpleasant.

Two tones must be in unison to produce absolutely perfect harmony. The nearest approach to it is an interval of an octave, and next in rank to the latter is a fifth.

That two notes sounded together may harmonize, it is essential not only that the pitch of their fundamental tones be so widely different that they cannot produce audible beats, but that no audible beats shall be formed by their overtones, or by an overtone and a fundamental.

For example, let the vibration numbers of the fundamentals of c' and its octave c'' be respectively 264 and 528; the number of beats that they give is 264 in a second. If, instead of c'', a

note the vibration number of whose fundamental is 527 be sounded with c′, the number of beats produced by their fundamentals would be 263, and no discord would result therefrom ; but there would be one beat per second between the first overtone of c′ and the fundamental of c″, and this would introduce a discord.

Observe that the relation between the vibration numbers of the fundamentals of c and c′, c and g, c and f, and c of any diatonic scale and any note in the same scale can be expressed in terms of small numbers, e.g. 1 : 2, 2 : 3, 3 : 4, etc. Generally, *those notes and only those harmonize whose fundamental tones bear to one another ratios expressed by small numbers ; and the smaller the numbers which express the ratios of the rates of vibration, the more perfect is the harmony of two sounds.*

Not only may *two* notes whose relative vibration frequency is expressible by a simple ratio harmonize, but three or four may concur with the same result. A sound produced by the coexistence of three or more notes is called in music a *chord*. A consonant chord is a *concord ;* a dissonant chord is a *discord.*

It follows, from what has been said, that only a limited number of notes can be sounded with any given note assumed as a prime without generating discord. Hence the musical scale is limited to certain determinate degrees, represented by the eight notes of the so-called *musical* or *diatonic scale.* This scale is not the result of any arbitrary or fanciful arrangement, but is composed of notes selected because they harmonize with the prime of the scale, both as regards their fundamental tones and their overtones.

QUESTIONS AND PROBLEMS

1. Prepare a table of the series of overtones of c and g respectively, as in § 189, and ascertain what overtones of the two series are in unison.

2. Arrange the notes in a single octave of the diatonic scale in the order of their rank with reference to their harmonizing with the prime of the scale, on the principle that "the smaller the numbers which express the ratios of vibration, the more perfect is the harmony of two sounds."

3. Verify your conclusions as follows : strike the c-key of a piano, together with each of the seven white keys above it, consecutively, and compare the results of the different pairs with reference to harmony.

SECTION IX

QUALITY OF SOUND

190. Simple sound waves can differ only in length and amplitude; consequently the sounds which they produce can differ only in pitch and loudness. Complex sound waves may differ, as we have seen, in *shape*, and this gives rise to a property of sound called *quality*. Quality is that property of sound, not due to pitch or intensity, that enables us to distinguish one sound from another.

Although the variety of sounds one hears appears well-nigh infinite, yet no two sounds can differ from each other in any other respect than pitch, loudness, or quality. The length, amplitude, and form of the wave completely determine the wave, and these three elements of a wave are mutually independent, i.e. any one may be changed without altering the other two. Loudness depends on amplitude of vibrations, pitch on vibration frequency, and quality on complexity of the motion of the vibrating particles.

Let the same note be sounded with the same intensity, successively, on a variety of musical instruments, e.g. a violin, cornet, clarinet, accordion, jew's-harp, etc.; each instrument will emit the same number of waves, and the waves from each will strike the ear with the same impulse, yet the ear is able to distinguish a decided difference between the sounds, — a difference that enables us instantly to identify the instruments from which they come. Sounds from instruments of the same kind, but by different makers, usually exhibit decided differences of character. For instance, of two pianos, the sound of one will be described as richer and fuller, or more ringing, or more "wiry," etc., than that of the other. No two human voices sound exactly alike.

SECTION X

ANALYSIS AND SYNTHESIS OF SOUND WAVES

191. Analysis. The unaided ear is unable, except to a very limited extent, to distinguish the individual tones that compose a note. Helmholtz arranged a series of resonators of brass nearly spherical in shape, each having two openings ; one, *A* (Fig. 155), large, for the reception of the sound waves, and the other, *B*, small and funnel-shaped, and adapted for insertion into the ear. Each resonator of the series was adjusted by its size and shape to resound powerfully to only a single tone of a definite pitch. When any musical sound is produced in front of these resonators, the ear, placed at the orifice of any one, is able to single out, from the total number of tones composing the note, that overtone, if present, to which alone this resonator is capable of responding. By applying one resonator after another to the ear a sound is analyzed into its components. It is thus found, for instance, that the notes of a clarinet are composed only of the odd harmonics, or of tones whose vibration numbers are in the ratios of 1 : 3 : 5 : 7 ; and that the notes of a flute are substantially those of a tone and its octave. It is found that when a note is produced on a given instrument, not only is there a great variety of intensity represented by the overtones, but all the possible overtones of the series are by no means present. Which are wanting depends very much, in stringed instruments, upon the point of the string struck ; for example, if a string be struck at its middle point, no node can be formed at that point, and consequently the two important overtones produced by two and four times the number of vibrations

FIG. 155

of the fundamental will be wanting. Strings of pianos, violins, etc., are generally struck near one of their ends, and thus they are deprived only of some of their higher and feebler overtones.

"Every vowel is a particular quality of sound." The mouth cavity acts as a resonator and reflector. To each vowel corresponds a different form of resonating mouth cavity. Upon the number of upper overtones which are reënforced, and the relative intensities of the reënforcement, depends the quality of the vowel sound produced. Vowel sounds may be analyzed in a very interesting manner, as follows: raise the cover of a grand piano, press down the "loud pedal," and sing strongly some vowel sound, projecting the voice upon the exposed strings. When you cease to sing, that vowel will be repeated by the strings. Each component of the complex vibration will be taken up by the string in unison with that component, and by noticing which strings vibrate a qualitative analysis of the sound is effected.

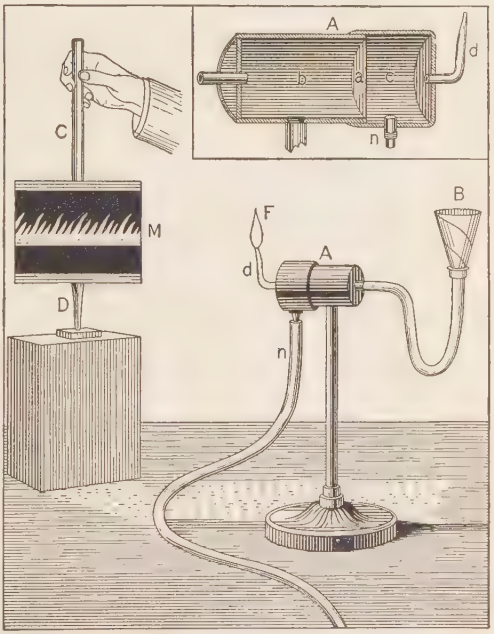


FIG. 156

192. Manometric flames. The pitch, intensity, and quality of a sound may be studied at the same time by causing sound waves to impinge directly upon some sensitive body without any

intermediate process of selection Apparatus like that shown in Fig. 156 will serve to illustrate.

The cylindrical box A is divided by the membrane a into two compartments, c and b. Illuminating gas is introduced into the

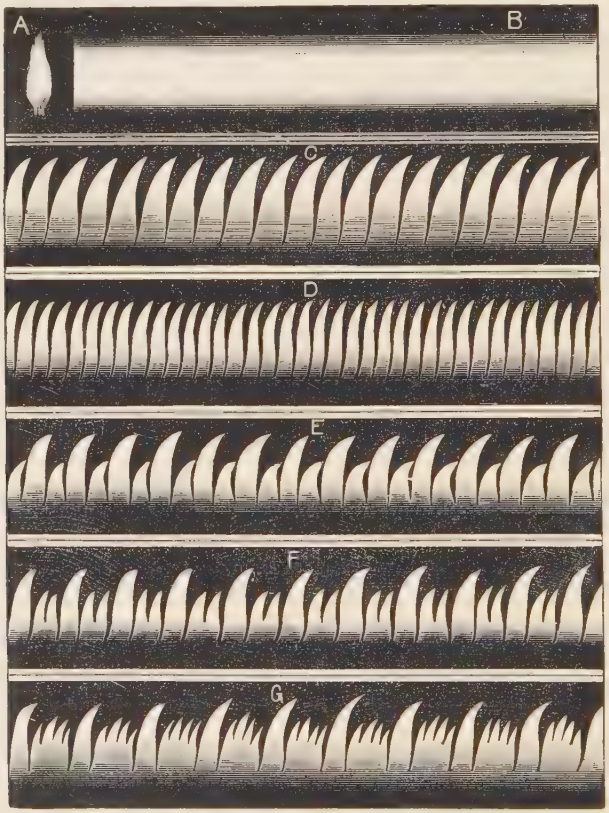


FIG. 157

compartment c through the rubber tube n, and burned at the orifice d. CD is a frame holding two mirrors, M, placed back to back, so that whichever side is turned toward the flame there is a reflection of the flame.

When the mirror is at rest an image of the flame will appear in the mirror, as represented by *A* (Fig. 157). If the mirror be rotated, the flame appears drawn out in a band of light, as shown in *B* of the same figure.

Sing into the cone *B* (Fig. 156) the sound of *oo* in tool, and waves of air will run down the tube and beat against the membrane *a*, causing it to vibrate. The membrane has impressed upon it a complex motion resembling the original compound vibration of the vocal chords or other sounding body. The membrane in turn acts upon the gas in the compartment *c*, varying its pressure in a manner corresponding to the vibrations of the membrane.

The result is, that instead of a flame appearing in the rotating mirror as a continuous band of light, as *B* (Fig. 157), it is divided into a series of tongues of light, as shown in *C*, each condensation being represented by a tongue, and each rarefaction by a dark interval between the tongues. The number and size of the greater tongues indicate the frequency and amplitude of the fundamental vibration; the subsidiary serrations correspond to the subsidiary vibrations or overtones.

If a note an octave higher than the last be sung, we obtain, as we should expect, twice as many tongues in the same space, as shown in *D*. *E* represents the result when the two tones are produced simultaneously, and illustrates in a striking manner the effect of interference. *F* represents the result when the vowel *e* is sung at the pitch of c''; and *G*, when the vowel *o* is sung on the same key. These are called *manometric flames.*

193. The phonograph. As the name indicates, this is an instrument for recording sound waves. As a matter of fact it is really more, since it is capable of reproducing these waves with all their original characteristics very little changed. It consists essentially of a rather hard cylinder of a wax composition against the surface of which rests a metal point attached to the middle of a flexible diaphragm *D* (Fig. 158) against which the waves impinge. The cylinder is rotated by an appropriate motor and at the same time the mouthpiece *B* advances the style longitudinally, thereby

tracing a spiral groove on the cylinder's surface. If now a sound be directed against the membrane, the latter causes the point to indent the wax groove in a manner characteristic of the particular sound waves employed, and in this manner a permanent record is made on the cylinder. In order to reproduce the sound it is only necessary to move the point with its diaphragm back to the place of starting, and again to cause the cylinder to rotate at the same speed as before. As the point now has to follow the same indented irregularities on the wax that were made during its first trip, the membrane is again thrown into identically the same series of vibrations that it had when receiving the sound waves from the air. The result is, that the air now receives these waves back again, and in the improved instruments of the present time the reproduction of the sound in all its properties is most faithful. In the original machine invented by Edison the recording surface was tin foil.

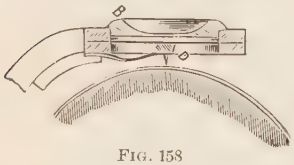


FIG. 158

194. Synthesis of sound waves. The sound of a tuning fork when its fundamental is reënforced by a suitable resonance cavity is very nearly a simple tone.

If two mounted forks forming the interval of an octave be sounded together, the tones proceeding from the separate forks soon blend together into one sound, to which we assign the pitch of the lower fork, with a quality richer than that of either. So strong is the illusion that we cannot believe the higher fork to be sounding, until we ascertain that placing a finger on its prongs so as to damp its vibrations at once changes the timbre, reducing it to the dull, uninteresting quality of a simple tone. If to these two forks there be added a fork whose interval is a fifth above the higher (i.e. one which gives the second harmonic of the first), the three tones blend as perfectly as did those of the two forks, the only difference perceptible being an additional increase of richness.

By sounding simultaneously several forks of different but appropriate pitch, and with the requisite relative intensities,

Helmholtz succeeded in producing sounds peculiar to various musical instruments, and even in imitating most of the vowel sounds of the human voice.

Thus it appears that he has been able to determine, both *analytically* and *synthetically*, that *the quality of a given sound depends upon what overtones combine with its fundamental tone, and upon their relative intensities:* or, more briefly, *upon the form of the sound wave*, since this form must be determined by the character and number of its components.

SECTION XI

MUSICAL INSTRUMENTS

195. Classification of musical instruments. Musical instruments may be grouped into three classes: (1) stringed instruments; (2) wind instruments, in which the sound is due to the vibration of columns of air confined in tubes; (3) instruments in which the vibrator is a membrane or plate. The first class has received its share of attention; the other two merit a little further consideration.

196. Wind instruments. By referring to § 173 it will be seen that the general theory of wind instruments has been anticipated and explained under the head of Resonance. The pitch of vibrating air columns, as well as that of strings, varies with the length, and (1) *in both stopped and open pipes the number of vibrations is inversely proportional to the length of the pipe:* (2) *an open pipe gives a note an octave higher than a closed pipe of the same length.* The terms *stopped* and *open* apply to only one end of the pipe: the other, in both kinds, is always open.

In Fig. 159 we have represented a set of Quincke's whistles consisting of tubes of the same diameter but of different lengths. To each is attached a smaller pipe, by blowing through which the larger tube may be sounded. By placing a finger on the bottom

of any tube it becomes "closed." With this simple apparatus the above laws of sounding air columns may be verified.

If the longer pipes be sounded with a side current considerably stronger than is necessary for sounding the lowest natural tone

FIG. 159

of the pipe, the first overtone may be produced. A still stronger blast will bring out one or more higher overtones.

197. Organ pipes. The common flue organ pipe differs from one of the above whistles only in the form of its mouthpiece. This is usually at the bottom of the pipe (Fig. 160), and through it the air issues upward in the form of a thin sheet, striking upon a sharp edge placed just above the opening, as is seen in the cut. By this means at the start a disturbance is created, which runs up the tube, is reflected at the upper end (whether open or closed),

and returns. When the returning pulse reaches the mouthpiece the effect is to deflect the steady blast outward. Immediately, of course, the blast returns, thereby sending another pulse up the tube and back. The blast is thus seen to become oscillating, the period being the interval required for the pulse to mount the tube and return, which is the period of the *stationary wave* in the air column. It follows then that the pitch is not dependent on the mouthpiece at all but on the length of the air column. That the character of the sound is independent of the material of which the pipe is composed may be shown by constructing, say, three different pipes of exactly the same size and shape, but one each of wood, brass, and pasteboard. On sounding them in succession little or no difference can be detected in the resulting sounds.

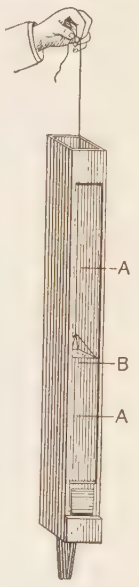


FIG. 160

Fig. 160 represents an open organ pipe provided with a glass window A in one of its sides. A wire hoop B has a membrane stretched over it, and the whole is suspended by a thread within the pipe. If the membrane be placed near the upper end, a buzzing sound proceeds from the membrane when the fundamental tone of the pipe is sounded; and sand placed on the membrane will dance up and down in a lively manner. On lowering the membrane the buzzing sound becomes fainter, till, at the middle of the tube, it ceases entirely, and the sand becomes quiet. Lowering the membrane still further, the sound and dancing recommence, and increase as the lower end is approached. This shows that *when the fundamental tone of an open pipe is produced, its air column divides into two equal vibrating sections, with the antinodes at the extremities of the tube and a node in the middle.*

If the pipe be stopped, there is a node at the stopped end; if it be open, there is an antinode at the open end; and in both cases there is an antinode at the end where the wind enters, which is always open to a certain extent.

A, *B*, and *C* of Fig. 161 show respectively the positions of the nodes and antinodes for the fundamental tone and first and second overtones of a closed pipe; and *A'*, *B'*, and *C'* show the positions of the same in an open pipe of the same length. The horizontal distances between the dotted lines show the relative amplitudes of the vibrations of the air particles at various points along the tube. Now the distance between a node and the nearest antinode is a quarter of a wave length. Comparing, then, *A* and *A'*, it will be seen that the wave length of the fundamental of the closed pipe must be twice the wave length of the fundamental of the open pipe; hence the vibration period of the latter is half that of the former, and consequently the fundamental of the open pipe must be an octave higher than that of the closed pipe, as was shown by the experiments with the whistles.

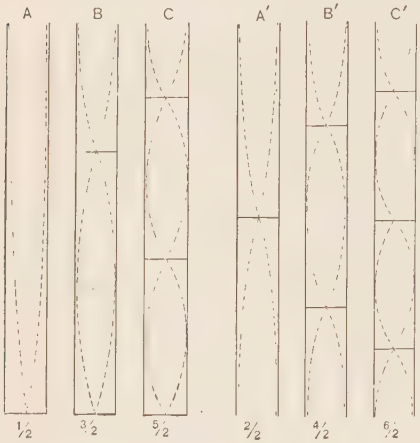


FIG. 161

In the three cases (*A*, *B*, and *C*) of the closed tube the length of the air column is divided into $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ segments respectively; hence the corresponding vibration numbers are as 1 : 3 : 5, etc. Hence, *in a closed tube, only those overtones whose vibration numbers correspond to the odd multiples of that of the fundamental are present.*

In the cases of the open tube the length of the air column is divided into $\frac{2}{2}$, $\frac{4}{2}$, and $\frac{6}{2}$ segments respectively; their vibration numbers are therefore as 1 : 2 : 3, etc. Hence, *in the sound from an open tube, the complete series of overtones corresponding to its fundamental may be present, and for this reason the tone is richer and pleasanter to listen to.*

198. Vibrating rods. The longitudinal vibration of rods is subject to just the same laws as that of air columns. Since in an open organ pipe the period is the time required for a sound wave to travel twice the length of the pipe, $\lambda = 2\,l$; and since $v = n\lambda$, $v = 2\,nl$. This relation at once suggests a very simple method for determining the speed of sound waves in various solid substances. By multiplying twice the length of a rod by the vibration frequency of the note emitted by it when clamped in the middle and vibrated longitudinally, the result is the speed. If the rod be fastened at one end, its behavior is similar to that of the air in a closed pipe, in which case $\lambda = 4\,l$, and $v = 4\,nl$.

The transversal vibration of rods is far less simple, the frequency being expressed by the formula

$$n = C\,\frac{\tau}{l^2}\sqrt{\frac{M}{\delta}},$$

in which τ is the thickness, l the length, M Young's modulus of the material, δ its density, and C a constant depending on the method of supporting the rod.

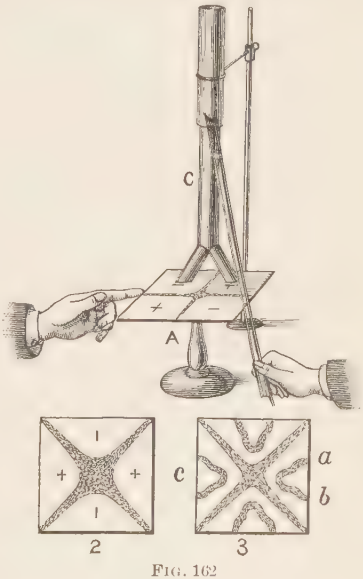


FIG. 162

199. Sounding plates. If an elastic brass plate A (Fig. 162) be fastened in a horizontal position to an upright support, one may, by sprinkling over its surface a thin layer of sand, study the various vibrations of the plate. The method of sounding it is to draw a rosined bass bow steadily and firmly over one of its edges near a corner, and at the same time touch the middle of one of its edges with the tip of the finger; a musical sound will be produced, and the sand will dance up and down, and quickly collect

(1) in two rows, extending across the plate at right angles to each other. Draw the bow across the middle of an edge, and touch with a finger one of its corners; the sand will arrange itself (2) in two diagonal rows across the plate, and the pitch of the note will be a fifth higher. Touch, with the nails of the thumb and forefinger, two points a and b on one edge, and draw the bow across the middle, c, of the opposite edge, and you will obtain (3) additional rows and a shriller note.

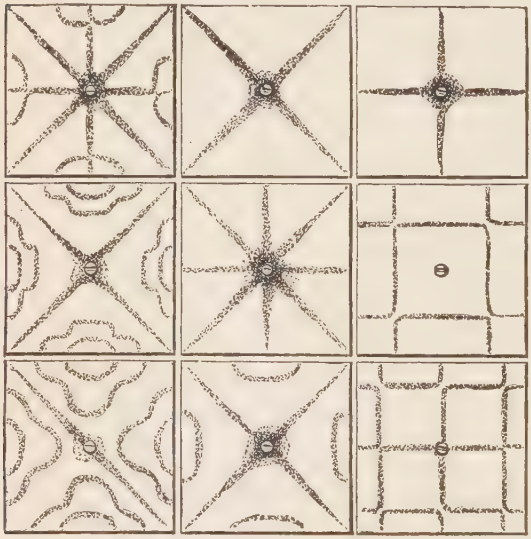


FIG. 163

By varying the positions of the points touched and bowed, a great variety of patterns can be obtained, some of which are represented in Fig. 163. It will be seen that the effect of touching the plate with a finger is to prevent vibration at that point, and consequently a node is there produced. The whole plate then divides itself up into segments with nodal division lines in conformity with the node first started. The sand rolls away from those parts which are alternately thrown into crests and troughs, to the parts that are at rest.

200. Interference. C (Fig. 162) is a tin tube made in two parts that telescope one within the other. The extremity of one of the parts terminates in two slightly smaller branches. Bow the plate as suggested above, place the two orifices of the branches over

the segments marked with the + signs, and regulate the length of the tube so as to reënforce the note given by the plate, and set the plate in vibration. Now turn the tube around, so that one orifice may be over a + segment and the other over a − segment; the sound due to resonance entirely ceases. It thus appears that the two segments marked + pass through the same phases together; likewise the phases of − segments correspond with one another, i.e. when one + segment is bent upward the other is bent upward, and at the same time the two − segments are bent downward; for, when the two orifices of the tube are placed over two + segments or two − segments, two condensations followed by two rarefactions pass up these branches and unite at their junction to produce a loud sound; but when one of the orifices is over a + segment and the other over a − segment, a condensation passes up one branch at the same time that a rarefaction passes up the other, and the two destroy each other when they come together; i.e. the two sound waves combine to produce silence.

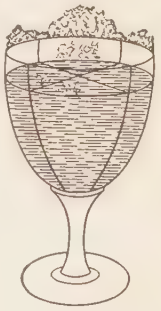


Fig. 164

201. Bells. A bell or goblet is subject to the same laws of vibration as a plate.

Nearly fill a large goblet with water, strew upon the surface lycopodium powder, and draw a rosined bow gently across the edge of the glass. The surface of the water will become rippled with wavelets (Fig. 164) radiating from four points 90° apart, corresponding to the centers of four venters into which the goblet is divided, and the powder will collect in lines proceeding from the nodal points of the bell. By touching the proper points of a bell or glass with a finger nail it may be made to divide itself, like a plate, into 6, 8, 10, etc. (always an even number), vibrating parts.

Remove the brass plate (Fig. 162) from its support and fasten the bell *B* (Fig. 165) on the support. Bow the edge of the bell at some point, and hold the open tube *C* in a horizontal position, with the center of one of its openings near that point of the edge

of the bell which is opposite the point bowed. The tube loudly reënforces the sound of the bell. Move the tube around the edge of the bell and find its nodes.

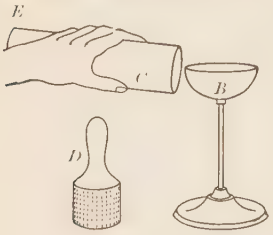


FIG. 165

Thrust the plunger *D* into the open end *E* of the tube, and find what part of the length of an open tube a closed tube should be to reënforce a sound of a given pitch.

202. Vocal organs. It is difficult to say which is more to be admired, the wonderful capability of the human voice or the extreme simplicity of the means by which it is produced. The organ of the voice is a reed instrument situated at the top of the windpipe, or trachea. A pair of elastic bands *aa* (Fig. 166), called the *vocal chords*, is stretched across the top of the windpipe. The air passage *b* between these chords is freely open while a person is breathing, but when he speaks or sings they are brought together so as to form a narrow, slitlike opening, thus making a sort of double reed, which vibrates when air is forced from the lungs through the narrow passage, somewhat like the little tongue of a toy trumpet. The sounds are grave or high according to the tension of the chords, which is regulated by muscular action. The cavities of the mouth and the nasal passages form a compound resonance tube. This tube adapts itself, by its varying width and length, to the pitch of the note produced by the vocal chords. Place a finger on the protuberance of the throat called "Adam's apple," and sing a low note; then sing a high note, and you will observe that the protuberance rises in the latter case, thus shortening the distance between the vocal chords and the lips. Set a tuning fork in vibration, open the mouth as if about to sing the

FIG. 166

corresponding note, place the fork in front of it, and the cavity of the mouth will resound to the note of the fork, but will cease to do so when the mouth adapts itself to the production of some other note. The different qualities of the different vowel sounds are produced by the varying forms of the resonating mouth cavity, the pitch of the fundamental tones given by the vocal chords remaining the same. This constitutes *articulation.*

Strictly speaking, consonants are not distinct sounds or the representatives of sound. They represent rather, when they precede vowels, the different positions of the organs of speech from which (like springboards, as it were) the vowels are attacked.[1] When following vowels, they represent the position of the organs of speech at the interruption of the vowel sounds, and the consequent modifications of these sounds. Only vowel sounds are musical, and any language is musical in proportion to the number of vowels it contains. Thus Greek is a more musical language than Latin, and Italian than German.

203. The ear. In Fig. 167, *A* represents the external ear passage; *a* is a membrane called the *tympanum*, stretched across the bottom of the passage, and thus closing the orifice of a cavity *b*; *c* is a chain of small bones stretching across this cavity, and connecting the tympanum with the thin membranous wall of the *vestibule e*; *f* is one of a series of semicircular canals opening into the vestibule; *g* is an opening into another canal in the form of a snail shell, hence called the *cochlea*; *d* is a tube (the *Eustachian tube*) connecting the drum with the throat; and *h* is the auditory nerve. The vestibule and all the canals opening into it are filled with a transparent liquid. The drum of the ear contains air, and the Eustachian tube forms a means of ingress and egress for air through the throat.

Now how does the ear hear? and how is it able to distinguish between the infinite variety of form, rapidity, and intensity of aërial sound waves so as to interpret correctly the corresponding quality, pitch, and loudness of sound? Sound waves enter the

[1] A common fault with young singers is attempting to "sing the consonants."

external ear passage *A* as ocean waves enter the bays of the sea-coast, are reflected inward, and strike the tympanum. The air particles, beating against this drumhead, impress upon it the precise wave form that is transmitted to it through the air from the sounding body. The motion received by the drumhead is transmitted by the chain of bones to the membranous wall of the vestibule. From the walls of the spiral passage of the cochlea

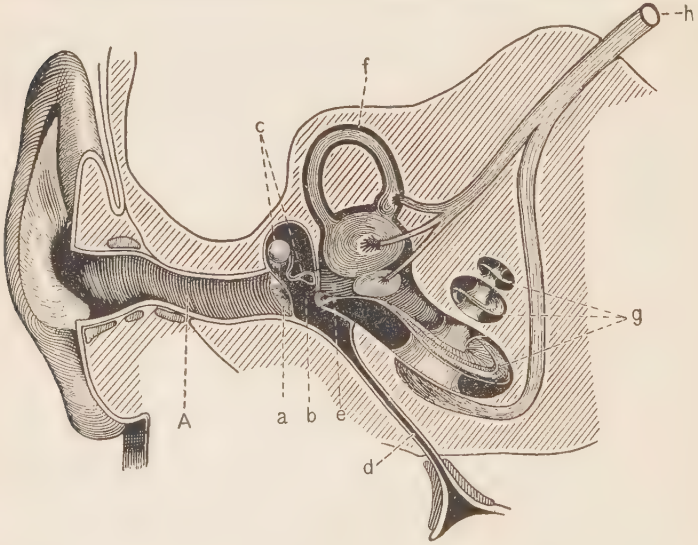


FIG. 167

project into its liquid contents thousands of fine elastic threads or fibers called *rods of Corti*. As the passage becomes smaller and smaller these vibratile rods become of gradually diminishing length and size (such as the wires of a piano may roughly represent), and are therefore fitted to respond sympathetically to a great variety of vibration periods. This arrangement is sometimes likened to a "harp of three thousand strings" (this being about the number of rods). The auditory nerve at this extremity is divided into a large number of filaments, like a cord unraveled at its end, and one of these filaments is attached to each rod. Now,

as the sound waves reach the membranous wall of the vestibule, they set it, and by means of it the liquid contents, into *forced vibration*, and so through the liquid all the fibers receive an impulse. Those rods whose vibration periods correspond with the periods of the constituents forming the compound wave are thrown into *sympathetic vibration*. The rods stir the nerve filaments, and the nerve transmits to the brain the impressions received. Much as a piano, when its dampers are raised and a person sings into it, may be said to analyze each sound wave, and show by the vibrating strings of how many tones it is composed, as well as their respective pitches, and by the amplitude of their vibrations their respective intensities; so, it is thought, this wonderful harp of the ear analyzes every complex sound wave into a series of simple waves. Tidings of the disturbances are communicated to the brain, and there, in some mysterious manner, these disturbances are interpreted as sound of *definite quality, pitch, and intensity.*

MOLECULAR DYNAMICS — HEAT

SECTION I

EARLY THEORIES

204. History. During the last century the scientific theory of heat has been completely changed. Heat was formerly thought to be an imponderable kind of matter, called *caloric*, very intimately associated with ordinary matter, and from which it could be extracted by pressure or friction. About 1798, however, Count Rumford performed a series of experiments in the arsenal at Munich, which demonstrated beyond question that heat cannot be matter at all but something produced by work, and hence energy.

Rumford's experiment consisted in boring with a blunt tool a cannon immersed in water, and noting that the water could be raised nearly to the boiling point while a very few chips were produced by the boring tool. His ready conclusion was that the heat evolved in such large quantities from the few borings could not possibly be a material substance. It was not, however, till fifty years later that Joule made his conclusive experiments to determine the quantitative relation of work to heat and thereby overthrew completely the material theory of heat.

In the preceding pages the modern theory of heat has been anticipated several times; we are now better qualified to judge of its validity.

205. Energy of mass motion convertible into heat. If some small steel tool be held upon a rapidly revolving dry grindstone, a shower of sparks flies from the stone. Place a tenpenny nail upon a stone and hammer it briskly ; it soon becomes too hot to be handled with comfort.

It is observed that in each case heat is generated at the expense of work or mass energy; i.e. *mass energy destroyed becomes heat.* When the brakes are applied to the wheels of a rapidly moving railroad train, its energy is converted into heat, much of which may be found in the wheels, brake blocks, and rails. The meteorites, or "shooting stars," which are seen at night passing through the upper air sometimes strike the earth, and are found to be stones heated to a light-giving state. They become heated when they reach our atmosphere in consequence of their motion being checked by the resistance of the air.

206. Heat convertible into mass energy. Take a thin glass flask A (Fig. 168), half fill it with water, and fit a cork air-tight into its neck. Perforate the cork, insert a glass tube B bent as indicated in the figure, and extend it into the water. Apply heat to the flask; soon the liquid rises in the tube and flows from its upper end.

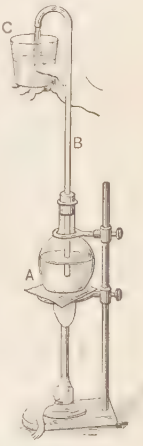


FIG. 168

Here heat produces mechanical motion, and does work in raising a mass in opposition to gravitation. Every steam engine is a *heat engine;* i.e. the power of steam is due to its heat. The steam which leaves the cylinder of an engine, after it has set the piston in motion, is cooler than when it entered.

It will be shown hereafter that in all cases when work is done by heat there is a definite quantitative relation between the heat and the work done.

If heat be consumed, and mechanical work thereby performed, we are justified in saying that *heat has been transformed into mass energy;* and, conversely, if mass energy be expended and heat thereby produced, we may say that *mass energy has been transformed into heat.*

Now, when the appearance of one thing is so connected with the disappearance of another that the quantity of the thing produced can be calculated from the quantity of that which disappears, we conclude that the one is formed at the expense of the

other, and that they are only different forms of the same thing. We have, therefore, reason to believe that *heat is of the same nature as mass energy;* i.e. *it is only another form of energy.*

207. Theory of heat. A body loses motion in communicating it. The hammer descends and strikes the anvil; its motion ceases, but the anvil is not sensibly moved; the only observable effect produced is heat. Instead of the motion of the hammer or anvil, there is now an increased *vibratory* motion of the *molecules* that compose the hammer and anvil, — *simply a change from molar to molecular motion.* Of course this latter motion is invisible. According to this view, *heat is but a name for the energy of vibration of the molecules of a body;* or, briefly, *heat is molecular kinetic energy.* The science which treats of heat as a form of energy is called *thermodynamics.*

A body is heated by having the motion of its molecules quickened, and cooled by parting with some of its molecular motion. *One body is hotter than another when the average kinetic energy of the molecules in it is greater than that in the other.*

Heat is often spoken of as the "lowest form of energy"; that is, all other forms of energy are more easily converted into heat than into anything else, and experience leads us to believe that finally all forms of energy will be degraded into heat.

208. Mechanical energy a source of heat. As heat is energy, so *all heat must originate in some form of energy,* i.e. *by the transformation of some other form of energy into heat.*

In the preceding section it was shown that heat may be generated at the expense of molar motion; i.e. molar motion checked usually results in molecular motion, the energy of which is heat. By friction, by compression, by percussion, or by any process by which mass motion is arrested, heat is mechanically generated.

209. Chemical union a source of heat. Take a glass test tube half full of cold water and pour into it one fourth its volume of strong sulphuric acid: The liquid at once becomes quite hot.

When water is poured upon quicklime heat is rapidly developed. The oxygen of the air combines with the constituents of the

various fuels, such as wood, coal, oils, and illuminating gas, and gives rise to what we call *burning*, or *combustion*, by which a large amount of heat is generated. In all such cases the heat is generated by the combination or clashing together of molecules of substances that have an affinity (i.e. an attraction) for one another. Before union the energy of the molecules is of the same kind as that of a stone on a shelf. When the shelf is withdrawn the potential energy of the stone becomes kinetic; so chemical affinity converts the potential energy of the molecules into kinetic energy of vibration, i.e. into heat.

When a definite mass, say of carbon or hydrogen, is burned the quantity of heat produced is definite, showing that the different fuels have a definite heat value.

In a majority of cases chemical union is attended by the evolution of heat; but in some cases work has to be done by heat upon separate elements to force them to combine either directly or indirectly; hence in such cases the union is attended by a consumption or disappearance of heat, and the decomposition of compounds thus formed is attended by an evolution of heat.

210. Origin of animal heat and muscular action. The plant finds its food in the air (principally the carbon dioxide in the air) and in the earth, in a condition analogous to that of a fallen weight; but, by the agency of the sun's radiation, work is performed upon this matter during the growth of the plant; potential energy is stored in the plant, — the weight is drawn up, as it were. The animal now finds its food in the plant, appropriates the potential energy stored in the plant, and, by chemical action, chiefly by the union of carbon and hydrogen with oxygen, this energy is converted into the energy of motion in the form of heat and muscular action, — the weight falls and its energy becomes kinetic. The plant, then, may be regarded as a machine for converting energy of motion received from the sun into potential energy; the animal, as a machine for transforming it again into energy of motion.

211. The sun as a source of energy. The sun is not only the source of the energy exhibited in the growth of plants, as well

as of the muscular and heat energy of the animal, but is also the source, directly or indirectly, of very nearly all the energy employed by man in doing work. Our coal beds, the results of the deposit of vegetable matter, are vast storehouses of the sun's energy, rendered potential during the growth of the plants many ages ago. Every drop of water that falls to the earth and rolls its way to the sea, contributing its mite to the immense water power of the earth, and every wind that blows, derives its power directly from the sun.

SECTION II

TEMPERATURE

212. Temperature defined. The words "hot," "warm," "cool," "cold," are associated in our minds with a series of sensations which we suppose to indicate a corresponding series of states of an object with respect to heat; that is, the agent which produces these sensations is heat. These are all temperature terms, and refer to the state of a body with reference to heat. *Temperature is the state of matter in respect to heat.* Assuming no change of state, when the quantity of heat in a body increases, its temperature is said *to rise*; and when this diminishes, its temperature is said *to fall*. The relation which temperature bears to heat is analogous to that which hydrostatic pressure bears to water. Water flows from a higher to a lower level; heat flows from a higher to a lower temperature. When we pour water into a vessel the level rises; so heat entering a body raises its temperature, unless it is transformed in doing work. It takes more water to fill a large vessel to a given depth than a small one; it takes more heat to raise the temperature of a body of large mass than to raise the temperature of a smaller mass of the same substance an equal amount. Depth of water in vessels of varying size is not proportional to the quantity of water they contain; temperature of bodies of varying mass is not necessarily proportional to the quantity of heat they

contain. For example, a pint of water at a given temperature does not contain the same quantity of heat as a gallon of water at the same temperature.

If body A be brought into contact with body B, and heat flows from A to B, then A is said to be at a higher *temperature* than B. *Temperature is sometimes spoken of as the state of a body with reference to its tendency to communicate heat to, or receive heat from, other bodies.* The direction of the flow of heat determines which of two bodies has the higher temperature. If the temperature of neither body rise at the expense of the other, then both have the same temperature, or are said to be in *thermal equilibrium.*

Temperature depends on the average kinetic energy of the molecules. Bodies have the same temperature when the average energy of the molecules of each is the same.

213. Temperature a relative term. As the term is now used, temperature is a relative expression, not an expression of absolute quantity. It is regarded rather as a *quality* capable of greater or less intensity, than as a *quantity* which may be added to or subtracted from other quantities of the same kind.

There is no propriety in the expression "twice as hot." The term "temperature" is similar to the term "hardness," insomuch as we are able to construct a scale for both, so that a body may have a definite place in the scale and be less hot or less hard than something above it.

SECTION III

THERMOMETRY

214. Since, then, temperature is not a quantity directly measurable, it can be indicated only by measuring some one of the effects produced by heat. Expansion is such an effect, and, other things being equal, it is closely proportional in general to the temperature change. The instrument used to indicate temperature and its changes is called a thermometer.

215. Construction of a thermometer. This instrument consists generally of a glass tube of uniform capillary bore, terminating at one end in a bulb. The bulb and part of the tube are filled with mercury, which for several reasons is eminently suitable for the purpose, and the space in the tube above the mercury contains only mercury vapor. On the tube is a scale to show the height of the mercurial column.

216. Graduation of thermometers. That a thermometer may indicate any definite temperature, it is necessary that its scale should relate to some definite and unchangeable temperatures. Fortunately nature furnishes us with two convenient standards. It is found that under ordinary atmospheric pressure ice always melts at the same temperature, called the *melting point*. Again, the temperature of steam rising from boiling water *under the same pressure* is always the same.

First, the bulb of a thermometer is placed in melting ice and allowed to stand until the surface of the mercury becomes stationary, and a mark is made upon the stem at that point, which indicates the *melting point*. Then the instrument is suspended in steam rising from boiling water, so that all but the very top of the column is in the steam. The mercury rises in the stem of the thermometer until its temperature becomes the same as that of the steam, when it becomes stationary, and a mark is placed on the stem to indicate the *boiling point*. This boiling point is the temperature of steam at a pressure of 760 millimeters of mercury, assuming that due consideration has been given to any variation from normal of the air pressure at the time. Then the space between the two points found is divided into a convenient number of parts of equal volume, called *degrees*, and the scale is extended above and below these points as far as is desirable.

Two methods of division are adopted in this country (see *a* and *b*, Fig. 169): in one, the space is divided into 180 equal parts, and the result is called the *Fahrenheit* scale, from the name of its designer; in the other, the space is divided into 100 equal parts, and the resulting scale is called *Centigrade*, which means

one hundred steps. The Centigrade scale, which is generally employed by scientists, has its melting and boiling points conveniently marked, respectively 0° and 100°, while on the Fahrenheit scale the melting point is 32° and the boiling point is 212° above zero. A temperature below 0° in either scale is indicated by a minus sign before the number; thus, — 12° F. indicates 12° below 0° (or 44° below melting point), according to the Fahrenheit scale. The Fahrenheit and Centigrade scales agree at — 40°, but diverge both ways from this point. It will be convenient to remember that this is the temperature at which mercury melts. If an instrument for still lower temperatures is desired, alcohol is used, which does not freeze till a temperature of about —80° C. is reached. Below this a gas thermometer must be used, or if this be not practicable, an instrument has been constructed of a coil of platinum wire.

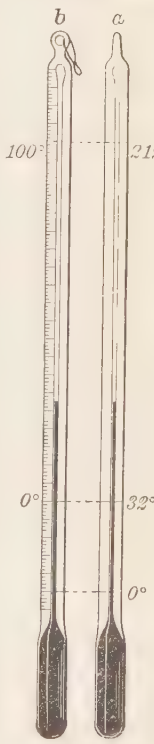


FIG. 169

217. Conversion from one scale to the other. Since 100° C. = 180° F., 5° C. = 9° F., or 1° C. = $\frac{9}{5}$ of 1° F. Hence, to convert Centigrade degrees into Fahrenheit degrees, we multiply the number by $\frac{9}{5}$; and to convert Fahrenheit degrees into Centigrade degrees we multiply by $\frac{5}{9}$. In finding the temperature on one scale that corresponds to a given temperature on the other scale, it must be remembered that the number that expresses the temperature on a Fahrenheit scale does not, as it does on a Centigrade scale, express the number of degrees above melting point; for example, 52° on a Fahrenheit scale is not 52° above melting point, but 52° — 32° = 20° above it.

Hence, to reduce a Fahrenheit to a Centigrade reading, *first subtract 32 from the given number and then multiply by* $\frac{5}{9}$; thus,

$$\frac{5}{9}(\text{F.} - 32) = \text{C.}$$

To change a Centigrade reading to a Fahrenheit reading, *first multiply the given number by* $\frac{9}{5}$ *and then add* 32; thus,

$$\tfrac{9}{5}\,\mathrm{C.} + 32 = \mathrm{F.}$$

218. Development of the thermometer. Though the invention of the thermometer has been ascribed to various scientific men, it did not assume a practical shape until 1620, at the hands of Drebel, a Dutch physician. Halley substituted mercury for spirit in 1730; Réaumur of Paris modified the instrument in 1730, and Fahrenheit of Danzig in 1749; Celsius of Upsala, Sweden, improved it in 1742 by adding the scale now known as Centigrade.

As the Réaumur thermometer is used generally in Germany, the Celsius in France, and the Fahrenheit in England, verily "A prophet is not without honor save in his own country."

219. Maximum and minimum thermometers. These are thermometers which enable us to ascertain the highest or lowest temperature to which they have been exposed in a given interval of time, ordinarily twenty-four hours. The *maximum* thermometer (the lower one, Fig. 170) is an ordinary mercurial thermometer, except that the bore of the tube near the bulb is reduced in such a way that while the expansion of the mercury is sufficient to force the liquid past the constriction, the cohesion of the liquid is insufficient to draw it back again when the temperature falls. To set the thermometer it is placed in a vertical position and shaken. This causes the mercury to return past the constriction to the bulb, and then the instrument indicates the same temperature as that of the air.

The *minimum* thermometer (the upper one, Fig. 170) contains alcohol. The bore of the tube contains a little index of metal or black glass, which moves with a little friction in the tube. It is entirely enveloped in the spirit, and the action is as follows: The instrument is set by placing it first in an inverted position to allow the index to run down the tube to the end of the liquid column, and it is then placed in a horizontal position. If the temperature rise, the spirit flows past the index without disturbing it. If the

temperature fall below that at which the instrument was set, the capillary action between the spirit and the index is such as to preclude its leaving the index; accordingly this is drawn back with the spirit, its upper end being always flush with the extremity of the liquid column, and ultimately marking the lowest temperature reached by the column.

220. Requirements of a thermometric substance. The substances most commonly employed for thermometric purposes are mercury, alcohol, and air. The requirements are as follows : (1) The substance should be uniformly expansible. In this respect the air thermometer most nearly meets the requirement; but there are great inconveniences attending its use. The expansion of mercury

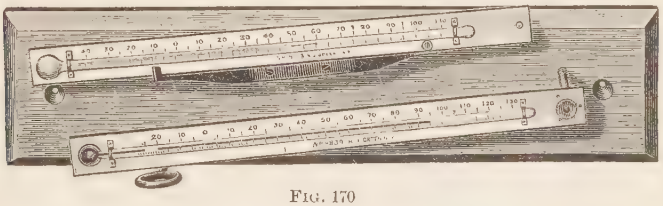


Fig. 170

is nearly uniform between the melting and boiling points on the scale and for a considerable range on each side of them. Alcohol, on account of lack of uniformity in its rate of expansion, is ill adapted for this use. (2) A thermometric liquid should have a high boiling point. A mercury thermometer will indicate temperature as high as about 327° C. (620° F.). The boiling point of alcohol is below that of water. (3) The freezing point of the liquid should be low. This furnishes the only reason for the use of alcohol. (4) The sensibility of the liquid to sudden changes of temperature is of importance. On account of the high conductivity of mercury and its low specific heat, a mercurial thermometer acquires thermal equilibrium with the surrounding body more quickly than any other liquid thermometer.

221. Standard thermometer. As its name indicates, this instrument is used as a standard for reference, and for testing from

time to time the accuracy of thermometers used for ordinary observations.

It is a thermometer which has been compared for every degree with an air thermometer, and has a table of corrections accompanied by a certificate of authority from the laboratory where it was compared. Owing to the fact that glass, after having been fused, does not immediately return to its normal density, it is necessary to use for a standard thermometer a tube which has lain several years after being filled. The contraction of the bulb causes a thermometer to read too high, if it was graduated before the contraction was completed. The result of this defect in a thermometer is called the *displacement of zero*. The graduation of a standard thermometer is made on the glass stem, and thus complication due to differences in expansibility of different substances is avoided.

222. Pyrometer. Any contrivance for determining temperature above the melting point of the best glass is called a *pyrometer*. There are several varieties. Some are constructed on the principle that a change of temperature affects the electrical resistance of a metal. A *hydropyrometer* indicates high temperatures by the number of degrees a given mass of water of known temperature is raised, by the immersion in it of a platinum ball of known mass after it has acquired the temperature, for instance, of a furnace or oven to be tested.

223. For a number of years it has been possible to make glass of very high fusing temperature. A thermometer of such glass, having nitrogen in the tube above the mercury, can measure temperatures much higher than without the gas. As the mercury rises it compresses the gas, and the pressure thereby produced on the mercury raises its boiling point. By this means temperatures of 500° to 600° C. can be measured, though under ordinary conditions mercury boils at a much lower temperature.

SECTION IV

CALORIMETRY

224. Distinction between the questions How hot and How much heat. The former, like the question How sweet, when applied to a solution of sugar, is answered only relatively. The latter, like the question How much sugar in the solution, is answered quantitatively. Sweetness and temperature are independent of the mass of the body. Quantity of sugar depends upon the sweetness and the mass of the liquid; quantity of heat in a body of any given substance depends upon the temperature and the mass of the body. A pint of boiling water is as hot as a gallon of the same, but the latter contains eight times as much heat. *Temperature depends on the average kinetic energy of the molecules. Quantity of heat is the product of the average kinetic energy of the molecules multiplied by the number of molecules.*

225. Thermal units. Heat is measured by the amount of effect it produces. The thermal unit generally adopted is the *calorie*, which is the quantity of heat necessary to raise the temperature of one gram of water from 4° to 5° C. Authorities do not agree on the temperature limits for this unit. Regnault chose 0° to 1° C., and this has been quite generally adopted in scientific treatises. There seem, however, to be good reasons for a departure from this custom, and we have chosen the limits proposed by Glazebrook in his treatise on heat, namely, 4° to 5° C.

In defining a thermal unit it is necessary to state the temperatures between which the water is raised, because, although the quantity of heat required to raise a given quantity of water one degree is very nearly the same at different temperatures, and in practice is usually regarded as the same, yet the quantity required is a very little greater at high temperatures than at low temperatures. The operation of measuring heat is called *calorimetry*.

226. Heat capacity; specific heat. The expression "heat capacity" applied to a body refers to the quantity of heat necessary

to raise the temperature of the body one degree. The expression "specific heat" is applied only to some particular *substance* and refers to the quantity of heat required to raise 1 gram of that substance from 4° to 5° C. It is apparent that *the specific heat of a substance is the heat capacity of 1 gram of that substance.*

Mix 1 kilogram of water at 0° with 1 kilogram at 20°; the temperature of the mixture becomes 10°. The heat that leaves 1 kilogram of water when it falls from 20° to 10° is just capable of raising 1 kilogram of water from 0° to 10°.

Take, say, 300 grams of sheet lead, make a loose roll of it, and suspend it by a thread in boiling water for about five minutes, that it may acquire the same temperature (100° C.) as the water. Remove the roll from the hot water, shake, and immerse it as quickly as possible in 300 grams of water at 0°, and introduce the bulb of a thermometer. Note the temperature of the water when it ceases to rise, which will be found to be about 3° (more accurately 3.3° +). The lead cools very much more than the water warms. The temperature of lead falls about 33° for every degree an equal mass of water is warmed.

From the first experiment we infer that a body in cooling a certain number of degrees gives to surrounding bodies as much heat as it takes to raise its temperature the same number of degrees. From the second experiment we learn that the quantity of heat that raises 1 kilogram of lead from 3° to 100°, when transferred to water, can raise 1 kilogram of water only from 0° to 3°. Hence we conclude that *equal quantities of heat, applied to equal masses of different substances, raise their temperatures unequally.*

If equal masses of mercury, alcohol, and water receive equal quantities of heat, the mercury will rise 30°, and the alcohol nearly 2°, for every degree the water rises. From this we infer that to raise equal masses of each of these substances 1° requires thirty times as much heat for the water as for the mercury, and twice as much as for the alcohol. Since a given quantity of heat affects the temperature of a given mass of water less than that of an equal mass of mercury or alcohol, water is said to have greater

specific heat than these substances. It is also apparent that a given mass of water in cooling imparts to surrounding bodies more heat than the same masses of mercury or alcohol would impart in cooling the same number of degrees.

227. Method of measuring specific heat. There are several methods practiced, only one of which, the "method of mixtures," will be considered. A known mass m (in grams) of the substance of which the specific heat is required is heated to a known temperature t_1; then it is mixed with (or immersed in) a known mass of water m_2 at a lower temperature t_2, and as soon as thermal equilibrium is established throughout, the temperature of the mixture t is taken. Let s represent the specific heat of the substance sought. Then the quantity of heat lost by the substance is $m \times s(t_1 - t)$ calories; while that gained by the water is $m_2(t - t_2)$ calories. Now if no heat be lost during the operation, $m \times s(t_1 - t) = m_2(t - t_2)$, whence $s = \dfrac{m_2(t - t_2)}{m(t_1 - t)}$. For example, taking the quantities obtained in the experiment above, we find for lead, $s = \dfrac{300(3.3 - 0)}{300(100 - 3.3)} = 0.034$ calorie.

In accurate measurements the amount of heat required to heat the calorimeter, the thermometer, etc., has to be taken into account. For more exact methods of measuring heat quantities the reader is referred to special laboratory manuals.

228. Specific heat of the same substances at different temperatures and in the three states of matter. The specific heat of solids and liquids usually increases slightly with the temperature, and diminishes with increase of density. Substances in the liquid state usually have a higher specific heat than in the solid or gaseous state. Thus, water has nearly double the specific heat of ice, and a little more than double the specific heat of steam.

REFERENCE TABLES

Table of mean specific heat between 0° C. and 100° C.

Sulphur	0.202	Copper	0.095
Glass	0.177	Mercury	0.033
Iron	0.113	Lead	0.031

Specific heat of the same substance in different states

	Solid	Liquid	Gaseous
Water	0.504	1.000	0.480
Bromine	0.083	0.106	0.055
Lead	0.031	0.040	——
Alcohol	——	0.55–0.77	0.45

229. Great capacity of water for heat. Water requires more heat to warm it, and gives out more in cooling through a given range of temperature, than any other substance except hydrogen. The quantity of heat that raises a kilogram of water from 0° to 100° C. would raise a kilogram of iron from 0° to 800° or 900° C., or above a red heat. Conversely, a kilogram of water in cooling from 100° to 0° C. gives out as much heat as a kilogram of iron in cooling from about 900° to 0° C.

"The vast influence which the ocean must exert as a moderator of climate here suggests itself. The heat of summer is stored up in the ocean, and slowly given out during the winter. This is one reason for the absence of extremes in an island climate."

The high specific heat of water is utilized in heating buildings by hot water.

230. Atomic and molecular heats. Many years ago Dulong and Petit noticed a very interesting reciprocal relation between the specific heats of elementary substances and their atomic masses. As a result of their experiments on about thirty of the elements, it was demonstrated that the product of the specific heat of each by its atomic mass was approximately 6.4 in every case.

Subsequently Neumann stated a similar law for compounds having an analogous chemical constitution. In this case the same constant equals the product of the specific heat and the molecular mass. The variation of this constant product is in few cases so great as not to be easily accounted for by the differences in the condition of the substances used, or of the temperatures at which the specific heats are measured.

231. Specific heat of gases. If during the heating process the volume of a gram of gas be maintained constant, the gas can do

no work against the confining pressure since nothing is moved though the pressure somewhat increases. If, however, the pressure instead of the volume be kept constant, the volume must increase against the confining pressure and a definite amount of external work will be done. In the latter case, therefore, heat enough must be supplied not only to raise the gram of gas one degree but also to supply energy equal to the external work done by the increasing volume.

A gas, then, is said to have two specific heats, one at constant volume and one at constant pressure, the latter being the greater. For example, the specific heat of air at constant pressure is 0.2375 calorie per gram, while that at constant volume is only 0.1691 calorie.

The ratio of these two specific heats is of much importance thermodynamically. For the simplest vapors and gases, as mercury vapor, argon, helium, and for monatomic gases generally, the value of the ratio is 1.67. For the diatomic gases, as air, hydrogen, and oxygen, it is 1.41, while for the more complicated molecular structures the ratio has a less value.

The value depends on the amount of intramolecular energy involved in the final temperature change.

SECTION V

EXPANSION

232. Examples illustrating expansion of solids, liquids, and gases. Having learned something of the nature of heat and the methods by which it is measured, we will next direct our attention to some effects it produces, namely, *expansion* and *change of state*. The first, as we have learned, furnishes a means of measuring temperature and leads to a fuller study of gases than has yet been made. Under the second effect we study *liquefaction* and *vaporization*.

Take two brass tubes, one of a size that will just permit it to enter the bore of the other. Heat the smaller tube; it will not in its expanded state enter the other. Thrust the heated tube into

cold water; its temperature falls, and it now enters the bore of the other tube. Heat expands, but cold does not contract. Cohesion, when a diminution of heat permits, causes a solid or liquid body to contract. Cold is a term of *negation*, signifying merely a greater or less deficiency of heat; it is not an entity, and hence cannot be the direct cause of any phenomenon.

FIG. 171

Fig. 171 represents a thin brass plate and an iron plate of the same dimensions riveted together so as to form what is called a *compound bar*. Place the bar edgewise in a flame, dividing the flame in halves so that both metals may be equally heated. The bar, which was at first straight, is now bent, owing to the *unequal expansion* of the two metals.

When heated above the normal temperature, the brass, which is more expansible, will be on the convex side; when cooled below the normal temperature, it will be on the concave side, since it contracts more rapidly than iron.

Metallic thermometers now in common use (Fig. 172) are constructed on this principle. They contain a compound ribbon of metal (Fig. 173) wound into a spiral, one end of which *a* is fixed, while the other is attached to a contrivance for multiplying motion, which moves the index. With a rise in temperature, the more expansible metal on the outside produces an increase of curvature, which causes the spiral to wind up closer. This motion is communicated to the index, which points on the dial to the corresponding temperature. With a fall of temperature the action is reversed.

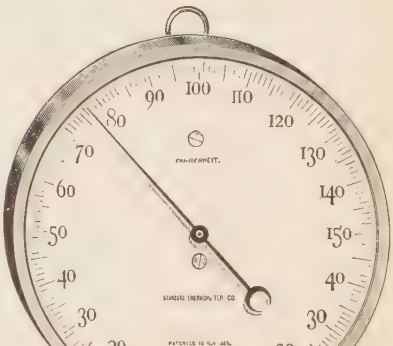


FIG. 172

The readings on the dial must be empirically located by comparison of each instrument with a mercury thermometer at various temperatures.

Advantage is taken of this principle also in the construction of balance wheels of chronometers. The rate of vibration of a chronometer balance wheel depends upon its mass and the distance of its circumference from the center. The parts BC and FG (Fig. 174) are made up of a compound strip, the more expansible metal being on the outside. As the temperature rises the radii AA lengthen, and the chronometer would lose time, but the heat causes the strips BC and FG to curve inwards. The masses D and D' are thus brought nearer the center, and this compensates for the expansion of A.

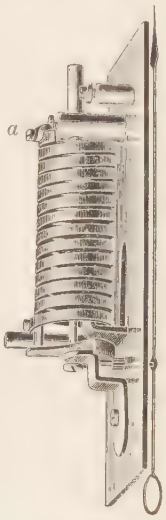


FIG. 173

Fit stoppers tightly in the necks of two similar thin glass flasks (or test tubes), and through each stopper pass a glass tube about 60 centimeters long. The flasks must be as nearly alike as possible. Fill one flask with alcohol and the other with water, and crowd in the stoppers so as to force the liquids in the tubes a little way above the corks. Set the two flasks into a basin of hot water, and note that, at the instant the flasks enter the hot water, the liquids sink a little in the tubes, but quickly begin to rise, until, perhaps, they reach the top of the tubes and run over.

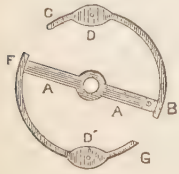


FIG. 174

When the flasks first enter the hot water they expand, and thereby their capacities are increased; meantime the heat has not reached the liquids to cause them to expand, consequently the liquids sink momentarily to accommodate themselves to the enlarged vessel. Soon the heat reaches the liquids, and they begin to expand, as shown by their rise in the tubes. The alcohol rises more than the water. *Different substances, in both the solid and*

the liquid states, expand unequally on experiencing equal changes of temperature.

Take a dry flask like one of those used above, invert, and insert the end of the tube in a bottle of colored water (Fig. 175), and apply heat to the flask; the inclosed air expands and comes out through the liquid in bubbles. After a few minutes withdraw the heat source, keeping the end of the tube in the liquid; as the air left in the flask cools, its pressure decreases, and the water is forced by atmospheric pressure up the tube into the flask, and partially fills it.

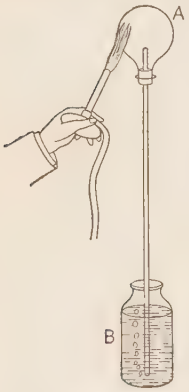


FIG. 175

233. Expansion coefficients. The expansion which attends a rise of temperature depends not only upon the size of the body, and upon the change of temperature, but upon a number peculiar to the substance itself called its *expansion coefficient.* This term is applied to *the increase of unit length per degree rise of temperature.*

Suppose that a rod of length l at 0° C. be heated through t degrees, so that its length becomes l_1; then, representing the linear expansion coefficient by c, we have, from the above definition,

$$c = \frac{l_1 - l}{lt}, \text{ whence } l_1 = l(1 + ct).$$

The expression $1 + ct$, called the *expansion factor*, is evidently the ratio of the final to the original length. Hence, multiplying the length of a solid at 0° C. by the expansion factor gives its length at t° above zero. Conversely, dividing its length at t° by the expansion factor gives its length at 0°.

In the expansion of fluids we have to do only with increase of volume, called *volume* or *cubical* expansion. A *volume expansion coefficient is the increase of unit volume per degree rise of temperature.* This is approximately $3c$, or three times the linear

expansion coefficient, and may be taken as such for most practical purposes. Likewise, the surface or superficial expansion coefficient is approximately $2c$.

Since liquids must be contained in a vessel of some sort, the observed expansion is usually not that of the liquid alone but a value less than the real expansion of the liquid by the increase of volume of the vessel. This observed expansion is often called the apparent expansion of the liquid.

Not only do the expansion coefficients of liquids and solids vary with the substance, but the coefficient for the same substance varies with the temperature, being greater at high than at low temperatures. Hence, in giving the expansion coefficient of any substance it is customary to give the *mean* coefficient through some definite range of temperature, as from 0° to 100° C.

234. Anomalous expansion. Water presents a partial exception to the general rule that matter expands on receiving heat and contracts on losing it. If a quantity of water at 0° C. be heated, it contracts as its temperature rises, until it reaches 4° C., when its volume is least, and therefore at this temperature it has its *maximum density*. If heated beyond this temperature it expands, and at about 8° C. its volume is the same as at 0°. On cooling, water reaches its maximum density at 4° C., and expands as the temperature falls below that point.

Water is said to have a negative expansion coefficient between 0° and 4° C. A few other substances, such as india rubber and iodide of lead, contract when heated, and have, therefore, negative coefficients.

235. Force exerted in expansion and contraction. The force which may be exerted by bodies in expanding or contracting may be very great, as shown by the following rough calculation. If an iron bar 1 square inch in section be raised from 0° C. (melting point of ice) to 500° C. (a dull red heat), its length, if allowed to expand freely, will be increased from 1 to 1.006, its expansion coefficient being about 0.000012. Now a force of about 90 tons is required to stretch a bar of iron of 1 square inch section this

amount, and this is very nearly the force that would be necessary to prevent *expansion* caused by the heat. It would require an equal force to prevent contraction, if the bar be cooled from 500° to 0°.

Boiler plates are riveted with red-hot rivets, which, on cooling, draw the plates together so as to form very tight joints. Tires are fitted on carriage wheels when hot, and, on cooling, grip them with very great force.

It is to be observed that while the force exerted in expansion and contraction is great, the distance through which it acts is very small, and hence the quantity of work performed is not very great.

236. Expansion of gases. The effect of a change of temperature upon a gas may be measured by noting the change in its volume when the pressure upon it is constant, or the change in its pressure when its volume is unchanged. Conversely, the changes in volume or pressure of a gas may be made to indicate changes of temperature. On this principle the air thermometer is constructed.

It is to Regnault that we owe the most accurate determination of the volume and pressure coefficients of a gas. He ascertained that when air is allowed to dilate with increase of temperature while the pressure remains constant, the *volume* at 100° is increased by 0.367 its volume at 0°. Dividing this number by 100, we obtain 0.00367 for the expansion coefficient of air between 0° and 100° C.

It is found that the expansion coefficient of all gases is approximately the same as long as they remain true gases, but as they approach the vaporous state the coefficient changes rapidly.

The apparatus employed will be understood by referring to Fig. 176.

Two glass tubes A and B vertically adjustable are joined by the rubber tube at the bottom, all being partly filled with mercury and mounted on a wooden frame furnished with a scale between the tubes. A glass bulb joined to A is shown at a.

The glass bulb a is placed first in melting ice and then in the steam from water boiling under normal conditions. The difference in the volume of the gas is measured by the change in the height of the mercury column in the tube A, the left-hand tube

B having been adjusted in height in both cases, so that the level of the mercury in it shall be the same as that in A. This adjustment insures the same pressure in the two experiments, i.e. that of the air at the time. Having measured the volume of the bulb and the tube, the ratio of the increase of volume under constant pressure to the original volume can easily be calculated.

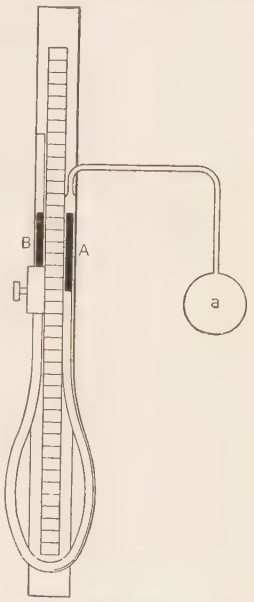


FIG. 176

To measure the pressure coefficient with the same apparatus, the tube B is raised till the height of the mercury in A is the same when the bulb is in the steam bath as when it is at 0°. The volume having thus been kept constant, the pressure change due to heating is measured by the excess of height of the mercury in B over that in A. This increase of pressure divided by the original pressure gives the pressure coefficient for 100°.

For the more perfect gases these coefficients are found to be the same, and according to the best authorities the value may be given as 0.003665.

The fact that the pressure and volume coefficients are approximately the same for all gases between 0° and 100° was first stated by Charles in 1787, and later more accurately by Gay-Lussac. It is often known as the *law of Charles*.

SECTION VI

EXPANSION. KINETIC THEORY OF MATTER. ABSOLUTE TEMPERATURE

237. Kinetic theory of matter. In the case of solids the molecules are thought to move in curved orbits the centers of which are fixed. In liquids the orbits are curved, but the molecules have,

besides the oscillating motion, a motion of translation. The theory that the molecules composing all bodies of matter are in perpetual relative motion is called the *kinetic theory of matter*. This theory claims that in gases the molecules are so far separated from one another that their motions are not generally influenced by molecular attractions. Hence, in accordance with the first law of motion, the molecules of gases move in *straight lines* and with uniform velocity until they collide with one another or strike against the walls of the containing vessel, when, in consequence of their elasticity, they at once rebound and start on a new path.

238. Pressure of a gas due to the kinetic energy of its molecules. Consider, then, what a molecular storm must be raging about us, and how it must beat against us and against every exposed surface. According to the kinetic theory, the pressure of a gas is entirely due to the striking of the molecules against the surfaces on which the gas is said to press, the impulses following one another in such rapid succession that the effect produced cannot be distinguished from constant pressure.[1] Upon the kinetic energy of these blows, and upon the number of blows per second, must depend the amount of pressure. But we have learned that on the energy of the individual molecules depends that condition of a gas called its *temperature*: so it is apparent, as stated above, that *the pressure of a given quantity of gas varies with its temperature*. Again, as at the same temperature the number of blows per second must depend upon the number of molecules in the unit of space, it is apparent that *the pressure varies with the density*.

"If the rarefaction of air can be carried so far that only one particle out of every million is left in the space exhausted, the mean path of the particles would then be about 4 inches. In our

[1] The following estimates made (by Maxwell, using a proposition formulated by Clausius) for hydrogen molecules at 0° C., and under a pressure of 760 mm., may prove interesting:

Mean velocity, 6100 feet per second.
Mean path without collision, 38 ten-millionths of an inch.
Collisions, 17,750 millions per second.
Mass, 216,000 million million million in 1 gram.
Number, 19 million million million fill 1 cubic centimeter.

atmosphere at a height of 210 miles the particles are relatively so few that each particle might travel through a uniform atmosphere of that density for sixty million miles without entering into collision" (Daniell).

239. Absolute temperature. If v_0 be the volume of a given mass of gas at 0° and v_t be its volume at $t°$, pressure remaining constant, we may write evidently

$$v_t = v_0(1 + \alpha t),$$

in which α is the coefficient of expansion, i.e. 0.003665, or about $\frac{1}{273}$.

If t be below 0°, v_t is less than v_0 and grows less as the temperature falls. Solving the above equation when v_t becomes zero, we find that $t = -\frac{1}{\alpha}$, or $-273°$; which means that 273° below 0° is the temperature at which the volume of a gas will become nothing.

Again, if the volume instead of the pressure had been constant during the change, the pressure would have fallen to zero at the same low temperature.

As it is not conceivable that a mass of matter can exist and not occupy some volume, we are led to the reasonable assumption that it is only the intermolecular volume that has contracted with the fall of temperature, and that, this having become zero, the molecules of the gas are in contact and therefore at rest.

As they are at rest they have no kinetic energy and exert no pressure; they are therefore appropriately said to be at the temperature of absolute zero.

Absolute temperature is that reckoned from the absolute zero, or $-273°$ C. *Temperatures measured from absolute zero are proportional to the pressure of a theoretically perfect gas of constant volume or density; this statement is merely a convenient expression of the laws of a perfect gas.*

The absolute temperature of any body is found by adding 273 to its temperature as indicated by a Centigrade thermometer.

240. Laws of gaseous masses. It follows from the above discussion (1) that *the volume of a given mass of gas at constant pressure is proportional to its absolute temperature.*

(2) *The pressure of a given mass of gas whose volume is kept constant is proportional to its absolute temperature.*

Boyle's law states that (3) *at a constant temperature the volume of a given mass of gas is inversely proportional to its pressure;* i.e. *the product of its pressure and its volume is constant.* Now, when both the pressure and the volume vary at the same time, it may be shown that (4) *the product of the pressure and the volume of a given mass of gas is proportional to its absolute temperature.* A gas is said to be *perfect* when it perfectly obeys these laws.

We may also state this law as follows: The product of the pressure and volume of a given mass of gas divided by its absolute temperature is a constant quantity, or

$$\frac{PV}{T} = C,$$

in which P = pressure, V = volume, T = absolute temperature of a given mass of gas, and C = a constant quantity, the value of which depends on the kind of gas in question, and on the mass taken.

QUESTIONS AND PROBLEMS

1. Find, in both Centigrade and Fahrenheit degrees, the absolute temperatures at which mercury boils and freezes.

2. At 0° C. the volume of a certain mass of gas under a constant pressure is 500 cc. (*a*) What will be its volume if its temperature be raised to 75° C.? (*b*) What will be its volume if its temperature become − 20° C. ?

3. If the volume of a mass of gas at 20° C. be 200 cc., what will be its volume at 30° C. ? *Solution:* 20° C. is equivalent to (20 + 273) or 293 absolute temperature; then 293 : 303 :: 200 : 206.8 cc. *Ans.*

4. To what volume will a liter of gas contract if cooled from 30° C. to − 15° C. ?

5. One liter of gas under a pressure of one atmosphere will have what volume, if the pressure be reduced to 900 g. per square centimeter, while the temperature remains constant?

6. The volume of a certain mass of air at a temperature of 17° C., under a pressure of 800 g. per square centimeter, is 500 cc. What will be its volume at a temperature of 27° C., under a pressure of 1200 g. per square centimeter? *Solution:* 17° C. is equivalent to 290° absolute temperature; 27° C. is equivalent to 300° absolute temperature. Then 290 : 300 : : 500 × 800 : x × 1200. Whence x = 344.8 cc. *Ans.*

7. If the volume of a mass of gas under a pressure of 1 kg. per square centimeter at a temperature of 0° C. be 1 l., at what temperature will its volume be reduced to 1 cc. under a pressure of 200 kg. per square centimeter ? 54.6° absolute temperature, or − 218.4° C. *Ans.*

8. Find the temperatures on the absolute scale at which the substances named in § 245 will melt.

9. If a cubic foot of coal gas at 32° F., when the barometer is at 30 in., has a mass of $\frac{1}{25}$ lb., what will be the mass of an equal volume at 68° F., when the barometer is at 29 in. ?

10. Explain the following statement : " To compare absolute temperatures, we may seal up a mercurial barometer in a tube, or an aneroid barometer in a preserving jar " (Whiting).

SECTION VII

EFFECTS OF HEAT. FUSION

241. Change of properties in solids attending change of temperature. " Every known property of a piece of matter, except its mass, varies with variation of temperature." Inasmuch as heat tends to weaken cohesion, the rigidity and tenacity of solids are generally lessened, and their plasticity is increased, by the addition of heat.

242. Fusion. As previously stated, whether a given substance exist in a solid, liquid, or gaseous state depends upon the temperature and the pressure it is under. Solids exposed to heat liquefy or fuse, unless previously decomposed. Some, like ice and tin, change their state abruptly ; others, like glass and wrought iron, become plastic prior to liquefaction. The temperature at which a substance melts is called its *fusion point*. The fusion points of different substances vary greatly, that of alcohol (− 130.5° C.) and that of iridium (1950° C.) being taken as extreme examples.

Experiment and experience teach that (1) *the melting points may vary widely for different substances, but for the same substance it is invariable when under the same pressure.*

(2) *The temperature of a solid or liquid remains constant at the melting point from the moment that melting or solidification begins until it ceases.*

If a lump of ice be kept in boiling water till half melted, the remaining piece on being taken out will be found to be still at a temperature of 0°. All the heat that entered the ice was used in melting a portion of it. Under ordinary pressure ice cannot be made warmer than 0° C.

243. Change of volume during fusion or solidification. During these changes of state there is usually a change of volume. Generally solids expand on melting, so that the volume is increased and the density is diminished by fusion. There are, however, certain important exceptions to this rule. Ice, cast iron, and type metal, for example, float in the denser liquids of these substances respectively. Such metals as expand on solidifying are especially adapted for casting, since they expand and fill every portion of the mold, and the resulting impression is sharp and clear.

244. Effect of pressure on the fusion point. In solids whose volume *increases* during fusion, the fusion is more difficult when the pressure is increased, since not only is heat required to change the state of the body, but an additional quantity is required to do additional external work against the pressure. In solids whose volume decreases during fusion, an increase of pressure facilitates the fusion, and the temperature is less as the pressure is greater. Briefly, then, *pressure lowers the fusion point of substances that expand on solidifying, and raises the fusion point of those that contract.*

245. Regelation. Press in contact for a few seconds smooth, dry surfaces of ice; remove the pressure, and they will be found to be firmly frozen together.

This phenomenon is called *regelation* (re-freezing), and is simply the consequence of lowering the fusion point by pressure. The fusion point being lowered, the ice at the surfaces of contact is melted, but as the water produced is below the normal fusion point of water, it freezes again as soon as the pressure is removed.

Regelation was applied by Tyndall to explain the motion of the glaciers, thought by some to be due to the viscosity of the ice. Under the enormous pressure from above, the ice layers in contact

with earth or rocks below liquefy at a temperature below normal, ooze out, and allow the ice mass above to settle a little. By a continuous process of this kind some of the Swiss glaciers have a speed of one to two feet per day.

TABLE OF FUSION POINTS

Alcohol	− 130.5° C.	Zinc	about 425° C.
Mercury	− 38.8°	Silver	" 954°
Sulphuric acid	− 34.4°	Gold	" 1200°
Ice	0°	Cast iron	" 1050–1250°
Phosphorus	44°	Wrought iron	" 1500–1600°
Sulphur	115°	Platinum	" 1775°
Tin	about 233°	Iridium (the most infusible metal)	" 1950°
Lead	" 334°		

246. Heat of fusion. The temperature of ice remains constant while melting, and generally heat imparted to a melting body affects its temperature very little, if any. Furthermore, ice and other solids are not instantly converted into liquids on reaching the fusion point, but absorb a quantity of heat depending on the substance and proportional to their mass before fusion is accomplished. Inasmuch as none of the heat applied during melting raises the temperature of the body, the question arises, *What becomes of the heat applied to the body?* The thermodynamical theory furnishes the only satisfactory answer to this important question. The answer is, about all the heat applied to a body during fusion is *consumed in doing internal work*, as it is called. The molecules that were held firmly in their places by molecular forces are, during fusion, loosened and moved from their places, and so work is done against these forces. Heat, the energy of motion, performs this work, and is thereby converted into *potential energy*, the energy of position, — energy of the same kind as that of a raised weight. The heat which disappears in melting is called the *heat of fusion.*

If it require a large quantity of heat and a long time to effect the fusion of a body, it must be inferred that the amount of work done is proportionately great. Fortunate is it that it does require

much heat to melt moderately small masses of ice and snow, else on a single warm day in winter all the ice and snow would melt, creating most destructive freshets.

247. Measurement of the heat of fusion. Let it be required to find approximately the quantity of heat that disappears during the melting of one gram of ice. This quantity is most readily determined by the *method of mixtures.*

Weigh out 200 grams of dry ice chips whose temperature in a room of ordinary temperature may be safely assumed to be 0° C. Weigh out 200 grams of boiling water whose temperature we assume to be 100° C. Pour the hot water upon the ice and stir it until the ice is all melted. Test the temperature of the resulting liquid.

Suppose its temperature is found to be 10° C. It is evident that the temperature of the hot water in falling from 100° to 90° would yield sufficient heat to raise an equal mass of water from 0° to 10° C. Hence it is clear that the heat which the water at 90° yields in falling from 90° to 10° — a fall of 80° — in some manner disappears. At this rate had you used one gram of ice and one gram of hot water, the amount of heat lost would be 80 calories. Careful experiments, in which suitable allowances are made for loss or gain of heat by radiation, conduction, absorption by the calorimeter, etc., have determined that about *80 calories of heat are consumed in melting one gram of ice.*

TABLE OF HEATS OF FUSION OF SUBSTANCES UNDER THE PRESSURE OF ONE ATMOSPHERE

	Calories per gram		Calories per gram		Calories per gram
Water	80.0	Silver	24.7	Sulphur	9.4
Sodium nitrate	63.0	Tin	14.2	Lead	5.4
Potassium nitrate	47.4	Bismuth	12.6	Phosphorus	5.0
Zinc	28.1	Iodine	11.7	Mercury	2.8

According to this table it is to be understood, for example, that one kilogram of ice at 0° under the pressure of one atmosphere, while changing to a liquid, absorbs without any change

of temperature 80,000 calories of heat, or as much heat as is required to raise 80 kilograms of water from 4° to 5° C. It will be seen that water possesses the greatest latent heat of fusion.

248. Transformation of heat reversible. As stated at the beginning of this chapter, work is transformable into heat, and potential energy is transformed into kinetic energy "by the return of the molecules to their original positions"; so when water freezes or any liquid is resolidified, the potential energy (latent heat of fusion) reappears as heat.

Water in freezing undergoes no change of temperature, hence, if heat be developed during the operation, it must become diffused or must be "given off" in order to allow the freezing to go on. As the diffusion is necessarily slow, so freezing must be slow; and this slow development of heat and its immediate dispersion accounts for the fact that we are seldom made conscious of the development of heat during solidification.

Farmers sometimes turn to practical use this well-known phenomenon. Anticipating a cold night, they carry tubs of water into cellars to be frozen. The heat generated thereby, although of a low temperature, is sufficient to protect vegetables, which freeze at a lower temperature than water.

Heat disappears in the process of melting ice; and, paradoxical as it may seem, heat is generated by freezing water. By freezing one kilogram of water 80,000 calories of heat are generated.

SECTION VIII

EFFECTS OF HEAT CONTINUED. VAPORIZATION

249. Evaporation; ebullition. The process of converting a liquid into a vapor is called *vaporization*. A comparatively slow vaporization which takes place only at the exposed surface of a liquid is called *evaporation*. A rapid process which may take place throughout the liquid, but usually is most rapid at the point where heat is applied, is called *boiling* or *ebullition*.

250. Kinetic theory of evaporation and condensation. According to this theory, some of the molecules in any liquid move faster than others. Those at the surface which have great velocities, if the direction of their motion be *from* the liquid, will break away from the forces that are able to retain the molecules moving more slowly, and will fly about as vapor in the space outside the liquid. This is *evaporation*. At the same time molecules of the vapor striking the liquid may plunge into it and become entangled in it, and thus there is a return to the liquid state. This is *condensation*. The number of molecules which passes from the liquid to the vapor increases with increase of temperature of the liquid. The number which passes from the vapor to the liquid depends upon the density of the vapor as well as upon its temperature. When the density of the vapor increases to such an extent that as many molecules are condensed as are evaporated, then the vapor is said to have its *maximum density* for that temperature, or to be *saturated*. The evaporation then appears to cease, because the proportions of liquid and vapor remain unchanged. Liquids which evaporate readily are called *volatile liquids* in distinction from those which do not, and which are called *fixed liquids*.

251. Boiling point. In evaporation, molecules fly from the surface of the liquid and mingle with the particles of the air, and drive only a certain small portion of them away. In boiling, the molecules which fly away from the surface drive all the air particles away a certain distance. Hence the vapor of a boiling liquid must exert a pressure at least as great as the atmospheric pressure. The greater the external pressure to be overcome, the greater must be the energy, i.e. the higher the temperature, of the vapor. When the saturated vapor of a liquid exerts a pressure equal to that of the atmosphere, the liquid begins to boil, and the temperature at which this occurs is called the *normal boiling point* of that liquid.

Half fill a thin glass flask with water. Boil the water over a Bunsen burner; the steam will drive the air from the flask. Withdraw the burner, quickly cork the flask very tightly, and plunge

it into cold water, or invert it and pour cold water upon the part containing steam, as in Fig. 177; the water in the flask, though cooled several degrees below the usual boiling point, boils again violently. The application of cold water to the flask condenses some of the steam and diminishes the pressure of the rest, so that the pressure upon the water is diminished and the water boils at a reduced temperature.

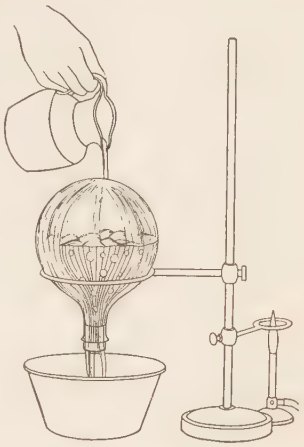


Fig. 177

If hot water be poured upon the flask, the water ceases to boil. Under the receiver of an air pump water may be made to boil at any temperature between 0° and 100° C.; indeed, if exhaustion be carried far enough, boiling and freezing may be going on at the same time. When high temperature is objectionable, apparatus is contrived for boiling and evaporating in a vacuum; as, for instance, in the vacuum pans used in sugar refineries. As water boils more easily under diminished pressure, so it boils with more difficulty when the pressure is increased; and the temperature to which water may be raised under the pressure of its own steam is limited only by the strength of the vessel containing it.

Fig. 178

Vessels adapted to resisting steam pressure, called *digesters*, are often employed to effect a complete penetration of water into solid and hard substances. By this means gelatin is extracted from the interior of bones. In a steam boiler, when the pressure is 150 pounds per square inch, the boiling point rises to about 185° C.

Place a test tube (Fig. 178) half filled with ether in a beaker containing water at a temperature of 60° C. Although the temperature of the water is 40° below its boiling point, it very quickly

raises the temperature of the ether sufficiently to cause it to boil violently. By introducing a thermometer into the test tube the boiling point of ether may be ascertained.

It is found that (1) *for a given pressure* (for example, that of the atmosphere, or 760 millimeters of mercury) *every liquid has a definite boiling point*; (2) *this temperature remains constant after boiling has begun*: (3) *salts dissolved in liquids raise their boiling points, but do not affect the temperature of the escaping vapor*. The last result is obviously due to the additional work required to be performed by the heat in order to overcome the increased cohesion due to the salt in solution.

Boiling Points under a Pressure of 760 mm.

Sulphurous anhydride	− 8° C.	Benzole	80.44° C.
Ether	34.89°	Water	100°
Carbon disulphide	46.8°	Acetic acid	117.28°
Bromine	63°	Butyric acid	157°
Wood spirit	65.50°	Sulphuric acid	337.77°
Alcohol	78.39°	Mercury	358°

Boiling Points of Water at Different Pressures

Barometer		Atmospheres	
680 mm.	96.9° C.	1	100° C.
700 "	97.7°	2	120.6°
720 "	98.5°	3	134°
740 "	99.26°	5	152.2°
760 "	100°	10	180.3°
770 "	100.3°		

The boiling point of water varies with the altitude of places, in consequence of the change in atmospheric pressure. Roughly speaking, a difference of altitude of 1000 feet causes a variation of 1° C. in the boiling point. The measurement of heights by means of the boiling point is called *hypsometry*. A *hypsometer* is simply a convenient portable apparatus for boiling water, provided with a thermometer sensitive to, say, 0.01°.

BOILING POINTS OF WATER AT DIFFERENT ALTITUDES

	Above the sea level	Mean height of barometer	Temperature
Mont Blanc	15,650 ft.	16.90 in.	86° C.
Quito	+ 9,500 "	21.53 "	91°
Mt. Washington	6,290 "	22.90 "	94°
Boston	0 "	30.00 "	100°
Dead Sea (below)	− 1,316 "	31.50 "	101°

252. Vaporization of solids; sublimation. The boiling point of a substance under ordinary atmospheric pressure may be below its fusion point; if so, the solid changes directly into a vapor without passing through the usual intermediate liquid state. For example, the maximum vapor pressure of carbon dioxide at its fusion point (− 65° C.) is three atmospheres; under any less pressure it vaporizes, but never melts. This phenomenon is called sublimation. Ice cannot be melted under less than 4.6 millimeters' pressure. It requires a greater pressure than that of a single atmosphere to melt arsenic and arsenious oxide. All of the substances named above evaporate far below their fusion points; hence arsenious oxide exposed in a room, in wall paper, for example, may render the air of the room dangerous for inhalation.

253. Heat of vaporization. Heat that is consumed in the process of vaporization is called the *heat of vaporization*. The quantity of heat required to convert a gram of water at 100° C. into steam without altering its temperature (which is the same as the quantity of heat generated by the condensation of one gram of steam at 100°) is called the *heat of vaporization of water*.

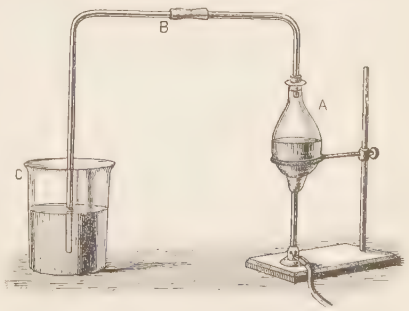


FIG. 179

In order to measure approximately this quantity, find the mass in grams of the glass beaker or calorimeter C (Fig. 179), and since

it will receive a small portion of the heat generated by the condensation of the steam, find its *water equivalent* by multiplying its mass by the specific heat of glass (0.177). Represent this quantity by m_1. Take in the calorimeter a known mass M of cold water at a known temperature t. When water in the flask A begins to boil, introduce the end of the delivery tube B into the water in C. The steam that passes through the tube is condensed on entering the cold water, and heats the water. When a considerable portion of the water in A has been vaporized, the temperature t_1 of the water in C is taken again, and the contents of the calorimeter are again weighed. The increase m in the mass of water in C is the mass of steam which has been condensed. Let L be the heat of vaporization. Then the whole quantity of heat generated by the condensation of m grams of steam is Lm, and the quantity of heat imparted to the cold water in falling from 100° to t_1° is $m\,(100 - t_1)$, or the total quantity of heat given to the calorimeter and its original contents is $Lm + m\,(100 - t_1)$. The heat required to raise the calorimeter and its original contents from t to t_1 is $(M + m_1)\,(t_1 - t)$. But these two quantities are equal, hence

$$Lm + m(100 - t_1) = (M + m_1)(t_1 - t);$$

whence

$$L = \frac{(M + m_1)(t_1 - t) - m(100 - t_1)}{m}.$$

In practice various precautions, which need not here be detailed, are necessary. Careful experiments have determined the value of L for steam to be 536 (Kohlrausch) calories; that is, it requires 536 calories of heat to convert one gram of water at 100° into steam at 100°, and when the process is reversed 536 calories per gram of steam are generated by the condensation.

When water is converted into steam, the larger portion of the heat which disappears is consumed in separating the molecules so far that molecular attraction is no longer sensible; a small portion — about $\frac{1}{15}$ — is consumed in overcoming atmospheric pressure. The amount of work done in boiling is very great, as shown by the

amount of heat consumed; hence it requires a long time for the water to acquire the requisite amount of heat. This is a protection against sudden changes.

Steam is a most convenient vehicle for the conveyance of heat of vaporization, i.e. potential energy, from the boiler to distant rooms to be heated. For example, for every kilogram of steam condensed in the pipes of the radiator, 536 calories, or heat enough to raise 5.36 kilograms (about 12 pounds) of ice water to the boiling point, are generated.

In this connection a brief discussion of some *molecular hypotheses* may prove of interest. The molecule is regarded as a collection of atoms. It may possess both translatory and rotary motion as a whole, and hence kinetic energy. It may possess potential energy, in virtue of the mutual attraction between it and other molecules and their relative distances, since the potential energy increases with both the force and the distance apart. This latter may be termed *intermolecular* energy. The atoms within the individual molecule may also possess all these kinds of motion and energy, and the total amount of this energy *within* the molecule may be called *intramolecular* energy. The work which is performed when either the intermolecular or the intramolecular energy is altered, or both are altered, is called *disgregation work*. The heat energy is regarded as the kinetic energy of the molecules in their vibratory motions, apart from any energy of rotation about their own centers of mass, and apart from intermolecular or intramolecular energy.

When heat is imparted to a body it is distributed, in general, so as to produce, in varying relative quantities, four effects, namely, (1) to raise the temperature by increasing the vibratory speed of the molecular motion, (2) to perform external work, (3) to perform intermolecular work, (4) to perform intramolecular work. (3) and (4) are classed as disgregation work.

254. Distillation. Vessel *A* (Fig. 180), called a *condenser*, contains a coil of tubing *B*, called a *worm*, terminating at *a*. The other end of the tube, *b*, projects through the side of the vessel

near its bottom. Near the top of the vessel projects another tube *c*, called the *overflow*, with which is connected a rubber tube *e*. This tube conveys the warm water which rises from the surface of the heated worm away to a sink or other convenient receptacle.

Fill a glass flask *C* three fourths full of pond or bog water. Connect the flask by means of a glass delivery-tube with the extremity *a* of the worm. Heat the water in the flask; as soon as it begins to boil, commence siphoning cold water through a small tube *d* from an elevated vessel *E* into the condenser. Inasmuch as the worm is constantly surrounded with cold water, the steam on passing through it becomes condensed into a liquid, and the liquid (called the *distillate*) trickles from the extremity *b* into a receiving vessel. The distillate is clear, but the water in the flask acquires a yellowish-brown tinge as the boiling progresses, due to the concentration of impurities (largely of vegetable matter) which are held in suspension and solution in ordinary pond water. The apparatus is called a *still*, and the operation *distillation*.

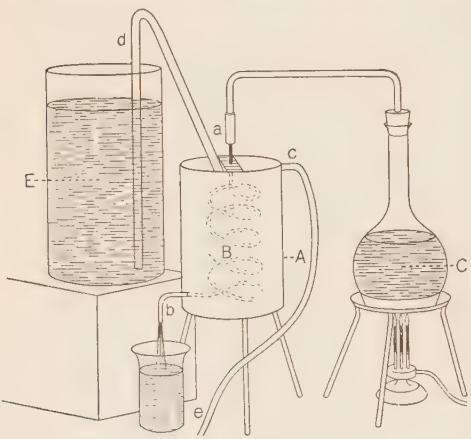


FIG. 180

When a volatile liquid is to be separated from water, — for example, when alcohol is separated from the vinous mash after fermentation, — the mixed liquid is heated to its boiling point, which is lower than that of water. Much more of the volatile liquid will be converted into vapor than of the water, because its boiling point is lower. Thus a partial separation is effected. By repeated distillations a high-grade alcohol is obtained.

SECTION IX

METHODS OF PRODUCING COLD ARTIFICIALLY

255. A body becomes cold only by losing heat. As heat passes only from warmer to colder bodies, it is evident that the temperature of a body cannot fall below that of surrounding bodies — for example, below the temperature of other bodies in the same room — by the natural process of imparting heat to its neighbors. The temperature of a body, then, can be reduced below that of its neighbors only by some artificial means.

The fact that heat must be consumed in the conversion of solids into liquids and liquids into vapors, because work is done, is turned to practical use in many ways for the purpose of producing *artificial cold*. The following will illustrate this process.

256. Heat consumed by solution; freezing mixtures. Prepare a mixture of 2 parts, by mass, of pulverized ammonium nitrate and 1 part of ammonium chloride. Take about 75 cubic centimeters of water (not warmer than 8° C.), and into it pour a large quantity of the mixture, stirring it, while dissolving, with a test tube containing a little cold water. The water in the test tube will be quickly frozen. A finger placed in the solution will feel a painful sensation of cold, and a thermometer will indicate a temperature of about −10° C.

One of the most common freezing mixtures consists of 3 parts of snow or broken ice and 1 part of common salt. The affinity of salt for water tends to produce liquefaction of the ice, and the resulting liquid dissolves the salt, *both operations consuming heat.*

257. Heat consumed in vaporization. The heat consumed in vaporization is greater than that consumed in liquefaction; for example, in the case of water it is greater in the ratio of 536 : 80. Hence vaporization is the more efficient means of producing extremely low temperatures. Whatever tends to hasten evaporation tends to accelerate the reduction of temperature. *Rapidity of evaporation increases with the temperature, extent of surface*

exposed, diminution of pressure, dryness of the atmosphere, and some other less important variables.

If the palm of the hand be wet with ether, the ether quickly evaporates and a sensation of cold results.

Place water at about 40° C. in a thin unglazed earthen cup, and the same amount of water at the same temperature in a glass beaker of about the same size as the porous cup. Introduce into each a thermometer. The comparatively large amount of surface exposed by means of the porous vessel will so hasten the evaporation in this vessel that in the course of ten or fifteen minutes a very noticeable difference of temperature will be indicated by the thermometers in the two vessels, that in the earthen cup being lower.

In warm climates water is frequently kept in porous earthen vessels in order that its temperature may be kept low enough by evaporation to render it suitable for drinking.

If a spray of ether be thrown continuously upon the bulb of a thermometer, in a very short time the temperature of the mercury will fall, in a warm room, to − 8° C. or lower.

Water may be frozen by its own boiling and consequent evaporation in the receiver of an air pump from which the air (and consequently the air pressure) is removed. A dish of sulphuric acid should be placed in the receiver to absorb the water vapor, the pressure of which, if present, would hinder the boiling.

By evaporating liquid ethylene and liquid air under a pressure of 4 millimeters, Olzewski produced a temperature of − 220° C., and Dewar has attained still lower temperatures by boiling hydrogen under low pressure. The cold produced by the evaporation of liquid carbon dioxide in the air, when it is relieved from pressure, is sufficient to freeze the greater part of it, producing a solid mass like snow, which evaporates slowly, producing a temperature of − 90° C.

258. Spheroidal state. A drop of water placed on a smooth metal surface heated above 200° C. will not come in contact with or moisten the surface, but assumes the form of a flat spheroid

and rolls about like a ball or spins on its axis. It is said to be in the *spheroidal state*. When in this state the liquid does not boil; indeed, its temperature is several degrees below its boiling point. The liquid globule rests upon a cushion of its own vapor, known as a Crookes' layer, and is held up by it. When, however, the heated metal cools and the vapor pressure is not great enough to sustain the globule, it comes in contact with the metal, its temperature rapidly rises to the boiling point, and it is quickly converted into steam. Showmen place red-hot irons in their mouths and dip their moistened hands into melted lead or even melted iron without injury. A layer of spheroidal fluid prevents contact of the flesh with the heated metal.

Boutigny placed liquid sulphur dioxide, whose temperature when in the spheroidal state is below zero, in a red-hot platinum crucible; it quickly assumed the spheroidal state, and drops of water let fall upon it quickly froze. Mercury can, in like manner, be frozen in a red-hot crucible by employing liquid nitrous oxide in the spheroidal state.

SECTION X

HYGROMETRY

259. Dew-point. *Hygrometry* treats of the state of the air with regard to the water vapor it contains. A given space, e.g. a cubic meter (it matters little whether there is air in the space or whether it is a vacuum), can hold only a limited quantity of water vapor. This quantity depends on the temperature. The capacity of a space for water vapor increases rapidly with the temperature. On the other hand, if air containing a given quantity of water vapor be cooled, it will continually approach and finally reach *saturation*, since the lower the temperature the less the capacity for water vapor. It is evident that air saturated with vapor cannot have its temperature lowered without some of the vapor being condensed into a liquid, which will appear, according to location and condition of objects within it, as *dew*, *fog*, or

cloud.[1] The temperature at which this condensation occurs is called the *dew-point.* The dew-point may be defined as the *temperature of saturation* for the quantity of water vapor actually present in the air. The greater the quantity of water vapor present in the air, the higher is its dew-point. Capacity for water vapor depends upon temperature ; dew-point depends upon quantity of vapor present.

If the existing temperature be far above dew-point, it indicates that the air can contain much more vapor than there is in it at the time, and the air is said to be *dry.* If the temperature of the air be little above dew-point, the air is said to be *humid,* which means that it can hold but little more vapor. The sensation of dryness experienced, especially in rooms heated artificially, does not therefore depend only upon the *absolute* quantity of water vapor present per cubic meter.

The heat of a stove, for instance, *dries* the air of a room without changing the hygrometric state. In such a room the lips, tongue, throat, and skin experience a disagreeable sensation of dryness, owing to the rapid evaporation which takes place from their surfaces. This should be taken as nature's admonition to keep water in the stove urns and in tanks connected with furnaces.

The quantity of water vapor present in the air is expressed either (1) by the hygrometric state, i.e. the mass of vapor per unit of volume ; or (2) by the relative humidity, i.e. the ratio between the quantity actually present and that which would be present if the air were saturated at the temperature of observation. The latter is the more common and more useful method. It is expressed in percentages. Thus, relative humidity = 75 per cent, or 0.75, denotes that the air contains three fourths the quantity of water vapor required to saturate it at the present temperature.

260. Wet- and dry-bulb thermometer. The relative humidity of air is measured in various ways and by various devices. The

[1] Clouds formed at temperatures above 0° consist of minute spherical drops of water, $\frac{1}{4000}$ to $\frac{1}{1000}$ of an inch or more in diameter. Clouds formed at temperatures below 0° consist of minute ice spicules, which may increase in size and become snowflakes.

instrument most commonly used is Mason's wet- and dry-bulb thermometer, or, as it is frequently called, *psychrometer*. It consists (Fig. 181) of two thermometers mounted side by side a short distance apart, one having a dry bulb and the other a bulb covered with muslin, kept moist by capillary action through conducting threads of lamp wick from a vessel of water below. The *dry bulb* indicates the temperature of the air itself, while the *wet bulb*, cooled by evaporation, shows usually a lower temperature according to the rapidity of evaporation. The difference in temperature of the two bulbs is greatest when the air is dryest.

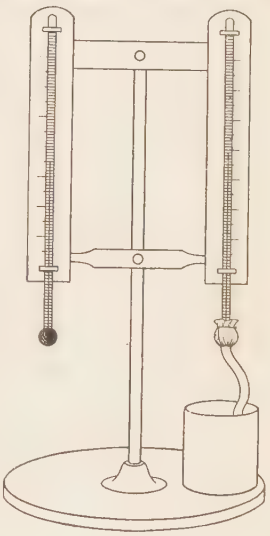


FIG. 181

By experiment has been ascertained the relative humidity corresponding to 1, 2, etc., degrees' difference between the two thermometers for any given temperature of the air. Empirical psychrometrical tables accompany this instrument. The observer reads the temperature of the air and ascertains the difference of the temperatures of the two bulbs, and from these two numbers determines by the table the relative humidity, and the dew-point of the air at that time and place.

Not only does hygrometry play an important part in the science of meteorology, and consequently have important bearings upon many branches of industry, but it also has an intimate relation to the hygienic qualities of the atmosphere. The human body is much affected by the hygrometric state of the air.

SECTION XI

DIFFUSION OR TRANSFERENCE OF HEAT

261. There is always a tendency to *equalization of temperature;* that is, heat has a tendency to pass from a warmer body to a colder, or from a warmer to a colder part of the same body, until there is an equality of temperature.

There are commonly recognized three processes of diffusion of heat, — *conduction, convection,* and *radiation.*

262. Conduction. The flow of heat through an unequally heated body, from places of higher to places of lower temperature, is called *conduction:* the body through which it travels is called a *conductor.*

If one end of a metal rod be held in a gas flame or other heat source, it will be found shortly that the remote end, if the rod be not too long, has become warm; also that the parts of the rod are progressively hotter as the source of heat is approached. The molecules of the rod in the flame have their motion quickened; they strike their neighbors and quicken their motion; the latter in turn quicken the motion of the next, and so on, until some of the motion is finally communicated to the far end.

Go into a cold room, and place the bulb of a thermometer in contact with various substances in the room; you will probably find that they have the same, or very nearly the same, temperature. Place your hand on the same substances; they appear to have very different temperatures. This is due to the fact that some substances conduct heat away from the hand faster than others. Those substances that appear coldest are the best conductors. If you go into a room warmer than your body, all this is reversed; those substances which feel warmest are the best conductors, because they conduct their own heat to your hand fastest.

By these and other experiments it appears that some substances conduct heat much more rapidly than others. The former are called *good conductors*, the latter *poor conductors*. Metals are the best conductors, though they differ widely among themselves.

Nearly fill a test tube with water, and hold it somewhat inclined (Fig. 182), so that a Bunsen flame may heat the part of the tube near the surface of the water. Do not allow the flame to touch the part of the tube that does not contain water. The water may be made to boil near its surface for several minutes before any change of temperature at the bottom will be perceived.

Liquids, as a class, are poorer conductors than solids. Gases are much poorer conductors than liquids. It is difficult to discover that pure, dry air possesses any conducting power. The poor conducting power of our clothing is due partly to the poor conducting power of the fibers of the cloth, but chiefly to the air which is confined by it.

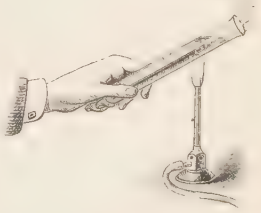


FIG. 182

Loose garments, and garments of loosely woven cloth, inasmuch as they hold a large amount of *confined air*, furnish a good protection from heat and cold. Bodies are surrounded with bad conductors, to *retain* heat when their temperature is above that of surrounding objects, and to *exclude* it when their temperature is below that of surrounding objects. In the same manner double windows and doors protect from cold.

263. Convection. Heat is transferred by convection when the hot body moves as a whole, or when there is relative motion between its parts, the heat being actually *carried*. Heat transference in fluids is chiefly by convection. If, for instance, the temperature of a mass of water be the same in all its parts, there is thermal as well as dynamic equilibrium, and there is no motion. If, however, the lower parts are heated, the water in this region will become less dense and will be buoyed upward by the denser water above, thus causing ascending and descending currents. This circulation continues till all parts of the fluid are at the same temperature. It is clear that the more expansible the fluid the more active will be the convection currents. For

this reason convection phenomena are much more violent in gases than in liquids.

The so-called *trade winds* originate in the torrid or heated zone of the earth. The air over the heated surface of the earth rises, and the colder air from the polar regions flows in on both sides, giving rise to a constant wind from the northeast[1] in the northern hemisphere, and from the southeast in the southern hemisphere. Convection currents on the surface of the sun often attain a velocity of a hundred miles per second.

264. Change of temperature in vertical currents ascending from the earth. The lower air in contact with the heated surface of the earth acquires a certain temperature and a corresponding expansion previous to its ascent. As it reaches higher altitudes the pressure upon it becomes less; it therefore expands, pushing away the surrounding air, until, as a result of its expansion, its expansive force is reduced to equality with the pressure upon it. It follows, from the dynamical theory of heat, that in doing this work the ascending air must expend some of its energy; i.e. the work is done by the expenditure of some of its heat; hence the ascending air is cooled by the very processes involved in its ascent. The rate of cooling thus produced is about 1° for 100 meters of ascent. Such changes are called *adiabatic*: i.e. they are produced *without a transfer* of heat to or from the working body.

265. Ventilation. Intimately connected with the topic *convection* is the subject (of vital importance) *ventilation*, inasmuch as our chief means of securing the latter is through the agency of the former. The chief constituents of our atmosphere are nitrogen and oxygen, with varying quantities of water vapor, carbon dioxide gas, ammonia gas, and other gases. The atmosphere also contains, in a state of suspension, varying quantities of small particles of free carbon in the form of smoke, microscopic organisms, and dust of innumerable substances. All of these constituents except the first three are called *impurities*. Carbon dioxide is the impurity that is usually the most abundant and most easily

[1] The easterly component is due to the earth's rotation.

detected: so it has come to be taken as the measure of the purity of the atmosphere, though not itself the most deleterious constituent. Its chief harm arises from its diluent effect upon the life-giving oxygen. Pure outdoor air contains about 4 parts of carbon dioxide by volume in 10,000. If the quantity rise to 10 parts, the air becomes unwholesome.

Carbon dioxide is about one and one-half times heavier than air at the same temperature; consequently, when both have the same temperature, and the former is very abundant, it tends to sink beneath the air, in which large quantities of this gas are generated.

The knowledge of this fact has led many to suppose that a means for the escape of impure air need be provided only near the floor of a room. But it should be remembered that (1) the tendency of carbon dioxide, unless present in excessive quantities, is to diffuse itself equally through a body of air; but (2) when it is heated to a temperature above that of the surrounding air, as when generated by flames, or when it escapes in the warm breath of animals, it is lighter than the air, and consequently rises. If this impure air could escape at the ceiling while fresh air entered at the floor, the ventilation would be good. But usually this fresh air must be warmed; and in passing over a stove, furnace, or steam radiator its temperature will generally become higher than that of the impure air, so that it will rise above the latter, and pass out at a ventilator in the ceiling, leaving the floor cold; hence, in high schoolrooms the most impure air is often found halfway up.

The quantity of fresh air introduced must be great enough to dilute the impurities till they are harmless. An adult makes about 18 respirations per minute, expelling from his lungs at each expiration about 500 cubic centimeters of air, over 4 per cent of which is carbon dioxide. At this rate, about 9000 cubic centimeters of air per minute become unfit for respiration; and to dilute this sufficiently, good authorities say that about 100 times as much fresh air is needed; or, for proper ventilation,

about *a cubic meter of fresh air per minute is needed for each person*, or, in British measures, 2000 cubic feet per hour.

Fig. 183 represents a scheme for heating a room by steam and ventilating it by convection. Steam is conveyed by a pipe from the boiler to a radiator box just beneath the floor of the room. The air in the box becomes heated by contact with and radiation from the coil of pipe in the box, and rises through a passage opening into the room by means of a register near the floor at *C*, a supply of pure air being kept up by means of a tubular passage opening into the box from the outside of the building. Thus the room is furnished with *pure warm* air, which, mingling with the impurities arising from the respiration of its occupants, serves to dilute them and render them less injurious. At the same time the warm and partially vitiated air of the room passes through the open ventilator *A* into the ventilating flue and escapes, so that in a moderate length of time a nearly complete change of air is effected. It is evident that on the coldest days of winter the convection is most rapid; indeed, it may be so rapid that the air cannot be heated sufficiently to render the room near the floor comfortable. At such times the ventilator *A* may be closed, while the ventilator *B* is always open. The heated air rises to the top of the room and, not being able to escape, crowds the colder air beneath out at the ventilator *B*. No system of ventilation dependent wholly on convection is adequate properly to ventilate crowded halls; air is too viscous and sluggish in its movements. In such cases

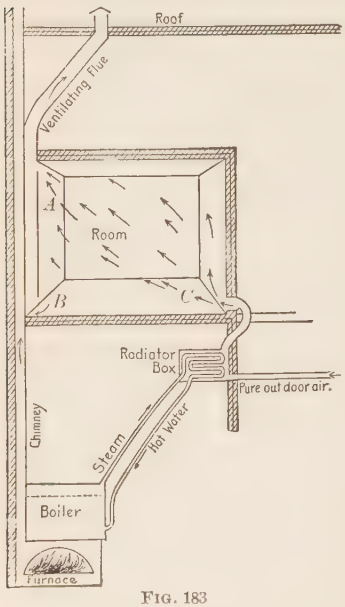


FIG. 183

ventilation should be assisted by some mechanical means, such as a blower or fan, worked by steam or water power.

266. Radiation. In *radiation* a hotter body loses heat, and a colder body is warmed, through the transmission of undulatory motion in a medium called the ether, *which is not itself heated thereby.* It is neither a mass nor a molecular transference of heat; in fact, heat itself is not transferred by radiation at all. Heat generates radiation (ether-wave motions) at one place, and the body which obstructs these waves transforms the energy of their motion, or, as it is commonly called, *radiant energy*, into heat. In this manner the earth is heated by the sun, though no heat passes between them. Similarly radiant energy passes through glass and slabs of ice without heating them much, since they offer little obstruction to the passage of ether waves. All bodies emit radiant energy, and there is an exchange of energy by radiation going on at all times between bodies. This mode of transmission of energy is the most important of all, and will be treated fully later.

SECTION XII

THERMODYNAMICS

267. Thermodynamics defined. *Thermodynamics treats of the relation between heat and mechanical work.* One of the most important discoveries in science is that of the *equivalence of heat and work*: that is, that *a definite quantity of mechanical work, when transformed without waste, yields a definite quantity of heat*; and conversely, *this heat, if there be no waste, can perform the original quantity of mechanical work.*

238. Transformation, correlation and conservation, of energy. The proof of the facts just stated was one of the most important steps in the establishment of the grand twin conceptions of modern science: (1) that *all kinds of energy are so related to one another that energy of any kind can be transformed into energy*

of any other kind, — known as the doctrine of CORRELATION OF ENERGY; (2) that *when one form of energy disappears, its exact equivalent in another form always takes its place, so that the sum total of energy is unchanged,* — known as the doctrine of CONSERVATION OF ENERGY.

These two doctrines are admirably summarized by Maxwell as follows: "*The total energy of any body or system of bodies is a quantity which can neither be increased nor diminished by any mutual action of these bodies, though it may be transformed into any of the forms of which energy is susceptible.*" Since all bodies of matter in the universe constitute a system, it follows from the above that *the sum total of energy in the universe is a constant quantity.* Neither creation nor annihilation of energy is possible through any agency known to man. These doctrines constitute the corner stones of modern physical science. Chemistry teaches that there is a conservation of matter, i.e. that matter is neither creatable nor annihilable through any known natural agency or process.

269. Joule's experiment; two laws of thermodynamics. The experiment to ascertain the "mechanical value of heat," as performed by Joule of England and later by Rowland of this country, is illustrated in principle by Fig. 184.

A copper vessel was provided with a paddle wheel (indicated by the dotted lines), which rotated about a vertical axle *A*. The axle was rotated by the weights *E* and *F*, the cord of each being so arranged that each weight, in falling, rotated the axle in the same direction. By turning the crank above *A* the weights were raised to any desired height measured on the scales *G* and *H*.

The resistance offered by the water to the motion of the paddles was the means by which the mechanical energy of the weights was converted into heat, which raised the temperature of the water. Taking two bodies whose combined mass was, say, 80 kilograms, he raised them a measured distance, say, 53 meters high; by so doing 4240 kilogrammeters of work were performed upon them, and consequently an equivalent amount of energy

was stored up in them, ready to be converted first into that of mechanical motion and then into heat. He took a definite mass of water to be agitated, say 2 kilograms, at a temperature of 0° C. After the descent of the weights the water was found to have a temperature of 5° C.; consequently the 2 kilograms of water must have received 10,000 calories of heat (careful allowance being made for all losses of heat), which is the number of calories that is equivalent to 4240 kilogrammeters of mechanical energy; *or one calorie is equivalent to 0.424 kilogrammeter of mechanical energy.*

In other words, *to produce the quantity of heat required to raise one kilogram of water through 1° C., 424 kilogrammeters of mechanical energy must be consumed.* What the experiment really shows

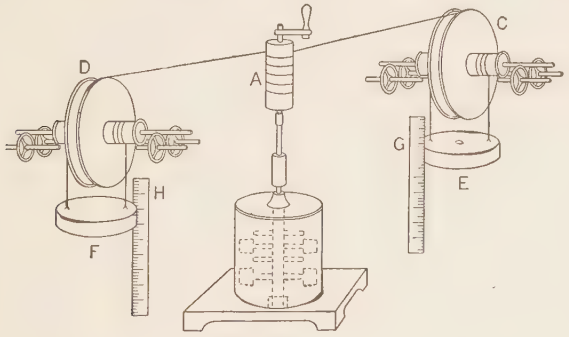


FIG. 184

is that whenever a certain quantity of mechanical energy is converted into heat, the number of thermal units produced is always proportional to the mechanical energy consumed, or to the work done. This is embodied in the first law of thermodynamics, which is expressed as follows: "*When equal quantities of mechanical effect are produced by any means whatever from purely thermal sources, or lost in purely thermal effects, equal quantities of heat are put out of existence, or are generated*": which means, in brief, that the principle of the conservation of energy applies to and so includes all heat effects. It is apparent that heat, being a form

of energy, may be measured in ergs. In this way the erg is regarded as the mechanical unit of heat. The advantage of this is found in the fact that we are often in the position of having to solve problems in which heat and work enter as terms to be added together. The existence of quantitative correlations between all the various forms of energy imposes upon men of science the duty of bringing all kinds of physical quantities to one common system of comparison, as is accomplished in the absolute system of measurement.

As we have seen, heat is the lowest form of energy, and the conversion of all other forms into heat is more easily and completely brought about than the reverse process. In fact, the conversion of heat into work is very inefficient even under the most favorable circumstances.

The subject is an extremely important one, however, since in the present age most of the power employed in commercial enterprises of all kinds is derived from the heat of combustion of some fuel.

270. Mechanical equivalent of heat. As has been stated, the mechanical equivalent of one calorie is about 0.424 kilogram-meter, which, reduced to absolute units, is 4.16×10^7 ergs, or 4.16 joules approximately. This value is known as *the mechanical equivalent of heat* and is denoted by J.

In general, if H denote a quantity of heat and W its work equivalent, then clearly $H = \frac{W}{J}$.

A knowledge of the exact value of J is of great scientific as well as of practical importance.

Rowland's work, following Joule's method, was productive of very accurate results, though more recently even better methods have been devised for measuring J by noting the heating effect of an electric current while passing through a conductor immersed in water.

From the results of all these experiments it may be said that the most probable value of J is 4.187×10^7 ergs.

SECTION XIII

THERMODYNAMICS CONTINUED — STEAM ENGINE

271. A steam engine is a machine in which steam pressure is the motive agent. Inasmuch as the elastic force of steam is entirely due to heat, *the steam engine is properly a heat engine*: that is, it is a machine by means of which heat is transformed into work, or the energy of mass motion.

The modern steam engine consists essentially of an arrangement by which steam from a boiler is conducted to each side of a piston alternately; and then, having done its work in driving the piston to and fro, is discharged from each side alternately, either into the air or into a condenser. The diagram in Fig. 185 will serve to illustrate the general features and the operation of a steam engine. The details of the various mechanical contrivances are purposely omitted, so as to present the engine as nearly as possible in its simplicity.

In the diagram, B represents the boiler, F the furnace, S the steam pipe through which steam passes from the boiler to a small chamber VC, called the valve chest. In this chamber is a slide valve V, which, as it is moved to and fro, opens and closes alternately the passages M and N leading from the valve chest to the cylinder C, and thus admits the steam alternately each side of the piston P. When one of these passages is open, the other is in general closed. Though the passage between the valve chest and the space in the cylinder on one side of the piston is closed, thereby preventing the entrance of steam into this space, the passage leading from the same space is open through the interior of the valve, so that steam can escape from this space through the exhaust pipe E. Thus, in the position of the valve represented in the diagram, the passage N is open, and steam entering the cylinder at the right drives the piston towards the left. At the same time the steam on the other side of the piston escapes through the passage M and the exhaust pipe E. While the piston

moves to the left, the valve moves to the right, and eventually closes the passage N leading from the valve chest and opens the passage M into the same, and thus the order of things is reversed.

Motion is communicated by the piston through the piston rod R to the crank G, and by this means the shaft is rotated. Connected with the shaft by means of the crank H is a rod R' which connects with the valve V, so that, as the shaft rotates, the valve

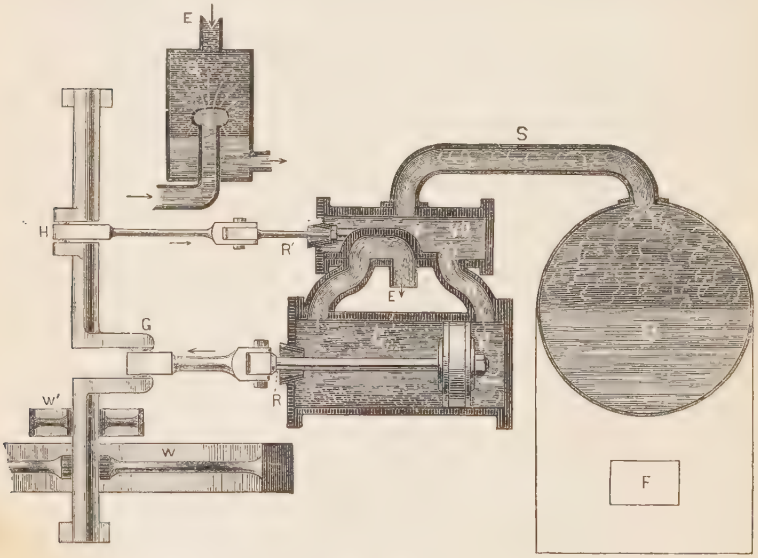


FIG. 185

for the greater part of its stroke is made to slide to and fro, in a direction in general opposite to that of the motion of the piston.

The shaft carries a fly wheel W. This is a large, massive wheel, having the larger part of its mass located near its circumference. It serves as a reservoir of energy, which is needed to make the rotation of the shaft and all machinery connected with it uniform, so that sudden changes of the driving power or of the resistances may be without effect on the speed. By means

of a belt over the pulley H' motion may be communicated to any machinery desirable.

272. Condensing and noncondensing engines. Sometimes steam, after it has done its work in the cylinder, is conducted through the exhaust pipe to a chamber Q, called a *condenser*, where, by means of a spray of cold water introduced through a pipe T, it is suddenly condensed. This water must be pumped out of the condenser by a special pump, called technically the *air pump*; thus a partial vacuum is maintained. Such an engine is called a *condensing engine*. Its advantage is obvious, for if the exhaust pipe, instead of opening into a condenser, communicate with the outside air, as in the *noncondensing engine*, the steam is obliged to move the piston constantly against a resistance from atmospheric pressure of 15 pounds for every square inch of the surface of the piston. But in the condensing engine a large portion of the pressure on the exhaust side of the piston is removed and an equivalent portion of the pressure on the steam side is utilized and made to do useful work. In well-proportioned condensing apparatus the pressure on the exhaust side may be reduced 90 per cent, so that the moving piston, instead of working against a resistance of 15 pounds, meets with a resistance of only 1.5 pounds per square inch.

273. Compound condensing or double-cylinder engine. This engine has two cylinders, each like that of a simple engine. One, A (Fig. 186), called the *high-pressure cylinder*, receives steam of very high pressure directly from the boiler through the orifice V. The steam, after it has done work in this cylinder, passes through the steam port E into cylinder B, *called the low-pressure cylinder*. Cylinder B is larger than cylinder A. The steam which enters cylinder B possesses considerable pressure, and is therefore capable of doing considerable work under suitable conditions. It should be borne in mind that in order that steam may do work in any cylinder it is necessary that an inequality in the pressure on the two sides of the piston should be maintained; just as an inequality of level, i.e. a *head*, is essential to water power.

The steam, after it has done its work in cylinder *B*, passes through a port *C* into a condenser (not represented in the figure), where it is suddenly condensed or let down to a very low pressure. If a vertical glass tube were led from the condenser to a vessel of mercury below, the mercury would ordinarily stand about 25 inches high in the tube, which would show that the pressure against which the steam does work when it

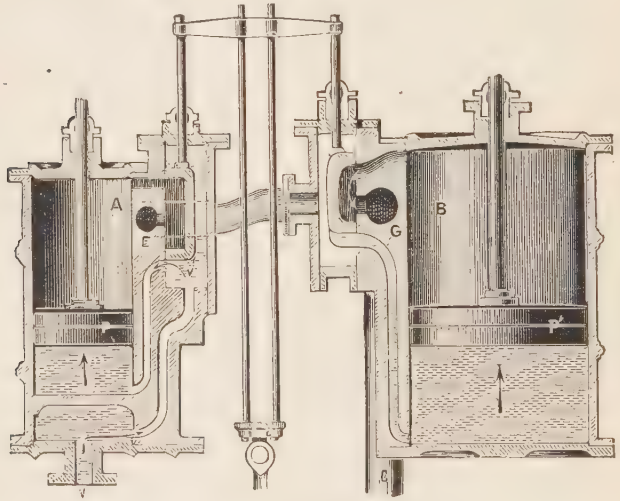


FIG. 186

enters cylinder *B*, is only about one sixth of an atmosphere. Much energy is economized by the compound engine.

274. The locomotive. The distinctive feature of the locomotive engine is its great steam-generating capacity relatively to its size and weight, which are necessarily limited. To do the work ordinarily required of it, about 20 tons of water must be converted into steam per hour. This is accomplished in two ways: first, by a rapid combustion of fuel (about 3 tons of coal per hour); second, by bringing the water in contact with a large extent (about 3000 square feet) of heated surface. The fire in the "fire box" is made

to burn briskly by means of a powerful draft which is created in the following manner: The exhaust steam, after it has done its work in the cylinders, is conducted by the exhaust pipe to the smoke box just beneath the smoke stack. The steam, as it escapes from the blast pipe, pushes the air above it and drags by friction the air around it, and thus produces a partial vacuum in the smoke box. The external pressure of the atmosphere then forces the air through the furnace grate and hot-air tubes and thus causes a constant draft. The large extent of heated surface is secured as follows: The water of the boiler is brought not only in contact with the heated surface of the fire box, but it surrounds the pipes. These pipes are kept hot by the heated gases and smoke, all of which must pass through them to the smoke box and smoke stack.

275. Power; efficiency. The horse power of a steam engine is calculated by means of the following formula: (Mean effective pressure in pounds per square inch on piston $\times$ area of piston in square inches $\times$ length of stroke in feet $\times$ number of strokes per minute) $\div$ 33,000.

The steam engine, with all its merits and with all the improvements which modern mechanical art has devised, is an exceedingly wasteful machine. The efficiency of any heat engine may be defined as the ratio of the mechanical energy utilized by the engine to the mechanical equivalent of the heat expended upon it.

QUESTIONS AND PROBLEMS

1. What kind of engine (i.e. condensing or noncondensing) is that which produces loud puffs? What is the cause of the puffs?

2. Why does the temperature of steam suddenly fall as it moves the piston?

3. What do you understand by a ten horse-power steam engine?

4. Upon what does the power of a steam engine depend?

5. Is the compound engine a condensing or a noncondensing engine? Which is the locomotive engine?

6. The area of a piston is 500 sq. in., and the average unbalanced steam pressure is 30 lb. per square inch; what is the total effective pressure?

Suppose that the piston travels 30 in. at each stroke, and makes 100 strokes per minute ; 40 per cent being allowed for wasted energy, what power does the engine furnish, estimated in horse power ?

7. A leaden bullet of mass 56 g. strikes a target with a velocity of 300 m. per second ; its temperature is 16° C. If two thirds of the energy of the bullet be used in raising its temperature, determine its final temperature.

8. Can ice at 0° C., and under ordinary atmospheric pressure, have its temperature raised ? Explain.

9. Find the resulting temperature (C.) of the following mixtures : (*a*) 5 kg. of snow at 0° with 25 kg. of water at 28° ; (*b*) 4 kg. of ice at − 10° with 30 kg. of water at 50° ; (*c*) 10 kg. of iron at 200° with 2 kg. of ice at 0°.

10. How many thermal units are required to change 5 kg. of ice at − 10° C. into water at 10° ?

11. If 30 g. of steam at 100° C. be passed into 400 g. of ice water at 0° C., what will be the temperature of the mixture ?

12. A building is heated by hot-water pipes. How does heat get from the furnace of the boiler to a person in the building ?

13. A building is heated by steam pipes. How does heat get from the furnace to objects in the building ?

14. A rod of copper at 0° C. measures 10 ft. ; its length at 100° C. is 0.191 in. greater. Find the coefficient of expansion of copper.

15. A silver rod at 0° C. is 10 ft. long. Find its length at 100° C.

16. A cubic meter of air at 100° C. is cooled down to 0°, and at the same time its pressure is halved. Determine its new volume.

17. A copper ball weighing 3 kg., taken out of a furnace and plunged into 8 kg. of water at 10° C., heated the water to 25°. Find the temperature of the furnace.

18. If the heat yielded by 1 kg. of water in cooling down from 100° to 0° C. were employed in heating 10 kg. of mercury, initially at 20°, to what temperature would the mercury be raised ?

19. A kilogram of ice at 0° C. is thrown into 6.3 kg. of water at 15° ; when the ice is melted the temperature of the water is 2°. Find the heat of fusion of ice.

20. A mass of 933 g. of copper at 80° C. is immersed in 560 g. of water at 10°, and raises the temperature of the water to 20°. Find the specific heat of copper.

ETHER DYNAMICS

ENERGY OF ETHER STRAIN; RADIATION; LIGHT

SECTION I

INTRODUCTION

276. Owing to the peculiarity of the subject to be treated in this, the third and final natural division of Physics, it is deemed expedient to state at the outset some leading propositions, whose truth must be assumed as the basis for the study of a large group of natural phenomena. The demonstrations of the validity of these several assumptions must, however, be deferred to their proper place in connection with the study of the phenomena themselves.

277. The ether. We know matter by its properties as perceived by means of our senses; in other words, the existence of any form of matter is to us only an inference from the phenomena to which it gives rise. By evidence of precisely similar nature we are led to believe in the existence of a medium called *the ether*, pervading all space and penetrating between the molecules of matter, and even between the atoms in the molecules. We cannot see, hear, feel, taste, smell, exhaust, or weigh it, and yet all this, paradoxical as it may seem, furnishes no proof that it does not exist. Briefly stated, the proof of its existence is this. *It furnishes the basis for the sole conceivable explanation of very many physical phenomena.*

Phenomena occur just *as* they would occur *if* all space were filled with an intangible and invisible medium capable of transmitting motion and energy, and we can account for all these phenomena on no other hypothesis; hence our belief in the existence of the medium. The evidence of the existence of ether is in some instances as strong and direct as that of the existence of air. The eye is an ether sense organ just as the ear may be called an air sense organ, or the hand a sense organ for the appreciation of grosser forms of matter. *The ether is a medium for the transmission of a special form of energy by means of transverse wave motion.*

In its structure the ether is assumed to be excessively fine grained, "differing from water, glass, and metals in being very much more finely grained in its structure" (Lord Kelvin). It possesses rigidity,[1] and in this respect is like a solid. Bodies of matter, even so large as the planets, pass freely through it, encountering little resistance; therein it is like a perfect fluid. It is almost perfectly elastic and incompressible.

278. Radiation. The transmission of energy by means of periodic disturbances in the ether is called *radiation*, energy so transmitted is called *radiant energy*, and the body emitting energy in this manner is called a *radiator*. The precise nature of the periodic disturbances, whether they be due to changes of position in the ether or to alternation between opposite conditions (e.g. such as successive local states of strain or distortion and release therefrom), is unknown to us. We do know, however, that the laws according to which these changes take place are those of *wave motion*. Space is traversed at all times and in all directions by myriads of ether waves of a very wide range of lengths. The all-pervading ether can be set in vibration by the motion of the molecules or atoms of ordinary matter. This local disturbance creates ether waves, and by these waves energy

[1] Calculation leads us to infer that its rigidity is about 10^{-9} that of steel, and its density 936×10^{-21} that of water at 4° C., or equal to that of our atmosphere at a height of 210 miles, — a density vastly greater than that of the same atmosphere in the interstellar spaces. — Maxwell.

is transferred from place to place by the process, as stated above, called radiation. Radiant energy can be transformed into any other form of energy, and therefore offers no exception to the doctrine of correlation of energy.

Just how vibrations of particles of matter create ether waves, and what constitutes a wave of ether, are things of which our knowledge is as yet very deficient. It must be remembered that ether is a substance very unlike ordinary matter, and therefore reasoning by analogy must often fail. It is generally supposed that ether waves are not waves of compression and rarefaction, like those of sound waves in air, but are transverse waves. Furthermore, the vibrations which occur in the ether are not longitudinal like those of the air particles during the passage of sound waves, but are transversal and somewhat analogous to the motions of particles of water in water waves.

279. Effects of radiant energy. When radiant energy is received upon the surfaces of our bodies, warmth is felt; when upon the bulb of a thermometer, rise of temperature is indicated; when by the eye, the sense of sight may be affected; if upon sensitive photographic plates, upon the leaves of plants, and upon various chemical mixtures, chemical changes may be promoted. Thus it seems that when ether waves impinge upon objects their energy is transformed, producing effects of different kinds, which are determined by the nature of the substance upon which they fall. The effect which we are now to study is that produced when the radiations strike the *eye* and become the means, through this organ, of awakening in the brain the sensation which we call light.

SECTION II

LIGHT

280. Light defined; hypotheses. Physiologically speaking, light is the *sensation of sight*. Physically considered, *it is that agent which, by its action on the retina of the eye, excites in us the*

sensation of vision. Two leading hypotheses regarding the nature of light have been propounded, which are totally different in character. One is the so-called *emission or corpuscular hypothesis* which was supported by Descartes (1629), by Newton (1672), and by most physicists up to the early part of the present century. It assumes that a luminous body, e.g. the sun, emits minute material particles (corpuscles) which travel through space in all directions with immense velocity ; these particles, by their impact upon the nerve-woven retina, produce the sensation of sight. As a rose emits minute particles which, reaching the nostrils, enable us to smell the rose, so a star was supposed to emit particles of light which, on reaching the eye, enable us to see the star.

This hypothesis is now discarded by scientists ; the reasons for its abandonment will appear further on. The theory which obtains at the present time, called the *undulatory* or *wave theory*,[1] is based upon the hypothesis that energy is transmitted from body to body, e.g. from the sun to the earth (and the reverse), in the form of vibrations or wave action in the all-pervading ether. In this connection it should be borne in mind that the evidence of the correctness of any theory is its exclusive competence to explain and coördinate phenomena. It is not claimed that all phenomena have been fully explained by the wave theory ; it is merely claimed that all we know at the present time about light is in perfect accord with it. It will be observed that both theories recognize the fact that light is essentially *dynamic*. According to the latter theory, *light is that wave motion of the ether which may be appreciated by the organ of sight.*[2]

[1] The first person who presented the wave theory of light in a definite shape was Huygens, in a work published in 1690 under the title of *Traité de la Lumière*. The theory was thoroughly established by Young and Fresnel between the years 1800 and 1820.

[2] It will be shown further on that not all ether waves are capable of affecting the sight, hence for the purpose of distinction we apply the term light waves to those ether waves only which are capable of producing vision. It is strongly recommended that the student, in beginning this branch of science, make use of the term *light waves* instead of light, except when such usage would lead to an inconvenient circumlocution, in order that he may have strongly impressed upon his mind the fact that when he is dealing with light he is dealing with *waves*.

281. Luminous and illuminated objects. Some bodies are seen by means of light waves which they generate and emit; e.g. the sun, a candle flame, and a "live coal"; they are called *luminous bodies*. Other bodies are seen only by means of light waves which they receive from luminous ones and reflect to the eye, and, when thus rendered visible, are said to be *illuminated*; e.g. the moon, a man, a cloud, and a "dead" coal.

282. Light itself invisible. Light makes visible to us luminous or illuminated objects, light waves from which actually reach our eyes; but if we look across the line of direction of a series of light waves, termed the *path* of the light, we cannot see the light. If we appear to see a sunbeam admitted through a keyhole or knot-hole, and traversing a darkened room, it is only because it is made to reveal its track by illuminating the dust motes floating in the air. If the air in a certain space be cleansed of dust, the path of a sunbeam through the space will be totally imperceptible.[1]

283. Light waves travel in straight lines. The path of light waves admitted into a darkened room through a small aperture, as indicated by the illuminated dust, is perfectly straight. *An object is seen by means of light waves which it sends to the eye.* A small object placed in a straight line between the eye and a luminous point may intercept the light waves in that path, and the point become invisible. Hence in general we cannot see around a corner or through a bent tube.

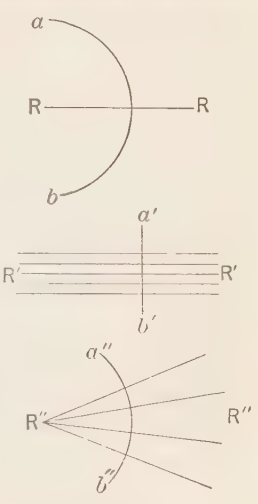


FIG. 187

284. Ray; beam; pencil. Any line *RR* (Fig. 187) which pierces the surface of an ether wave *ab* perpendicularly is called a *ray*. The term "ray" *is but an expression for the direction in which motion is propagated, and along which the successive effects of*

[1] See Tyndall's *Fragments of Science*, p. 277.

ether waves occur.[1] If the wave surface $a'b'$ be a plane, the rays $R'R'$ are parallel, and a collection of such rays is called a *beam*. If the wave surface $a''b''$ be spherical, the rays $R''R''$ have a common point at the center of curvature, and a collection of such rays is called a *pencil*.

285. Transparent, translucent, and opaque substances. Substances are *transparent*, *translucent*, or *opaque* according to the manner in which they act upon the light waves which are incident upon them. Generally speaking, those substances are *transparent* that allow other objects to be seen through them distinctly, e.g. air, glass, and water. Those substances are *translucent* that allow light waves to pass, but in such a scattered condition that objects are not seen distinctly through them, e.g. fog, ground glass, and oiled paper. Those substances are *opaque* that apparently cut off all the light waves incident upon them. When bodies intercept light they are said to cast shadows.

286. Every point of a luminous body an independent source of light waves. Place a candle flame in the center of a darkened room; each wall and every point of each wall becomes illuminated. Place yourself in any part of the room, i.e. in any direction from the flame; you are able to see not only the flame but every point of the flame; hence every point of the flame must emit light waves in every direction. *Every point of a luminous body is an independent source of light waves, and emits them in every direction.* Such a point is called a *luminous point.* In Fig. 188 there are represented a few of the infinite number of pencils of light emitted by three luminous

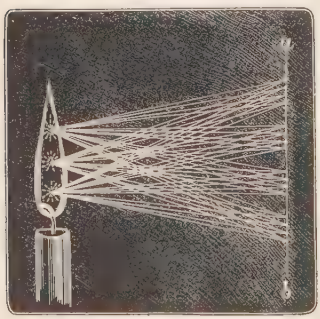


FIG. 188

[1] In dealing with certain phenomena, e.g. reflection of light, we may, to facilitate our study, consider the light as propagated in straight lines or rays; but we must bear in mind that a ray has no material or physical existence, for it is a wave that is propagated, not a ray.

points of a candle flame. Every point of an illuminated object *ab* receives light from every luminous point.

287. Images formed through small apertures. If light waves from objects illuminated by the sun — e.g. trees, houses, clouds, or even an entire landscape — be allowed to pass through a small aperture in a window shutter and strike a white screen or wall in a dark room, inverted images of the objects in their true colors will appear upon the screen. The cause of these phenomena is easily understood. When no screen intervenes between the candle and the screen *A* (Fig. 189), every point of the screen receives light from every point of the candle; consequently, at every point on *A*, images of the infinite number of points of the candle are formed. The result of the confusion of images is that no image is distinguishable. But let the screen *B*, containing a small hole, be interposed; then, since light travels only in straight lines, the point *Y'* can receive an image only of the point *Y*, the point *Z'* only of the point *Z*, and so for intermediate points; hence a distinct image of the object must be formed on the screen *A*.

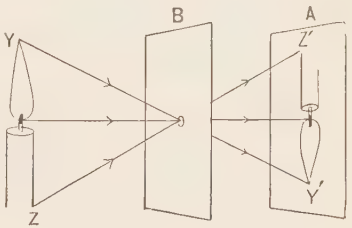


FIG. 189

The brightness of the image decreases as the opening is made smaller, since less light can pass through it. The aperture, if small, may have any shape without affecting the outline of the image. The image of the sun is a circle, irrespective of the shape of the aperture, if its rays strike the screen perpendicularly; but elliptical, if they strike the screen obliquely.

288. Shadows. Place a card 18 centimeters square between a white wall and a candle flame in a darkened room. The opaque card intercepts the light that strikes it, and thereby excludes light from a space behind it.

This space is called a *shadow*. That portion of the surface of the wall that is darkened is a section of the shadow, and in general

represents in form a cross section of the body that intercepts the light. For convenience a section of a shadow is frequently called the shadow. Notice that the shadow is made up of two distinct parts, — a dark center bordered on all sides by a much lighter fringe. The dark center is called the *umbra*, and the lighter envelope is called the *penumbra*.

Carry the card nearer the wall, and notice that the penumbra gradually disappears and the outline of the umbra becomes more distinct. Employ two candle flames, a little distance apart, and notice that two shadows are produced. Move the card toward the wall and the two shadows approach each other, then touch, and finally overlap. Notice that where they overlap the shadow is deepest. This part gets no light from either flame, and is the umbra, while the remaining portion gets light from one or the other, and is the penumbra, considering now that the light source is the two candles combined.

Just so *the umbra of every shadow is the part that gets no light from any part of the luminous body, while the penumbra is the part that gets light from some portion of the body, but not from the whole.*

Let the above experiments be repeated, employing a card 3 centimeters square, and note all differences in phenomena that occur. Hold a hair in the sunlight, about a centimeter in front of a sheet of white paper, and observe the shadow cast by the hair. Then gradually increase the distance between the hair and the paper, and note the change of phenomena.

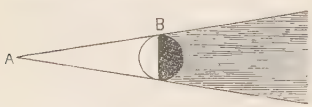


FIG. 190

If the source of light were a single luminous point, as *A* (Fig. 190), the shadow of an opaque body *B* would be of infinite length, and would consist only of an umbra. But if the source of light have a sensible size, as all physical sources have, the opaque body will intercept just as many separate pencils of light as there are luminous points, and consequently will cast an equal number of independent shadows.

Let AB (Fig. 191) represent a luminous body, and CD an opaque body. The pencil from the luminous point A will be intercepted between the lines CF and DG, and the pencil from B will be intercepted between the lines CE and DF. Hence the

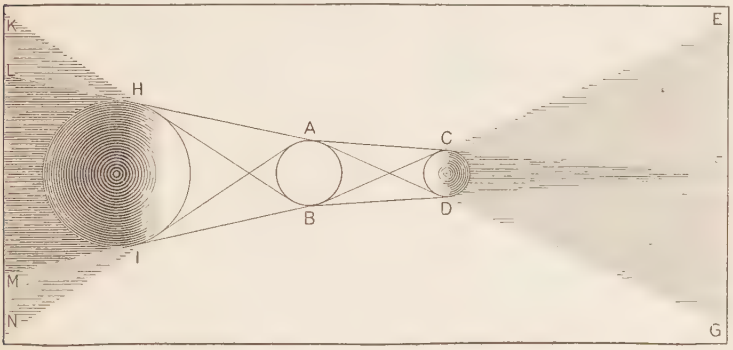


FIG. 191

light will be wholly excluded only from the space between the lines CF and DF, which inclose the umbra. The enveloping penumbra, a section of which is included between the lines CE and CF, and between DF and DG, receives light from certain points of the luminous body, but not from all.

QUESTIONS AND PROBLEMS

1. Why are images formed through apertures inverted?
2. Why is the size of the image dependent on the distance of the screen from the aperture?
3. Why does an image become dimmer as it becomes larger?
4. Why do we not imprint an image of our person on every object in front of which we stand?
5. Upon what fact does a gunner rely in taking sight?
6. Explain the umbra and penumbra cast by the opaque body HI (Fig. 191).
7. When will a transverse section of the umbra of an opaque body be larger than the object itself?
8. When has an umbra a limited length?

9. What is the shape of the umbra cast by the sphere *CD* (Fig. 191)?

10. If *CD* should become the luminous body, and *AB* a nonluminous opaque body, what changes would occur in the umbra and the shadow cast?

11. Why is it difficult to determine the exact point on the ground where the umbra of a church steeple terminates?

12. What is the shape of a section of the shadow cast by a circular disk placed obliquely between a luminous body and a screen? What is its shape when the disk is placed edgewise?

13. The section of the earth's umbra on the moon in an eclipse always has a circular outline. What does this show respecting the shape of the earth?

14. Describe the shadow cast by the earth.

15. Why does the electric arc lamp cast well-defined shadows?

SECTION III

SPEED OF LIGHT

289. Astronomical method. That light travels with finite speed was first established in 1676 by the Danish astronomer Roemer, then engaged in Paris in observing the eclipses of Jupiter's moons. He made observations on that one of the five of Jupiter's satellites which is nearest to the planet, and which revolves round this planet as the moon does round the earth. At regular intervals the satellite passes behind the planet and is eclipsed within its shadow. The observed intervals, however, were found to be shorter than the mean value when the earth and Jupiter were approaching each other, and longer when they were receding from each other. It was evident that this difference was due to the time consumed by the light in crossing the intervening spaces. From the results of these observations it was calculated that light required 16 minutes and 36 seconds to traverse the diameter of the earth's orbit, approximately 185,000,000 miles, from which it follows that the speed of light through free ether is about 185,700 miles per second.

Later this constant has been determined with great care by purely physical methods by Fizeau, Foucault, Michelson, Newcomb,

and others. The mean of all the best values up to the present time may be stated as 2.999880×10^{10} centimeters per second, with a probable error of about 20 kilometers.

Sound creeps along at the comparatively slow pace of about one fifth of a mile (or 0.3 kilometer) per second. The former is the speed with which waves in ether are transmitted; the latter, the speed with which waves in air move forward. This great difference can be accounted for only on the supposition that *the ether is far less dense and much more elastic than air.*

Notwithstanding its great speed, light requires more than three years to reach us from the nearest fixed star (α Centauri), and from those more distant it may require centuries. It is thus possible, through the instrumentality of light, faintly to conceive of the vastness of space.

SECTION IV

INTENSITY OF ILLUMINATION

290. Unit of measurement. The unit generally employed for the measurement of the intensity of the illuminating power of light emitted by a luminous body is the British *candle power.* It is the illuminating power of a sperm candle $\frac{7}{8}$ inch in diameter, burning 120 grains to the hour.

The French unit is the *carcel* (the name given to a lamp), which is equal to $9\frac{1}{2}$ candles. The unit adopted at the International Congress of Electricians in 1884 is the light emitted by a square centimeter of molten platinum at the temperature of solidification, or about 2.08 carcels, or 19.8 candles. This was called the platinum standard; and the method, the Violle method. Subsequently (1889), $\frac{1}{20}$ of this unit was adopted as the practical standard.

The following is found in Young's *Elements of Astronomy:*

"By photometric methods it is found that the sun gives us 1575 billions of billions times as much light as a standard candle would do at that distance.

"The intensity of sunlight, or the intrinsic brightness of the sun's surface, is quite a different matter from the total quantity of its light expressed in candle power. By intensity we mean the amount of light per square unit of luminous surface. From the best data we can get we find that the sun's surface is about 190,000 times as bright as that of a candle flame, and about 150 times as bright as the lime of a calcium light.

"The brightest part of an electric arc comes nearer sunlight in intensity than anything else that we know, being from one half to one quarter as bright as the solar surface itself."

And Langley says:

"If there were an electric light of 2000 candle power on each square foot of the surface of the earth, the whole light from the earth would be less than one billionth that from the sun."

The earth intercepts an extremely small part of the whole quantity of light emitted by the sun.

291. Diminution of intensity of illumination with distance. Light diminishes in its power to illuminate an object as it recedes from the object. *The illuminating power of light diminishes as the square of the distance from the light source increases.* Calling the quantity of light falling upon a visiting card at a distance of 2 feet from a lamp flame 1, the quantity falling upon the same card at a distance of 4 feet is $\frac{1}{4}$, at a distance of 6 feet it is $\frac{1}{9}$, and so on. This is the meaning of the law of inverse squares, as applied to light.

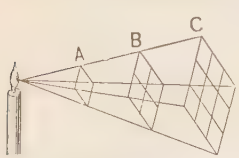


FIG. 192

This law may be illustrated thus: A square card placed, say, 1 foot from a certain point in a candle flame, as at *A* (Fig. 192), receives from this point a certain quantity of light. The same light, if not intercepted, would go on to *B*, at a distance of 2 feet, and would there illuminate four squares, each of the size of the card; and, being spread over four times the area, can illuminate each square with only one fourth the intensity. If allowed to proceed to *C*, 3 feet distant, it illuminates nine such squares, and has but one ninth its intensity

at A. The law is strictly true only when distance from individual points is considered.

292. Photometry. The law just established enables us to compare the illuminating power of one light with that of another,

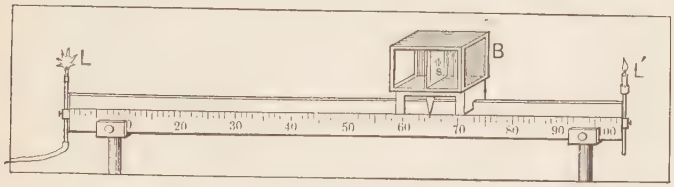


FIG. 193

and to express by numbers their relative illuminating powers. The process is called *photometry* (light measurement); and the instrument employed, a *photometer*.

293. The Bunsen photometer (Fig. 193) has a screen of paper S mounted in a box B, open in front and at the two ends. The box slides on a graduated bar. The screen has a circular central spot saturated with paraffin, which renders the spot more translucent than other portions of the screen. One side of the screen is illuminated by the light L, whose intensity is to be measured, and the other side by a standard candle L'. When the screen is so placed that the two sides are equally illuminated by the two lights, the paraffined spot becomes nearly invisible. When one side is more strongly illuminated than the other, the spot appears dark on that side and light on the other. *The candle power of the two lights is directly proportional to the square of their respective distances from the screen when it is equally illuminated on both sides.*

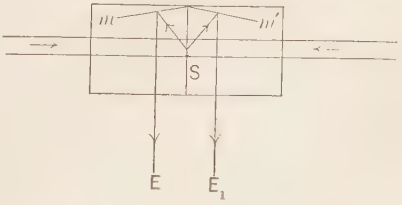


FIG. 194

In order to render both sides of the disk simultaneously visible, two mirrors, m and m' (Fig. 194), are placed in the box in a

vertical position, so as to reflect images of the circular spot in the screen S to the eyes at EE_1.

QUESTIONS AND PROBLEMS

1. Suppose that a lighted candle is placed in the center of each of three cubical rooms, respectively 10, 20, and 30 ft. on a side. Would a single wall of the first room receive more light than a single wall of either of the other rooms, or less?

2. Would 1 sq. ft. of a wall of the third room receive as much light as would be received by 1 sq. ft. of a wall of the first room? If not, what difference would there be, and why?

3. If a board 10 cm. square be placed 25 cm. from a candle flame, the area of the shadow of the board cast on a screen 75 cm. distant from the candle will be how many times the area of the board? Then the light intercepted by the board will illuminate how much of the surface of the screen if the board be withdrawn?

4. Give a reason for the law of inverse squares.

5. To what besides light has this law been found applicable?

6. The two sides of a paper disk are illuminated equally by a candle flame 50 cm. distant on one side and a gas flame 200 cm. distant on the other side. (*a*) Compare the illuminating powers of the two lights at equal distances from their sources. (*b*) If the candle be a standard candle, what is the illuminating power of the gas flame?

SECTION V

APPARENT SIZE OF AN OBJECT

294. Visual angle. Prick a pin hole in a card, place an eye near the hole, and look at a pin about 20 centimeters distant. Then bring the pin slowly toward the eye, and the dimensions of the pin will appear to increase as the distance diminishes.

Why is this? We see an object by means of its image formed on the retina of the eye, and its apparent magnitude is determined by the extent of the retina covered by its image. Rays proceeding from the extremities of an object, as AB (Fig. 195), meet and cross each other within the eye. Now, as the distance between the points of the blades of a pair of scissors depends

upon the angle that the handles form with each other, so the size of the image formed on the retina depends upon the size of the angle, called the *visual angle*, formed by these rays as they enter the eye. But the size of the visual angle diminishes approximately as the distance of the object from the eye increases, as

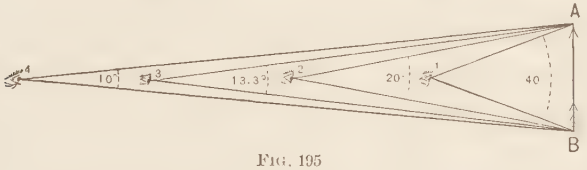


FIG. 195

shown in the diagram; e.g. at twice the distance the angle is about one half as great; at three times the distance the angle is one third as great, and so on. Hence, *distance affects the apparent size of an object.* Our judgment of the size of objects is, however, somewhat influenced by other things besides the visual angle which they subtend.

SECTION VI

REFLECTION OF LIGHT

295. Mirrors and images. Objects having polished surfaces which reflect light regularly (i.e. do not scatter the light), and show images of objects presented to them, are called *mirrors.* The mirror itself, if clean and smooth, is scarcely visible. An image is a picture of an object. According to their shape mirrors are called *plane*, *concave*, *convex*, *spherical*, *parabolic*, etc.

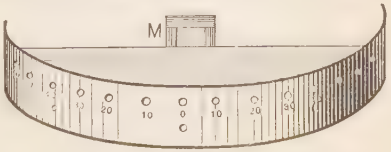


FIG. 196

Look at the mirror *M* through the hole marked 0 in the band (Fig. 196). In the mirror is seen an image of the hole through which you look, but you do not see the images of any of the other holes. Rays that pass through this hole strike the mirror

perpendicularly and are said to be normal to the mirror. Rays falling upon a surface are called *incident rays*. The point where a ray strikes is called the *point of incidence*. The *reflected rays* in this case are thrown back in the same lines and through the same hole that the incident rays travel. *Rays normal to a mirror simply retrace their own course after reflection.* Next hold a candle flame at one of the other holes, e.g. at the hole marked 30. You can see the image of the candle flame only through the hole of the same number and at an equal distance on the other side. The angle which an incident ray makes with a line normal at the point of incidence is called the *angle of incidence*, and the angle made by a reflected ray with the normal is called the *angle of reflection*. From these observations is demonstrated the law of reflection, namely, *that the angles of incidence and reflection are in the same plane, and are equal.*

296. The wave theory applied to reflection. The following is an explanation of reflection in accordance with the wave theory. Suppose KA, ND, etc. (Fig. 197), to be parallel rays of a beam of light falling on a plane mirror $H\,I$. $K\,L\,M\,N$ may represent the plane front of one of the waves. As soon as the wave reaches A that point becomes the origin of a disturbance in the ether, which spreads out in the form of a sphere having its center at A. Let the arc of a circle described around A as a center denote the boundary which the new wave has reached during the interval between the arrival of the plane wave at A and D respectively; then the radius of this circle is equal to the excess of ND over KA, because so long as light travels in the same isotropic medium its speed in all directions is the same. Similarly let arcs

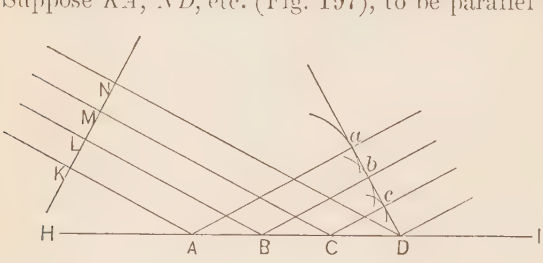


FIG. 197

be described around points *B*, *C*, and *D*, with radii determined in the same manner. A straight line *abcD*, drawn tangent to these circles, represents a plane reflected wave front corresponding to the plane incident wave. It is inclined to *II* at the same angle but on the other side of the normal.

297. Intensity of reflected light. The intensity of reflected light increases with the polish of the reflecting surface and with the obliquity of the incident rays. It also depends largely upon the nature of the medium from which reflection takes place. For example, at perpendicular incidence water reflects about the fiftieth part of the incident light, while mercury reflects about two thirds; but at an incidence of 89.5° each reflects about 72 per cent of the incident light. The varnished surfaces of furniture appear much brighter when viewed obliquely than when seen by light reflected less obliquely. Light reflected from the surface of a pond just before the sun sets is much more dazzling than at noon when the sun is overhead. This is due in part to the fact that we are in a more suitable position to observe it.

298. Diffused light. Introduce a small beam of light into a darkened room by means of a porte-lumière, and place in its path a mirror. The light is reflected in a definite direction. If the eye be placed so as to receive the reflected light, it will see not the mirror, but the image of the sun, and the light will be

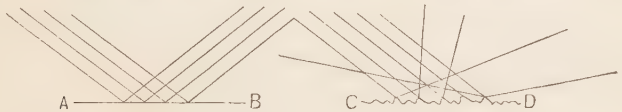


FIG. 198

painfully intense. Substitute for the mirror a piece of unglazed paper. The light is not reflected by the paper in any definite direction, but is scattered in every direction, illuminating objects in the vicinity and rendering them visible. Looking at the paper, you see not an image of the sun, but the paper, and you may see it about equally well in all directions.

The *dull* surface of the paper receives light in a definite direction, but reflects it in every direction; in other words, it scatters or *diffuses* the light. Much of the difference in the phenomena in the two cases is caused by the difference in the smoothness of the two reflecting surfaces. AB (Fig. 198) represents a smooth surface, like that of glass, which reflects nearly all the rays of light in the same direction, because nearly all the points of reflection are in the same plane. CD represents a surface of paper having its roughness greatly exaggerated. The diffusion clearly results from the many different directions in which the incident light is reflected by the rough surface. Thus the dull surfaces of various objects around us reflect light in all directions, and are consequently visible from every side. Objects rendered visible by reflected light are said to be *illuminated*.

By means of regularly reflected light we see images of objects in mirrors, but only from definite positions, i.e. in definite directions; by means of diffused light we see the object itself from every direction. Whether we see the image of the source of the light (the eye being situated so as to receive the regularly reflected light), or the object on which the light falls, or both at the same time, depends largely upon the degree of smoothness possessed by the surface that reflects the light. Polished metals are the best mirrors. Surfaces of liquids at rest are excellent mirrors. It is sometimes difficult to see a smooth surface of a pond surrounded by trees and overhung by clouds, as the eye is occupied by the reflected images of these objects; but a faint breath of wind, slightly rippling the surface, will reveal the water.

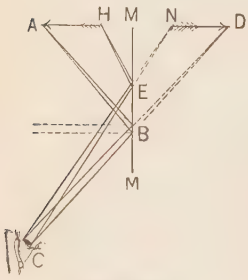


FIG. 199

299. Reflection from plane mirrors; virtual images. MM (Fig. 199) represents a plane mirror, and AB a pencil of divergent rays proceeding from the point A of an object AH. By erecting perpendiculars at the points of incidence, and making the angles of reflection equal to the

angles of incidence, the paths BC' and EC' of the reflected rays from B and E respectively are found.

Every visible point of an object sends a cone of rays to the eye. The point always appears at the place whence these rays seem to emerge, i.e. at the real apex of the cone. If the direction of these rays be changed by reflection, or in any other manner, the point will appear to be in the direction of the rays as they enter the eye; thus the point A appears to lie in the direction CD; and the point H, in the direction CN. The exact location of these points may be found by continuing the rays $C'B$ and $C'E$ behind the mirror till they meet at the points D and N. Thus the pencils EC' and BC' appear to emanate from the points N and D, and the whole body of light waves received by the eye seems to come from an *apparent object* ND behind the mirror. This apparent object is called an *image*. An image is a point, or a series of points, from which a diverging pencil of rays comes or appears to come. As of course no real image can be formed back of a mirror, such an image is called a *virtual* image. It will be seen, by construction, that *an image in a plane mirror appears as far behind the mirror as the object is in front of it, and is of the same size and shape as the object.*

It appears from the above diagram that divergent incident rays remain divergent after reflection from a plane mirror. In a like manner it may be shown by construction that parallel incident rays are parallel after reflection, and convergent incident rays are convergent after reflection, i.e. *reflection from a plane mirror does not change the mutual angle between rays.*

300. Reversion of images. When we look at our own faces in a mirror we discover a lateral reversion. The right cheek is the left cheek in the image; the hair parted on the left is parted on the right in the image.

If the plane of the mirror be vertical, objects appear in their proper relations to the horizon; but if the mirror have any other position, objects assume unnatural postures. Thus, turn this book so that the mirror MM (Fig. 199) may represent a

horizontal mirror, and AH a vertical object above it, and it will be seen that the image appears inverted. To verify this, place a mirror in a horizontal position, and set on it a goblet of water. The image of the goblet will appear upside down. In a mirror inclined at an angle of 45° to the horizon the image of an erect object appears horizontal, while the image of a horizontal object appears erect.

· **301. Multiple reflection; images of images.** When light is reflected successively from two plane mirrors, the image in the first becomes the object for the second mirror, and the second image is found in precisely the same manner as the first one. Again, the second image serves as an object for a third image, and so on. If the two mirrors be parallel, as A and B (Fig. 200), the series of virtual images appears at regularly increasing distances from the mirrors on the same straight line normal to their surfaces. Thus a' is the primary image of the object a in mirror A. (To avoid confusion, a pencil from only one point, o, is drawn.) The light reflected at e enters the eye as though it came from o'. Other rays reflected from A at e diverge as though they emanated from o', and are reflected from B at e', and may be regarded as proceeding from a *real* object at o', whose image is o'', as far back of B as o' is in front of B. The light reflected from B to A again diverges as though it really came from o'', and o'' being regarded as a *real* object, *its* image would be formed at o''', and the pencil which enters the eye seems to proceed from o''', having been reflected at e'' as though it came from o''. The pencil which would enter the eye from a third image at the left of a'' may be traced through all its reflections in like manner. As some light is lost at each reflection, the images decrease in brightness as they recede.

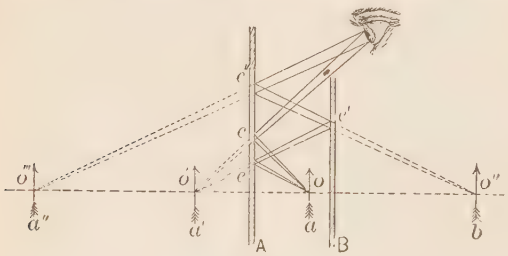
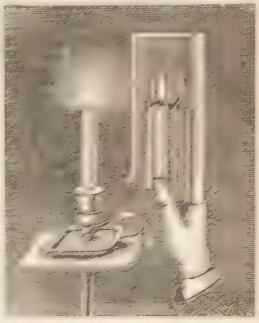


FIG. 200

A kaleidoscope is constructed on the principle of multiple reflection. It consists of a tube containing three mirrors running lengthwise facing the inside and placed at angles of 60°. Pieces of colored glass, free to move at one end of the tube, are viewed through an eyepiece at the oppo[illegible] [illegible] by repeated reflections.



[illegible]

Multiplied images of a small, [illegible] object, as of a candle flame (Fig. 201), often seen in a thick glass mirr[illegible] [illegible] by repeated reflections be[illegible] the anterior surface and the silvered [illegible] while another portion is reflected [illegible] back, and thence forward again, show[illegible]

302. Reflection from concave mirrors. [illegible] 202 [illegible] sent a section of a concave spherical mirror, which may be regarded as a small part of a hollow spherical shell having a polished interior surface. The distance MM' is called the *diameter of the mirror.* C is the center of the sphere, and i[illegible] called the *center of curvature.* G is the *vertex* of the mirror. A straight line [illegible] through the center of curvature and the vertex is called the *principal axis* of the mirror. A concave [illegible]

[illegible]

surfaces. All radii of the mirror, as CA, [illegible] dicular to the small [illegible] point, it is evident that all light waves em[illegible] [illegible]

Let E be any luminous point in front of a concave mirror. To find the direction that rays emanating from this point take after reflection, draw any two lines from this point, as EA and EB, representing two of the infinite number of rays composing the divergent pencil that strikes the mirror. Next draw radii to the points of incidence A and B, and draw the lines AF and BF, making the angles of reflection equal to the angles of incidence. Place arrowheads on the lines representing rays, to indicate the direction of the motion. The lines AF and BF represent the direction of the rays after reflection.

It will be seen that in this case the rays after reflection are convergent, and meet at a point F. This point is the focus of all reflected rays that emanate from the point E. It is obvious that if F were the luminous point, the lines AE and BE would represent the reflected rays, and E would be the focus of these rays. Since the relation between the two points is such that light waves emanating from either one are brought by reflection to a focus at the other, these points are called *conjugate foci. Conjugate foci are two points so related that the image of either is formed at the other.* The rays EA and EB, emanating from E, are less divergent than rays FA and FB, emanating from a point F less distant from the mirror, and incident at the same points. Rays emanating from D will be still less divergent; and if the point D were removed to a distance of many miles, the rays incident at these points would be very nearly parallel. Hence rays may be regarded as practically parallel when their source is at a very great distance, e.g. the sun's rays. If a sunbeam, consisting of a bundle of parallel rays, as EA, DG, and HB (Fig. 203), strike a concave mirror in a direction parallel with its principal axis, these rays become convergent by reflection, and meet at a point F in the principal axis. This point, called the *principal focus, is about halfway between the center of curvature and the vertex of the mirror.*

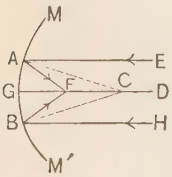


FIG. 203

The distance FG is called the *principal focal length* of the mirror.

On the other hand, it is obvious that *divergent rays emanating from the principal focus of a concave mirror become parallel by reflection.*

If a small piece of paper be placed at the principal focus of a concave mirror, and the mirror be exposed to the parallel rays of the sun, the paper will quickly burn.

Construct a diagram and show that *rays proceeding from a point between the principal focus and the mirror are divergent after reflection, but less divergent than the incident rays.* On reversing the direction of the rays, the same diagram will show that *convergent rays are rendered more convergent by reflection from concave mirrors.*

The general effect of a concave mirror is to increase the convergence or to decrease the divergence of incidence rays.

303. Spherical aberration of mirrors. The statement that parallel rays after reflection from a concave mirror meet at the principal focus is only approximately true. It is strictly true only of paraboloic mirrors such as are used in the headlights of locomotives. Consequently paraboloic mirrors are used when it is desired to bring the plane-fronted light waves of a distant star accurately to a focus, or to change a divergent pencil to a parallel beam; in the latter case the source of light is placed at the focus of the paraboloid. In spherical mirrors, when the pencil is broad, the outside rays, or those which are incident upon the mirror farthest from its vertex, are brought to a focus nearer the mirror than the inner rays; consequently the image furnished by a luminous point is a circle brightest toward its center. This phenomenon is called the *spherical aberration of a mirror.* It renders the definition of the images of objects, especially of broad objects, very bad. In consequence of this it is often necessary to cut off the outside rays by a diaphragm, which improves the definition at the expense of the brightness of the image.

By constructing a number of rays (Fig. 201) we may show that all rays after reflection are tangent to a characteristic curve called a *caustic.* The light emitted from a single point, as A, is spread

over the surface produced by the revolution of the line ST about the axis AM. This curve formed in milk by reflection from the interior surface of a bright tin pan or pail is commonly called *the cow's foot.*

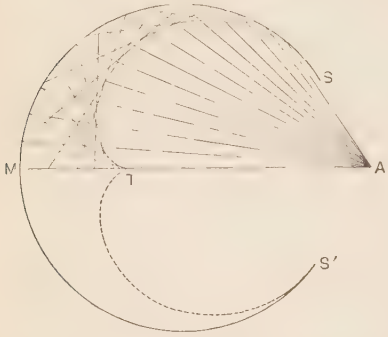


FIG. 204

304. Formation of images. To determine the position and kind of images formed in concave mirrors of objects placed in front of them, proceed as follows: Locate the object, as DE (Fig. 205). Draw lines, EA and DB, from the extremities of the object through the center of curvature of the mirror, to meet the mirror. These lines are called *secondary axes.* Incident rays along these lines will return by the same paths after reflection. Draw another line from D to any point in the mirror, e.g. to F, to represent another of the infinite number of rays emanating from D. Make the angle of reflection CFD' equal to the angle of incidence CFD, and the reflected ray will intersect the secondary axis DB at the point D'. This point is the conjugate focus of all rays proceeding from D. Consequently an image of the point D is formed at D'. This image is called a *real image* because rays actually meet at this point. In a similar manner, find the point E', the conjugate focus of the point E. The images of intermediate points between D and E lie between the points D' and E', and consequently the image of the object lies between those points as extremities.

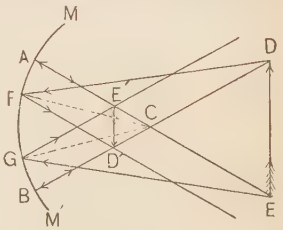


FIG. 205

If, for the second ray to be drawn from any point, we select that ray which is parallel with the principal axis, as AG (Fig. 206), it will not be necessary to measure angles. For this ray, after

reflection, must pass through the principal focus F; and consequently the conjugate focus A' is easily found, and so for the point B' and intermediate points.

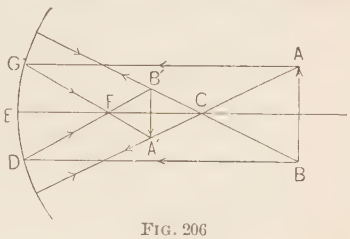


Fig. 206

It thus appears that *an image of an object placed beyond the center of curvature of a concave mirror is real, inverted, smaller than the object, and located between the center of curvature and the principal focus of the mirror.*

An eye placed in a suitable position to receive the light, as at H (Fig. 207), will receive the same impression from the reflected rays as if the image $E'D'$ were a real object; for a cone of rays originally emanates from, say, the point D of the object, but it enters the eye as if emanating from D', and consequently appears to originate from the latter point. A person standing in front of such a mirror, at a distance greater than its radius of curvature, will see an inverted image of himself suspended, as it were, in mid-air. Or, if in a darkened room an illuminated object be placed in front of the mirror, and a small oiled-paper screen or one of ground glass be placed where the image is formed, a large audience may see the image projected upon the screen.

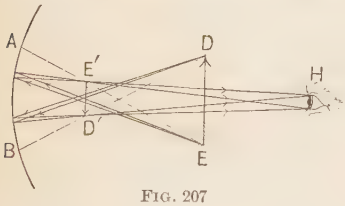


Fig. 207

If $E'D'$ (Fig. 207) be taken as the object, then the direction of the light in the diagram will be reversed, and ED will represent the image. Hence *the image of an object placed between the principal focus and the center of curvature is also real and inverted, but larger than the object, and located beyond the center of curvature.*

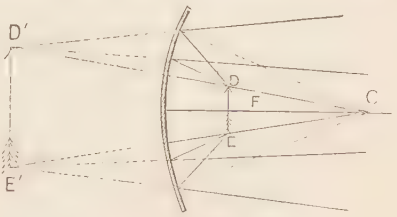


Fig. 208

The image in this case may be projected upon a screen, but it will not be as bright as in the former case, because the light is spread over a larger surface.

If now the object *DE* be placed between the principal focus and the mirror, the image will no longer be a real one. It will be seen in this case (Fig. 208) that a pencil of rays proceeding from any point of the object, e.g. *D*, has no actual focus, but appears to proceed from a *virtual* focus *D'*, back of the mirror; and so with other points, as *E*. *The image of an object placed between the principal focus and the mirror is virtual, erect, larger than the object, and back of the mirror.*

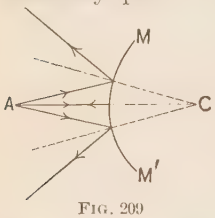


FIG. 209

The diagram in Fig. 209 suggests the method of finding the disposition of a pencil of rays emanating from any point (e.g. *A*) after reflection from a convex mirror.

A carrying out of this suggestion will enable the student to construct the images of objects in the case of a convex mirror. It will be noted that all such images are virtual, smaller, and erect.

305. Illustrative experiments. Hold some object, e.g. a rose, as *ab* (Fig. 210), a few feet in front of a concave mirror. Looking in the direction of the axis of the mirror you see a small inverted image *AB* of the object, between the center of curvature *C* of the mirror and its principal focus *F*.

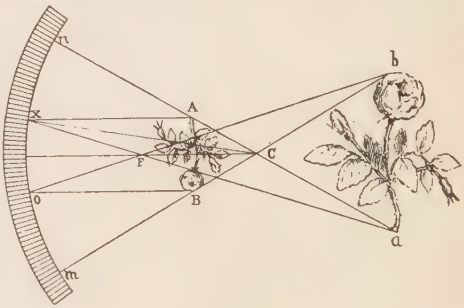


FIG. 210

Evidently the process is exactly reversible so that if *AB* represent an object placed between the principal focus and the center of curvature, then *ab* will represent the *image* of the object.

Place a candle in an otherwise dark room 20 feet from a mirror of rather short focal length, catch the focused light waves upon a paper screen, and show that the focus is about halfway between the vertex and the center of curvature of the mirror.

Move the distant candle flame toward the mirror, moving it up and down. (1) Show that the image advances to meet the flame, and that when the flame is raised the image is depressed, and vice versa. (2) Show that when the flame is at the center of curvature the image is also there. (3) Show that when the flame is between the center of curvature and the principal focus, the image of the flame is farther away than the center of curvature. (4) Show that when the flame is at the principal focus, the reflected rays are parallel, or the image is at an infinite distance. (5) Show that when the flame is still nearer, the reflected rays diverge and appear to come from a point behind the mirror, and that the image is therefore virtual. (6) Notice that in all cases except the last the images are real and inverted, and that in all cases where a real image is formed, the flame and the image may change places.

306. Derivation of formulas. It has already been mentioned that the principal focus of a concave mirror is approximately halfway from the vertex to the center of curvature.

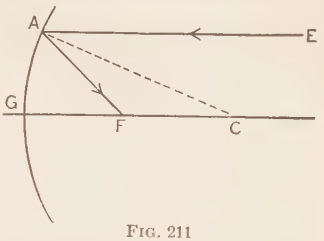


FIG. 211

To prove this let EA (Fig. 211) be an incident ray parallel to the principal axis, which is reflected to F, the principal focus. Now since in all practical optical mirrors the semidiameter AG is not great compared with the radius of curvature, FA and FG are approximately equal. Since, too, in the triangle ACF the angles adjacent to AC are equal (each being equal to EAC), FC is equal to FA and therefore approximately to FG, which proves F to be about midway between G and C. The nearer the ray EA is to the axis, i.e. the smaller the aperture of the mirror, the nearer does F come to the exact middle of CG.

When light emitted by a luminous point E (Fig. 202) at a distance f is reflected by a concave spherical mirror, it meets the principal axis at an approximate focus at F, a distance f'. It may easily be shown that the relation between f, f', and r, the radius of curvature, is given by the simple formula

$$\frac{1}{f} + \frac{1}{f'} = \frac{2}{r}.$$

To prove this let EAF be the path of the incident and reflected ray; then, since the radius AC bisects the angle EAF, EF is divided by C into parts proportional to EA and FA, or $EC : FC :: EA : FA$. Substituting now for EA and FA the approximate values EG and FG, we get at once $f - r : r - f' :: f : f'$, which reduces readily to the desired form, namely,

$$\frac{1}{f} + \frac{1}{f'} = \frac{2}{r}.$$

From this it immediately follows that the principal focal length is $r/2$. For if f be infinite $1/f$ is 0, and $1/f'$ equals $2/r$, or $f' = r/2$. Since, then, this is the case in which the incident rays are parallel to the axis, f' becomes the principal focal length or half the radius. If either f or f' be made equal to r, the other must also be equal to r, and the object and image both meet at C. If either be less than $r/2$, the other becomes negative and the image is behind the mirror and therefore virtual.

QUESTIONS AND PROBLEMS

1. When an object is located at a distance from a concave mirror equal to its radius, will any image be formed? Why?

2. What is the effect of placing the object at the principal focus?

3. (*a*) When is the real image formed by a concave mirror smaller than the object? (*b*) When is it larger?

4. (*a*) When is the image formed by a concave mirror real? (*b*) When is it virtual?

5. (*a*) Is the image of an object formed by a convex mirror real or virtual? (*b*) Is it larger or smaller than the object? (*c*) Is it erect or inverted?

6. Is the general effect of a convex mirror to collect or to scatter rays?

7. The radius of a concave spherical mirror is 20 in. Determine the conjugate focus for a point on the principal axis 15 in. from the mirror.

8. Why do images formed by a surface of water appear inverted?

9. (*a*) What kind of wave front has a beam of parallel rays? (*b*) What change in front of such a beam occurs when it strikes respectively a plane, a concave, and a convex mirror?

10. Where is the conjugate focus of light emanating from each of the following points: (*a*) the center of curvature of a concave spherical mirror; (*b*) a point on the principal axis at an infinite distance from the mirror; (*c*) a point on the principal axis beyond the center of curvature and at a finite distance from the mirror; (*d*) a point on the principal axis between the center of curvature and the principal focus; (*e*) a point between the principal focus and the vertex?

11. How may the radius of curvature of a concave spherical mirror be found by optical means alone?

SECTION VII

REFRACTION

307. When a ray of light passes from one medium into another of different density (Fig. 215), it is bent or *refracted* at the interface between the two media unless it meet this surface perpendicularly. In the latter case there is no change of direction. If it pass into an optically denser medium, it is refracted toward the perpendicular to this surface; if into a rarer medium, it is refracted from the perpendicular. It is not universally true that media of greater mass density are the more highly refracting. The refractive power of water is less than that of alcohol or oil of turpentine. A substance which has a higher refractive power than another is said to be *optically denser*.

The angle RAD is called the angle of incidence; $R'AH$, the angle of refraction; and $R''AR'$, the angle of deviation.

308. The wave theory applied to refraction. Foucault and others have proved by careful experiments that the speed of light is less in water than in air. It is less in glass than in water, and much less in diamond than in glass. Every transparent substance has its own rate of transmission. It would

seem that there is an interaction between the ether and the molecules of matter such that in different media the ether waves are unequally retarded.

Let the series of parallel lines AB (Fig. 212) represent a series of wave fronts leaving an object C, passing through a rectangular piece of glass DE, and constituting a beam. Every point in a wave front moves with equal velocity as long as it traverses the same medium; but the point a of a given wave ab enters the glass first, and its velocity is impeded, while the point b retains its original velocity; so that, while the point a moves to a', b moves to b', and the result is that the wave front assumes a new direction (very much in the same manner as a line of soldiers executes a wheel), and a ray drawn perpendicularly through the series of waves is turned out of its original direction on entering the glass. Again, the extremity c of a given wave front cd first emerges from the glass, when its velocity is immediately quickened; so that, while d advances to d', c advances to c', and the direction of the ray is again changed. The direction of the waves after emerging from the glass is parallel to their direction before entering it, but there has been a lateral displacement.

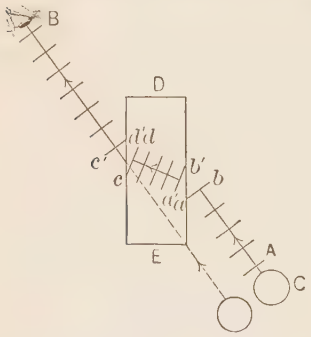


FIG. 212

It is evident that if the waves enter the new medium in a direction perpendicular to its surface, i.e. with wave front parallel to this surface, all parts of it will be retarded simultaneously and no deviation will take place.

Very beautiful diagrams were constructed by Huyghens to demonstrate the laws of reflection and refraction of light according to this wave theory, and are to be found and explained in most advanced books on the subject.

309. Failure of the emission theory to account for the refraction of light. To explain refraction from a rare to a denser medium

according to the emission theory of light it is necessary to assume that when a light particle shot from a luminous body comes within a very small distance of the surface of separation between two mediums, it begins to be attracted towards the surface so that its component velocity perpendicular to the surface gradually increases till it reaches a limited distance on the other side of the surface; that is, the speed of light should be, by the theory, greater in dense than in rarer mediums: whereas the reverse is found to be true. Hence the failure of the emission theory to account for the phenomenon of refraction.

310. Familiar examples of refraction. Place a coin (A, Fig. 213) on the bottom of an empty basin so that, as you look through a small hole in a card BC over the edge of the vessel, the coin is just out of sight. Then, without moving the card or basin, fill the latter with water. Now, on looking through the aperture in the card, the coin is visible. The beam AE, which formerly moved in the straight line AD, is now bent at E, where it leaves the water, and, passing through the aperture in the card, enters the eye. Observe that as the beam passes from the water into the air it is turned farther from a vertical line EF; in other words, *the beam is farther from the vertical than before.*

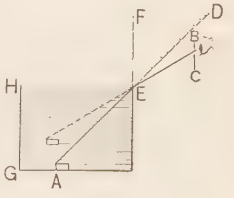


FIG. 213

From the same position as in the last experiment, direct the eye to the point G in the basin filled with water. Reach your hand around the basin, and place your finger where that point appears to be. On examination it will be found that your finger is considerably above the bottom. Hence *the effect of the bending of rays, as they pass obliquely out of water, is to cause the bottom to appear more elevated than it really is*; in other words, *to cause the water to appear shallower than it is.*

Place a piece of wire (Fig. 214) vertically in front of the eye, and hold a narrow strip of thick plate glass horizontally across the wire, so that the light waves from the wire may pass

obliquely through the glass to the eye. The wire will appear to be broken at the two edges of the glass, and the intervening section will appear to the right or left according to the inclination of the glass; but if the glass be not inclined to the one side or the other, the wire does not appear broken.

FIG. 214

311. Index of refraction. The deviation of light waves in passing from one medium into another depends upon the media and the angle of incidence. It diminishes as the angle of incidence diminishes, and is zero when the incident ray is normal. It is highly important, knowing the angle of incidence, to be able to determine the direction which a ray will take on entering a new medium. Describe a circle around the point of incidence A (Fig. 215) as a center; through the same point draw HI perpendicular to the surfaces of the two media, and to this line drop perpendiculars BD and CE from the points where the circle cuts the ray in the two media. Then suppose that the perpendicular BD is $\frac{8}{10}$ of the radius AB. Now this fraction $\frac{8}{10}$ is called (in trigonometry) the *sine* of the angle DAB; hence $\frac{8}{10}$ is the *sine of the angle of incidence*. Again, if we suppose that the perpendicular CE is $\frac{6}{10}$ of the radius, then the fraction $\frac{6}{10}$ is the *sine of the angle of refraction*. The sines of the two angles are to each other as $\frac{8}{10} : \frac{6}{10}$, or as 4 : 3. The quotient (1.33+) obtained by dividing the sine of the angle of incidence by the sine of the angle of refraction, generally expressed decimally, is called the *index of refraction*. It has been shown to be equal to the ratio of the velocity of the incident waves to that of the refracted waves, and is constant for the same two media. This process is also reversible, as will be shown.

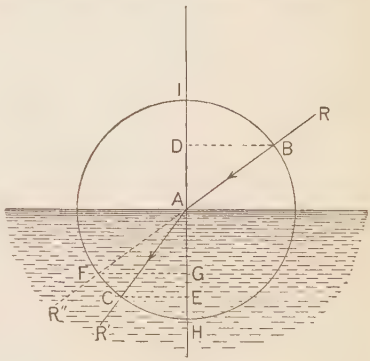


FIG. 215

This constant sine ratio was discovered by Snell in 1621 and is called Snell's *law of sines*. To sum up now, the laws of refraction may be stated as follows. "The incident and the refracted rays are in the same plane with the normal to the surface; they lie on opposite sides of the normal, and the sines of their inclinations to it bear a constant ratio to each other."

312. Absolute indices of refraction. When a ray passes from a vacuum into a medium the refractive index is greater than unity, and is called the *absolute index of refraction. The relative index of refraction, from any medium A into another B, is found by dividing the absolute index of B by the absolute index of A.*

It will be shown later that the refractive index varies with wave length. The following table is intended to represent *mean indices* for light waves:

TABLE OF ABSOLUTE INDICES

Lead chromate	2.97	Crown glass (about)	1.53
Diamond (about)	2.5	Spirits of turpentine	1.48
Carbon disulphide	1.64	Alcohol	1.37
Flint glass (about)	1.61	Humors of the eye (about)	1.35
Agate	1.54	Pure water	1.33
Canada balsam	1.53	Air at 0°C. and 760^{mm} pressure	1.000294

313. Construction. *Given the direction of the incident ray and the refractive index, to determine the direction of the refracted ray.* Let LO (Fig. 216) be the incident ray; draw a circle with the point of incidence O as a center. Divide OC by the refractive index, and set off the quotient OD on the other side of O. Draw DB perpendicular to the surface at D, meeting the circumference at B. Then OB is the direction of the refracted ray required; for it is apparent that $\frac{OC}{OD}$ is the same as the ratio of the sines, and this ratio is, by construction, equal to the refractive index.

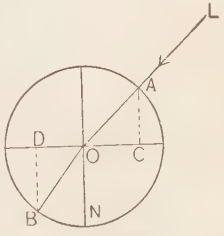


FIG. 216

314. Some phenomena of refraction. A stick partly immersed in water appears to be bent upwards and shortened unless its position is vertical, when the part immersed appears simply shortened. Fig. 217 explains this. The dotted lines represent the real position of the submerged part of the stick, and dotted lines diverging from a point at the bottom of the stick show the course of the rays which reach the eye from that point. On reaching the surface they are bent from the perpendicular, and the bottom of the stick is seen in the direction from which the rays actually enter the eye.

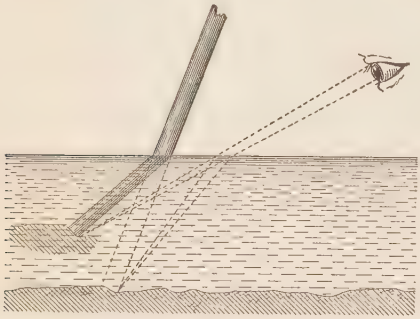


FIG. 217

Viewed obliquely, the depth of water cannot appear greater than $\frac{3}{4}$ its real depth; hence the shoaling effect of still water in which the bottom is visible. To an eye under water the surface must appear at least $\frac{4}{3}$ of its real distance.

315. Critical angle; total reflection. Let SS' (Fig. 218) represent the boundary surface between two media, and AO and BO incident rays in the more refractive medium (e.g. glass); then OD and OE may represent the same rays respectively after they enter the less refractive medium (e.g. air). It will be seen that, as the angle of incidence is increased, the refracted ray

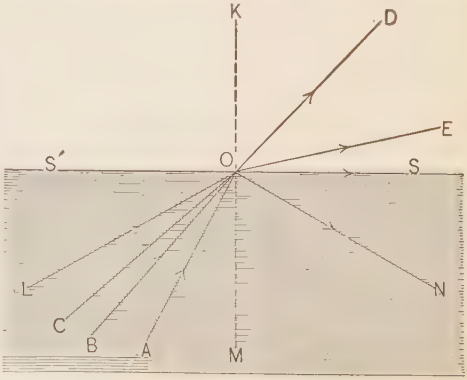


FIG. 218

rapidly approaches the surface *OS*. Now there must be an angle of incidence (e.g. *COM*) such that the angle of refraction will be 90°; in this case the incident ray *CO*, after refraction, will just graze the surface *OS*. This angle (*COM*) is called *the critical angle*. Any incident ray, as *LO*, making a larger angle with the normal than the critical angle, cannot emerge from the medium, and consequently is not refracted. Experiment shows that all such rays undergo internal reflection; e.g. the ray *LO* is reflected in the direction *ON*. Reflection in this case is perfect, and hence is called *total reflection*. *Total reflection occurs when rays in the more refractive medium are incident at an angle greater than the critical angle.*

Surfaces of transparent media, under these circumstances, constitute the best mirrors possible. The critical angle diminishes as the refractive index increases. For water it is about 48.5°; for flint glass, 38.6°; and for the diamond, 23.6°. Light waves cannot, therefore, pass out of water into air with a greater angle of incidence than 48.5°. The brilliancy of gems, particularly the diamond, is due in part to their extraordinary power of internal reflection, arising from their high indices of refraction.

It is evident on inspection of Fig. 218 that a ray traveling in the direction *SO* will be refracted in the direction *OC*. The angle *SOK* is a right angle, and the sine of a right angle is unity. Therefore the index of refraction of the medium $= \frac{1}{\sin \alpha}$, in which α is the critical angle. Thus, to get the index of refraction of any substance it is only necessary to find the critical angle of the substance. This principle has been applied by Kohlrausch in his total reflectometer, in determining indices of refraction of crystals.

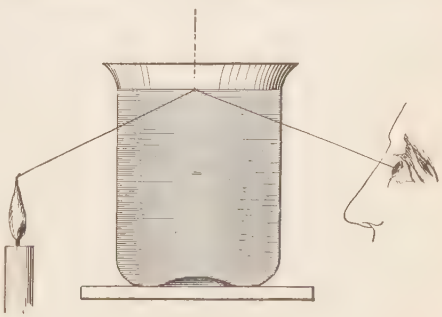


FIG. 219

316. Illustrations of refraction and of total reflection. Observe the image of a candle flame reflected by the surface of water in a glass beaker, as in Fig. 219.

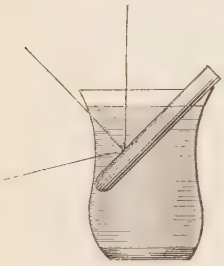


FIG. 220

Thrust the closed end of a glass test tube (Fig. 220) into water, and incline the tube. Look down upon the immersed part of the tube, and its upper surface will look like burnished silver, or as if the tube contained mercury. Fill the test tube with water and immerse as before; the total reflection which before occurred at the surface of the air in the submerged tube now disappears. Explain.

A glass prism of 90° is often used as a reflector. Light passes through the surface AB (Fig. 221) and meets the surface AC at an angle of 45°, which is greater than the critical angle for glass. It is therefore totally reflected, and the device is consequently more effective than an ordinary mirror.

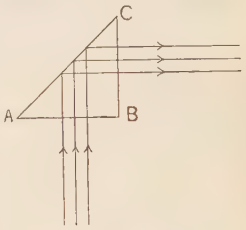


FIG. 221

317. Luminous cascade. If water be siphoned through a glass tube having an open side tubule a (Fig. 222), and the tube below a be exposed to the direct rays of the sun or placed in the path of a beam of light from a lantern or porte-lumière, the stream of water mingled with air which enters the tubule will appear like a "stream of living fire," and has received the name of *luminous cascade*. This is due to total reflection. The light in passing through the water meets the surface of the air bubbles at angles greater than the critical angle, and is therefore totally reflected from side to side all down the stream, rendering it vividly bright.

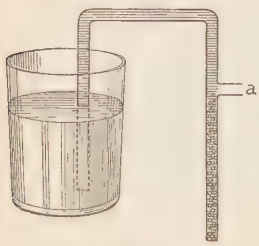


FIG. 222

Place uncolored glass beads, or glass broken into small pieces, in a test tube. They appear not only white, because of diffused reflection, but quite opaque, because of refraction and internal reflection. Pour some water into the tube, and it becomes somewhat translucent. Substitute spirits of turpentine for the water, and the translucency is increased.

By mixing a small quantity of carbon bisulphide with the turpentine, or olive oil with oil of cassia, a liquid can be obtained whose refractive index is about the same as that of glass, when the light will pass through the liquid without obstruction, and the beads become transparent and nearly invisible. The last illustration shows that *one transparent body within another can be seen only when their refractive indices differ.* Place your eye on a level with the surface of a hot stove, and you may observe a wavy motion in the air, due to the mingling of currents of heated and less refractive air with cooler and more refractive air.

QUESTIONS AND PROBLEMS

1. Draw a straight line to represent a surface of flint glass, and draw another line, meeting this obliquely, to represent a ray of light passing from a vacuum into this medium. Find the direction of the ray after it enters the medium.

2. (*a*) Determine the relative index of refraction for light in passing from water into diamond ; (*b*) in passing from water into air.

3. How must one modify his aim in shooting or spearing fish from the bank of a stream ?

4. A ray is incident on a surface of crown glass at an angle of 40° ; find the angle of refraction.

5. Find the refractive index for light passing from water into crown glass.

SECTION VIII

PRISMS AND LENSES

318. Optical prisms. An optical prism is a portion of a transparent medium bounded by surfaces at least two of which are plane and inclined to each other. Fig. 223 represents a transverse

section of a common form of prism. Let AB be a ray of light incident upon one of its surfaces. On entering the prism it is refracted *toward* the normal, and takes the direction BC.

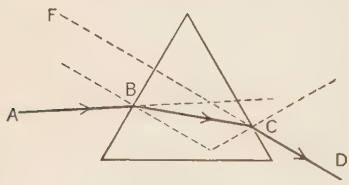


FIG. 223

On emerging from the prism it is again refracted, but now *from* the normal in the direction CD. The object that emits the ray will appear in the direction DF. Observe that the ray AB, at both refractions, is bent toward the thicker part, or base, of the prism.

319. Minimum deviation. Furthermore, it can be shown that when the prism is so placed that the angle of emergence becomes equal to the incident angle, the angle of deviation D of the ray has its least value. Such a position is shown in Fig. 224, and is called the position of *minimum deviation*. It is easily obtained in practice by turning the prism until a certain position is obtained where the beam of light comes to a standstill and begins to move back, no matter which way the prism is rotated.

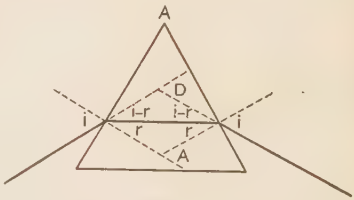


FIG. 224

320. Measurement of refractive index. Having obtained by experiment the position of minimum deviation, it is a very simple matter to determine the index of refraction by measuring the angle of the prism A and of the deviation D. From the figure it is seen that the angle between the normals is equal to $A = 2\,r$; therefore $r = A/2$. But $D = 2\,i - 2\,r = 2\,i - A$; or $i = \dfrac{D + A}{2}$.

But since $n = \dfrac{\sin i}{\sin r}$, we have, finally, $n = \dfrac{\sin \frac{1}{2}(D + A)}{\sin \frac{1}{2} A}$.

In this connection it must be noted that n is different for light of different colors or wave lengths (§ 332).

321. Lenses. Any transparent medium bounded by surfaces of which at least one is curved is a *lens*.

Lenses are of two classes, converging and diverging, according as they collect rays or cause them to diverge. Each class comprises three kinds (Fig. 225):

Class I		Class II	
1. Biconvex	Converging or convex lenses, thicker in the middle than at the edges	4. Biconcave	Diverging or concave lenses, thinner in the middle than at the edges
2. Plano-convex		5. Plano-concave	
3. Concavo-convex (or meniscus)		6. Convexo-concave	

A straight line passing through the centers of curvature of a lens, as AB, is called its *principal axis*. There is a point in the principal axis of every lens called its *optical center*. This point is so placed that a ray whose direction within the lens passes through it suffers no angular deviation, but at most only a slight lateral displacement. In lenses 1 and 4 it is halfway between their respective curved surfaces.[1]

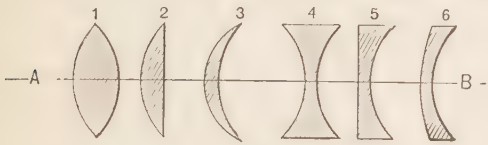


Fig. 225

A ray from any point of an object which passes through the optical center, as Aa (Fig. 232), is called the secondary axis of this point.

322. Effect of lenses. We may, for convenience of illustration, regard a biconvex lens as composed, approximately, of a series of prisms of gradually increasing angles arranged around an axis, as represented in section in Fig. 226. It is apparent that the parallel rays farthest from the principal axis, meeting

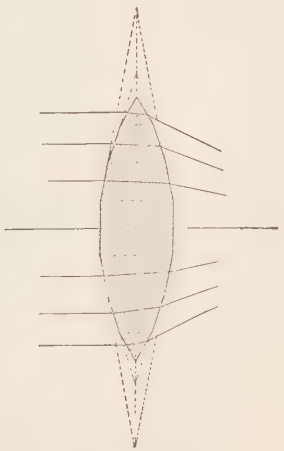


Fig. 226

[1] In lens 2 the optical center is in its convex surface; in lens 5 it is in its concave surface; in lenses 3 and 6 it is without the lens.

prisms of greater and greater angles of incidence, are more deflected than those nearer the axis; and if the curvatures be properly adjusted, all may be made to converge to one point.

On the other hand, if the lens be thinnest at the center and gradually increase in thickness outward, exactly the opposite effect would be expected. Parallel incident rays, being bent toward the thicker part of the component prisms, would become separated.

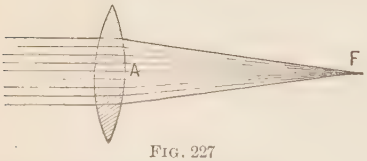


FIG. 227

The general effect of all convex lenses is to cause transmitted rays to converge; that of concave lenses, to cause them to diverge. Incident rays parallel to the principal axis of a convex lens are brought to a focus *F* (Fig. 227) at a point in the principal axis. This point is called the *principal focus*. It may be found approximately by holding the lens so that the rays of the sun may fall perpendicularly upon it, and then moving a sheet of paper back and forth behind it until the image of the sun formed on the paper is brightest and smallest. Or, in a room, it may be found less accurately by holding a lens at a considerable distance from a window,

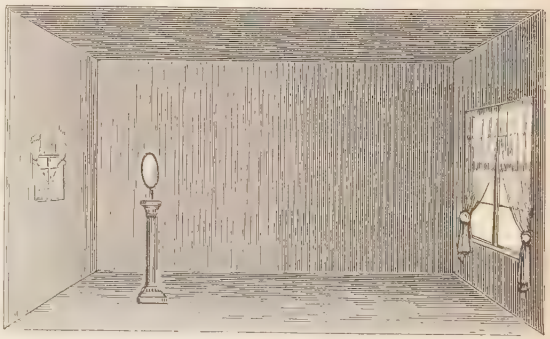


FIG. 228

and regulating the distance so that a distinct image of the window will be projected upon the opposite wall, as in Fig. 228. The focal length is the distance from the optical center of the lens to the center of the image on the paper. The shorter the focal length the more powerful is the lens; that is, the more

quickly are the parallel rays that traverse different parts of the lens brought to cross one another.

If the paper be kept at the principal focus for a short time, it will take fire. The reason is apparent why convex lenses are sometimes called *burning glasses*. A pencil of rays emitted from the principal focus F (Fig. 227) as a luminous point becomes parallel on emerging from a convex lens. If the rays emanate from a point nearer the lens, they diverge after egress, but the divergence is less than before; if from a point beyond the principal focus, the rays are rendered convergent. A concave lens causes parallel incident rays to diverge as if they came from a point, as F (Fig. 229). This point is therefore its principal focus. It is, of course, a *virtual focus*.

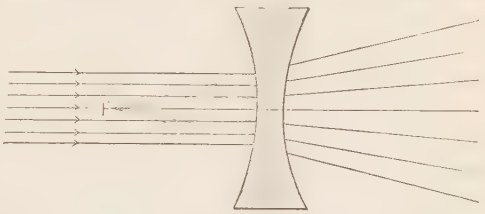


FIG. 229

Every lens has a *principal focus;* it is the point to which parallel rays are caused to converge, or from which, after deflection, they appear to diverge, as the case may be.

323. Conjugate foci. When a luminous point S beyond the principal focus (Fig. 230) sends waves to a convex lens, the emergent waves converge to another point S'; waves sent from S' to the lens would converge to S. Two points thus related are called *conjugate foci*. The fact that waves which emanate from one point are caused by convex lenses to collect at one point gives rise to real images, as in the case of concave mirrors.

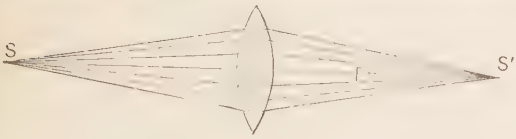


FIG. 230

324. Relation of conjugate foci. Lenses, like mirrors, have conjugate foci at distances p and p' from the optical centers. In

converging lenses the principal focal distance and the distance of their conjugate foci (or distance of object and image) are related according to the formula

$$\frac{1}{p} + \frac{1}{p'} = \frac{1}{f}.$$

Hence the law of converging lenses: *The reciprocal of the principal focal length is equal to the sum of the reciprocals of any two conjugate focal lengths.*

When a pencil of light comes from an infinite distance (i.e. when its rays are parallel), $p = \infty$; then $p' = f$, and the rays converge at the principal focus. Conversely, if a pencil come from the principal focus, $p = f$; hence $p' = \infty$; that is, no image is formed, and the rays after transmission are parallel.

If the object (i.e. the source of light) be at a distance less than infinity, but greater than $2f$, the image is real, and is on the other side of the lens at a distance greater than f and less than $2f$. Conversely, if the object be at a distance greater than f, but less than $2f$, the image is at a distance greater than $2f$.

If the object be at a distance $2f$, the image is also at the distance $2f$, and object and image are of equal size. This suggests a simple way of finding f. Adjust an object, convex lens, and screen, so that the image on the screen is equal in size to the object. Half the distance of either the object or its image from the center of the lens is the focal length of the lens.

325. Diverging lenses. The formula for these is

$$\frac{1}{p} - \frac{1}{p'} = \frac{1}{f}.$$

When $p = \infty$, $p' = -f$ (a virtual image at the principal focus).
When $p = f$, $p' = -\infty$ (no image).

When p is of any value greater than f, and less than ∞, p' is greater than f.

326. Images formed. Fairly distinct images of objects may be formed through *very small* apertures; but owing to the small amount of light that passes through the aperture, the images are

very deficient in brilliancy. If the aperture be enlarged, brilliancy is increased at the expense of definition. *A convex lens enables us to obtain both brilliancy and definition at the same time.*

Place a candle or gas flame at such a distance on one side of a convex lens that there will be produced a distinct and well-defined image on the wall (Fig. 231). Observe the size and kind of image. Advance the flame toward the lens, regulating at the same time the distance between the lens and wall, so as to preserve a distinctness of image. Note the changes which the image undergoes. When the image and the flame become of the same size, measure and note the distance of each from the lens. Advance the flame still nearer, and note the changes in the image, until it is impossible to obtain an image on the wall.

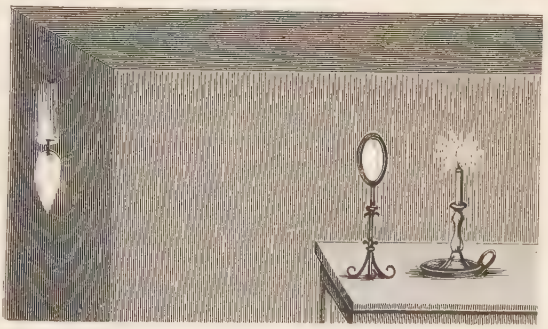


FIG. 231

Measure the distance of the flame from the lens, and compare this distance with the focal length of the lens. Move the flame still nearer. Note whether the rays, after emerging from the lens, are divergent or convergent. See whether an image and an object may change places. Form images of the flame on the wall at different distances from the lens; measure the distances, also the linear dimensions (e.g. the width or the vertical height) of the images, and determine that *the linear dimensions of images are proportional to their distances from the lens.*

327. Construction of images. Given the lens L (Fig. 232), whose principal focus is at F, and object AB in front of it; any two of the many rays from A will determine where its image a is formed. Two that can be traced easily are one along the secondary axis

AO, and one AA' parallel to the principal axis; the latter will be deviated so as to pass through the principal focus F, and will afterward intersect the secondary axis at some point a; therefore this is the conjugate focus of A. Rays can be similarly traced for B and all intermediate points along the arrow. Thus a *real inverted image* is formed at ab.

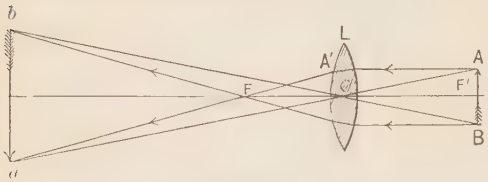


Fig. 232

The linear dimensions of an object and of its image formed by a convex lens are proportional to their respective distances from the optical center of the lens. The image is virtual or real, erect or inverted, according as it is on the same side of the lens with the object or on the opposite side.

328. Virtual images. Since rays that emanate from a point nearer the lens than the principal focus diverge after egress, it is evident that their focus must be virtual and on the same side of the lens as the object. Hence *the image of an object placed nearer the lens than the principal focus is virtual, magnified, and erect*, as shown in Fig. 233. A convex lens used in this manner is called a *simple microscope.*

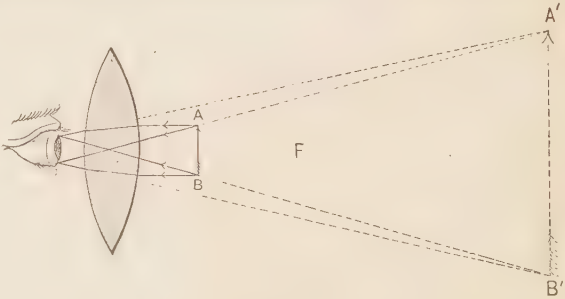


Fig. 233

329. Simple microscope. As its name implies, the microscope is an instrument for viewing minute objects. The simple microscope consists of a single converging lens so placed that the object is between the principal focus and the lens. It magnifies by increasing the visual angle.

The *magnifying power* of the lens is simply the ratio between the apparent diameter of the image and the diameter of the object, e.g. $A'B' : AB$ (Fig. 233), or it is approximately the ratio between the visual angles under which the eye would see image and object, if both were placed at the distance of distinct vision.[1]

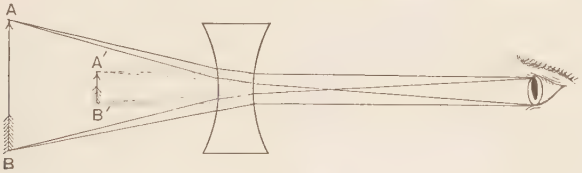


FIG. 234

If the lens be of short focus, as is usually the case, the magnifying power is approximately the ratio of the distance of distinct vision to the focal length. This is because for the best effect AB is placed nearly out to F. Thus a lens of one-half-inch focal length would magnify from twenty to twenty-four diameters.

330. Diverging lenses. Since the effect of concave lenses is to render transmitted rays divergent, pencils of rays emitted from A and B (Fig. 234) diverge after refraction, as if they came from A' and B', and the image will appear to be at $A'B'$. Hence *images formed by concave lenses are virtual, erect, and smaller than the object.*

331. Spherical aberration. In all ordinary convex lenses the curved surfaces are spherical, and the angles which incident rays make with the little plane surfaces of which we may imagine the spherical surface to be made up, increase rapidly toward the edge of the lens. Thus, while those rays from a given point of an object which pass through the central

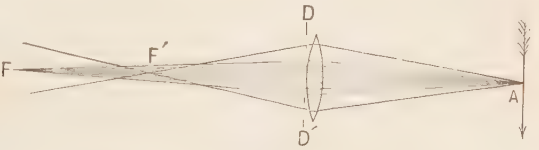


FIG. 235

[1] For normal eyes, an object to be seen most distinctly must be placed at a distance of 10 to 12 inches; hence this is regarded as the distance of distinct vision.

portion, as A (Fig. 235), meet approximately at the same point F, those which pass through the marginal portion are deflected so much that they cross the axis at nearer points, e.g. at F', so that a blurred image results. This wandering of the rays from a single focus is called *spherical aberration.*

No lens with spherical surfaces can bring all the rays to the same focus. The evil may be in a measure corrected by interposing a diaphragm DD' provided with a central aperture smaller than the lens, so as to cut off those rays that pass through the marginal part of the lens. But it can be wholly corrected only by properly modifying the curvature of the surfaces of the lens.

QUESTIONS AND PROBLEMS

1. What must be the position of an object with reference to a converging lens, that its image may be real and magnified?

2. A transparency whose dimensions are 3×4 in. is placed 16 in. from the lens; at what distance from the lens must the screen be, that it may receive a distinct image of the transparency that shall cover a surface 3×4 ft.?

3. What is the focal length of the lens used in the last case?

4. With a converging lens the image of a candle is thrown on a screen 6 ft. from the candle, and the focal length of the lens is 16 in. Find the distances of the candle and of the screen from the lens.

5. A luminous point is 3 in. from a convex lens having a focal length of 5 in. Find the position of the image.

6. If the candle and screen be 3 ft. apart, and the lens midway between them, what is the focal length?

7. Find the focal length of a lens which throws the image of an object 5 ft. distant on a screen 3 ft. distant.

8. A double concave lens having a focal length of 3 in. is held at a distance of 2 in. from a small object. Find the position of the image.

9. If an object be at twice the focal distance of a convex lens, how will the length of the image compare with the length of the object?

10. To an eye whose distance of distinct vision is 25 cm., how many diameters will a lens of 1 cm. focus magnify?

11. Show that a concave air lens in water has the same effect on incident light as a convex water lens in air.

SECTION IX

PRISMATIC ANALYSIS OF LIGHT

332. Dispersion. Place a disk with an adjustable slit in the aperture of a porte-lumière, so as to exclude from a darkened room all light waves except those which pass through the slit. Near the slit interpose a double convex lens of, say, 10-inch focus. A narrow sheet of light will traverse the room and produce an image AB (Fig. 236) of the slit on a white screen placed

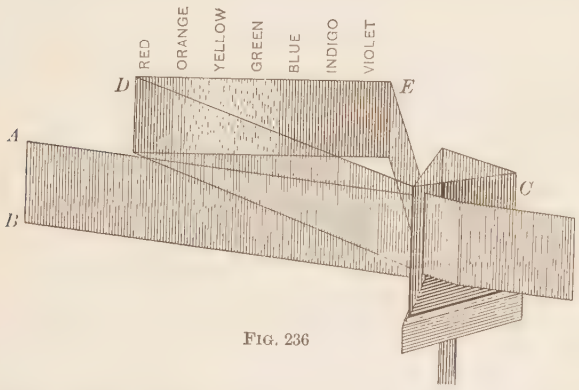


FIG. 236

in its path. Now place a glass prism C in the path of the narrow sheet of light and near to the lens, with its edge vertical. (1) The light now is not only turned from its former path, but that which before was a narrow sheet is, after emerging from the prism, spread out fanlike into a wedge-shaped body, with its thickest part resting on the screen. (2) The image, before only a narrow, vertical band AB, is now drawn out into a long horizontal ribbon DE. (3) The image, before white, now presents "all the colors of the rainbow," from red at one end to violet at the other; it passes gradually through all the gradations of orange, yellow, green, blue, and violet.

From this experiment we learn (1) that *white light is not simple* in its composition, *but the result of a mixture of colors.*[1] (2) *The colors of which white light is composed may be separated by refraction.* (3) *The separation is due to the different degrees of deviation which colors undergo by refraction.* Red, which is always least deviated, is the least refrangible color. Then follow orange, yellow, green, blue, and violet, in the order of their refrangibility. The many-colored ribbon of light *DE* is called the *solar spectrum.* This separation of white light into its constituents is called *dispersion.* The number of colors of which white light is composed is really infinite, but we have names for only seven of them, namely *red, orange, yellow, green, blue, indigo,* and *violet;* and these Newton called the *primary* or *prismatic colors.*

The spectrum may be projected upon a screen or it may be received directly by the eye suitably placed.

333. Synthesis of white light. The composition of white light has been ascertained by the process of analysis; it can be verified by *synthesis,* i.e. the colors after dispersion may be reunited, and the result of the reunion is white light.

If a second prism be placed in such a position, △△, that light which has passed through one prism, and been refracted and decomposed, may be refracted back, the colors will be reblended, and a white image of the slit will be restored on the screen.

If a large convex lens or a concave mirror (better a concave cylindrical mirror) be placed so as to receive the colors after dispersion by a prism, the rays will be brought to a focus on a screen, and the image produced will be white.

334. The rainbow. The rainbow is a solar spectrum on a grand scale. It is the result of refraction, total reflection, and dispersion of sunlight by raindrops. Let spheres 1 and 2 (Fig. 237) represent drops at the extreme opposite edges of the bow. The eye is in a position to receive, after the dispersion and internal reflection of the light waves within drop 1, only the red waves;

[1] Newton (1666) was the first to recognize the true import of this phenomenon, i.e. to refer the colors to the heterogeneity of white light.

consequently this part of the bow appears red. So, likewise, from drop 2 the eye receives only violet; consequently this part of the bow appears violet. In like manner, the intermediate colors of the bow are sifted out.

Outside the primary bow a *secondary bow* is sometimes seen. Drops 3 and 4 (Fig. 237) are supposed to be at the opposite edges of the secondary bow. It will be seen that the light waves undergo two internal reflections within the drops which produce this bow. The colors of this bow are in reverse order to those of the primary bow, and less brilliant.

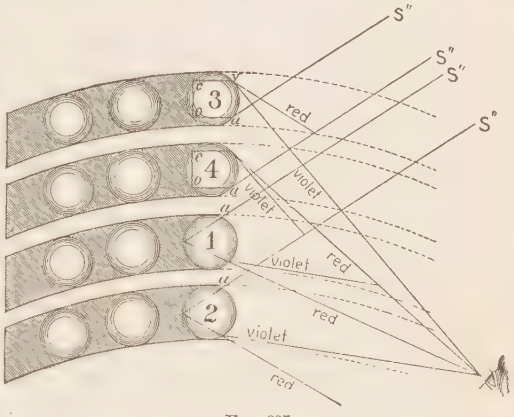


FIG. 237

For a more detailed explanation of this subject the reader is referred to more advanced works. Here the figure should be accepted as simply a diagram to assist the explanation, and not at all as indicating the real relations of distances and directions. In the real case the sun, the eye, and the center of the bow are on the same straight line; in other words, no two people ever see the *same* bow.

335. Chromatic aberration. There is also in ordinary lenses a serious defect, to which we have not before referred, called *chromatic aberration*, the correction of which has demanded the highest skill. The convex lens both *refracts* and *disperses* the light waves that pass through it. The tendency, of course, is to bring to a focus the more refrangible waves, as the violet, much sooner than the less refrangible waves, such as the red. The result is a disagreeable coloration of the images that are formed

by the lens, especially by those portions of the light waves that pass through the lens near its edges. This evil has been overcome very effectually by combining with the convex lens a plano-concave lens. Now since the refractive and dispersive powers of crown and flint glass are not proportional mutually, if a crown-glass convex lens be taken, a flint-glass concave lens may be prepared that will correct the dispersion of the former without neutralizing all its refraction. A compound lens composed of these two lenses cemented together (Fig. 238) constitutes what is called an *achromatic lens.*

FIG. 238

336. Cause of color and dispersion. The color of light is determined by *vibration frequency*, or, in other words, by the corresponding *wave length.* The light waves diminish in length from the red to the violet. As pitch depends on the frequency with which aërial waves strike the ear, so color depends upon the frequency with which ether waves strike the eye. The difference between violet and red is a difference analogous to the difference between a high note and a low note on a piano.

The speed of propagation in a vacuum appears to be the same for all wave lengths, but in a refracting medium the short waves are more retarded than the longer ones, and hence are more refracted. This is the cause of dispersion. Each wave length has its own refractive index, or, since vibration frequency corresponds to color, every simple color has its special refractive index. Light composed of waves all of the same length is called *homogeneous* or *monochromatic* light. The yellow light emitted by the flame of a Bunsen burner or alcohol lamp when common salt is sifted upon it is approximately monochromatic. Ordinary white light is a mixture of ether waves of various lengths.

From well-established data, determined by a variety of methods, physicists have calculated the number of waves that succeed one another per second for each of the several prismatic colors, and the corresponding wave lengths; the following table contains the

results. The letters *A*, *C*, *D*, etc., refer to Fraunhofer's lines (see § 342).

		Length of waves in millimeters	Number of waves per second
Dark red	*A*	0.000760	395×10^{12}
Orange	*C*	0.000656	458×10^{12}
Yellow	*D*	0.000589	510×10^{12}
Green	*E*	0.000527	570×10^{12}
Blue	*F*	0.000486	618×10^{12}
Indigo	*G*	0.000431	697×10^{12}
Violet	*H*	0.000397	760×10^{12}

"That man should be able to measure, with certainty, such minute portions of space and time is not a little wonderful; for, whatever theory of light we adopt, it may be observed that these periods and these spaces have a real existence, being, in fact, deduced by Newton himself from direct measurements, and involving nothing hypothetical but the names here given them" (Sir John Herschel).

If science in the future shall be able to dispense with the ether of space, the vibration periods and what corresponds to wave lengths will necessarily remain.

There is a limit to the sensibility of the eye as well as of the ear. The limit in the number of vibrations appreciable by the eye lies approximately within the range of numbers given in the above table; i.e. if the succession of waves be much more or much less rapid than is indicated by these numbers, the sensation of sight is not produced.

"Our knowledge of ether waves is at present limited to those which lie between 107 trillions and 40,000 trillions per second, — a range, in musical parlance, of about eight and a half octaves" (Langley). Of these our eyes are sensitive to scarcely one octave.

It is evident that *the frequency of the waves emitted by a luminous body, and consequently the color of the light emitted, must depend on the rapidity of the vibratory motions of the molecules or atoms of that body*, i.e. *upon its temperature*. This

has been shown in a convincing manner as follows: The temperature of a platinum wire is slowly raised by passing a gradually increasing current of electricity through it. At a temperature of about 540° C. it begins to emit light; and if the light be analyzed by a prism, it is shown that only red light is emitted. As the temperature rises there will be added to the red of the spectrum first yellow, and then green, blue, and violet successively. When it reaches a white heat it emits all the prismatic colors. It is significant that a white-hot body emits more red light than a red-hot body, and likewise more light of every color than at any lower temperature. The conclusion is, that *a body which emits white light sends forth simultaneously waves of a variety of lengths.*

337. Continuous spectra. The spectrum produced by the platinum is continuous; that is, the band of light is unbroken. If the spectrum be not complete, as when the temperature is too low, it will begin with red, and be continuous as far as it goes. *All luminous solids and liquids give continuous spectra.*

A gas, kerosene, or candle flame does not give the spectrum of a vapor, but gives that of the solid particles of carbon in a state of incandescence; hence the continuous spectra which these flames afford.

338. The spectroscope. Instruments for the observation of spectra are called *spectroscopes*. The essential part of the apparatus is the dispersion piece, which is either a prism or a diffraction grating. Instead of looking at the spectrum with the naked eye it is usually better to view it through a small telescope. Fig. 239 represents the simplest form of the Kirchhoff and Bunsen spectroscope. A flint-glass prism P on the table A receives light through an adjustable slit S at the end of a tube called the *collimator*. At the opposite end of this tube is a converging lens C, and the slit is located at its principal focus so that rays diverging from the slit are rendered parallel by the lens.

It is often necessary to have some means of determining the positions of certain lines (to be described hereafter) observed in

the spectrum. The usual method is to have a second tube L, somewhat like the collimating tube, so placed that the rays from a candle flame, after passing through a transparent plate at the end of the tube on which a fine scale is engraved, and through a lens, by which they are made parallel, are reflected from the nearest face of the prism, and pass into the telescope T along with the beam of light under analysis. Thus the eye

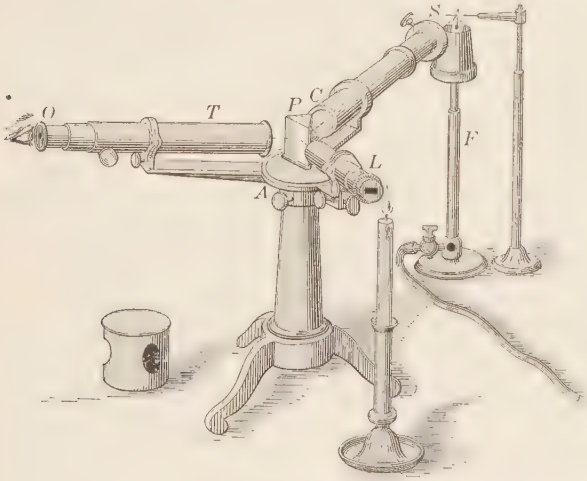


FIG. 239

at O while viewing the spectrum sees also a magnified image of the scale in the same plane as the spectrum. By reference to this scale, lines, bands, or other peculiarities in the spectrum may be localized or identified.

339. The direct-vision spectroscope. Just as it has been shown possible, by the combination of crown and flint glass in the make-up of an achromatic lens, to destroy the color effect without doing away with the deviation entirely, so by the proper grouping of prisms of different kinds of glass cut at different angles, it has been found possible to destroy on the average the deviation and yet retain considerable dispersion. Such a combination is known

as a direct-vision prism and has an obvious advantage for certain kinds of work. In an instrument of this kind the collimator and the telescope have practically coincident axes, the telescope being laterally adjustable within narrow limits for viewing different parts of the spectrum. The scale attachment is the same as in the Bunsen form.

Take a piece of platinum wire a couple of inches long. Seal one end by fusion to a short glass tube for a handle. Form a small loop at the other end. Dip the loop into a strong solution of common salt or, better, into a little powdered borax, and support it by a clamp in the midst of the almost invisible and colorless flame of a Bunsen burner F (Fig. 239). Instantly the flame becomes luminous and colored a deep yellow. Examine it with a spectroscope, and you will find, instead of a continuous spectrum, only a bright narrow line of yellow in the yellow part of the spectrum, next the orange. The spectrum consists essentially of a closely double[1] bright yellow line on a comparatively dark ground.

Heat the platinum wire until it ceases to color the flame; then dip it into a solution of chloride of lithium, and repeat the last experiment. You obtain a carmen-tinted flame, and see through the spectroscope a bright red line and a faint orange line.

Use potassium hydrate and you obtain a violet-colored flame and a spectrum consisting of a red line and a violet line (the latter very difficult to see even with the best instruments). Use strontium nitrate and obtain a crimson flame and a spectrum consisting of several lines in the red and the orange, and a blue line.

Use a mixture of several of the above chemicals, and you will obtain a spectrum containing all the lines that characterize the several substances.

Every chemical compound used in the above experiments contains a different metal; e.g. common salt contains the metal sodium. The other substances used successively contain respectively the metals lithium, potassium, and strontium. These

[1] It is not a hypothesis, but a reality, that sodium vapor has two independent vibrations whose periods differ by about $\frac{1}{1000}$ of each other. — Lord Kelvin.

metals, when introduced into the flame, are vaporized, and we get their spectra when in a gaseous state. *All incandescent gases, unless under great pressure, give discontinuous, or bright-line, spectra, and no two gases give the same spectrum.*

340. Spectrum analysis. The vibrating atoms or their electrons embedded in ether emit waves, the length of which depends on the rate of vibration, and the waves of different lengths produce the different color sensations. Now, like a tuning fork, a free atom of every substance has its own definite period or periods of vibration, and accordingly sends out light of a certain definite color or of a few definite colors, just as the sounding body emits sound of a certain definite pitch, with sometimes harmonics. For example, every atom of sodium or of lithium vibrates in the same way, and always has vibrated in the same way, whether it exists in the sun, in the earth, or in a distant star. The same is true of every other kind of matter; each has its own rates of vibration, and hence each produces its own bright-line spectrum corresponding with its peculiar rates of vibration. Hence has arisen a new method of analysis, wherein substances are detected simply by observing the rates of vibration of their atoms (i.e. the bright lines of their spectra), — a branch of physics called *spectrum analysis.*

It is only in the gaseous state, however, that the atom is free to exhibit its special rate of vibration; when molecules and atoms are packed closely together in a solid or liquid, their motions are cramped, their periodicity is lost, and all manner of vibrations are induced. Hence spectra of solids and liquids are continuous, i.e. the rates of vibrations are so many in number as to leave no gaps in their spectra.

Many compounds are decomposed into their elements, and the elements are rendered gaseous at a temperature that is at, or below, the temperature necessary for incandescence. In that case the spectrum given is the combined spectra of the components. A compound gas that does not suffer dissociation at the temperature of incandescence gives its own spectrum, which is generally totally different from the spectrum of its elements.

341. Reversed or dark-line spectrum. Arrange apparatus in a dark room, as in Fig. 240. N is the flange nozzle of a projecting lantern containing only the condensing lens; T and S are two tin plates, in the latter of which a slit is cut. Allow a beam of light from an electric arc lamp to pass through the slit in S, and thence through the converging lens L and the prism P, and form a spectrum on a screen H. Hold in the flame of a Bunsen burner B a pellet of sodium; it burns vividly, and the light has to pass

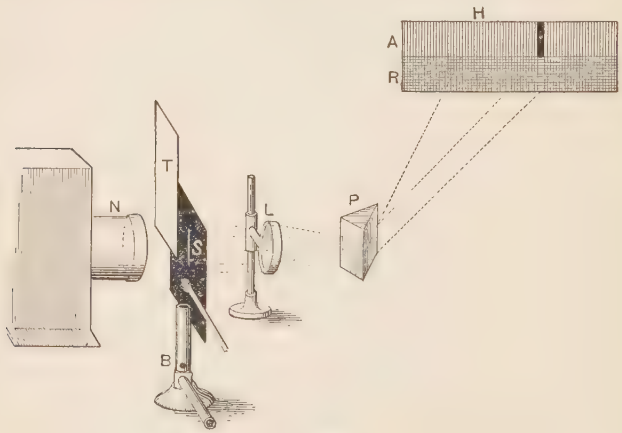


FIG. 240

through the intensely yellow flame. We should naturally expect that the yellow of the spectrum would now be more intensely illuminated, but, instead, a *dark* band in the yellow now appears. It is not really black, but *comparatively* dark.

Next hold the plate T between the burner and the condensers so that the electric light may be cut off from the upper portion of the slit, leaving the light from the sodium flame alone to pass through this part of the slit. The spectrum R formed by this part consists of a bright yellow line on a dark ground, being the radiation spectrum of sodium. (It should be borne in mind that the image of the slit is inverted.) The other half A shows a dark

line on the continuous spectrum. We thus have, contiguous to each other, the bright-line spectrum of sodium and its *reversed dark-line,* or *absorption,* spectrum. If the experiment could easily be tried with lithium, potassium, strontium, etc., it would be found that in a similar manner the spectra would be crossed by dark lines where one might expect to find bright ones.

It thus appears that *the vapors of different substances absorb or quench the very same rays that they are capable of emitting when made self-luminous ;* very much, it would seem, as a given tuning fork selects from various sounds only those of a definite wave length corresponding to its own vibration period. The dark places of the spectrum receive light in full force from the vapor flame ; but the light is so feeble, in comparison with that of those places illuminated by the electric light, that the former appear dark by contrast.

Light transmitted through certain liquids (as sulphate of quinine and blood) and certain solids (as some colored glasses) produces band spectra. These spectra are obtained only when light passes through media capable of absorbing rays of certain wave length; hence they are commonly called *absorption spectra.* Since a given vapor causes dark lines precisely where it would cause bright lines if it were itself the only radiator of light, dark-line spectra are frequently called *reversed spectra.* There are then three kinds of spectra: *continuous spectra,* produced by luminous solids, liquids, or, as has been found in a few instances, gases under great pressure ; *bright-line spectra,* produced by luminous vapors; and *absorption spectra,* produced by light that has been sifted by certain media.

342. Fraunhofer's lines. The spectrum of sunlight is observed to contain a large number of dark lines transverse to its length. These were first observed by Wollaston (1802), and were mapped by Fraunhofer (1814), who distinguished several of the more prominent ones by letters of the alphabet; hence the dark lines of the solar spectrum have received the name of *Fraunhofer's lines.*

So far as discovered, no two substances have a spectrum consisting of the same combination of lines; and, in general, different simple substances very rarely possess lines appearing to be common to both. Hence, when we have once observed and mapped the spectrum of any substance, we may ever after be able to recognize the presence of that substance when emitting light, whether it is in our laboratory or in a distant heavenly body.

The spectroscope, therefore, furnishes us a most efficient means of detecting the presence (or absence) of any elementary substance, even when it is combined or mixed with other substances. It is not necessary that the given substance should exist in large quantities; for example, the fourteen-millionth part of a milligram of sodium can be detected by the spectroscope. Substances which are not easily converted into vapors at low temperatures may be placed between the secondary terminals of an induction coil or other high-potential electric apparatus. The heat generated by electricity will vaporize most substances. Thus the spark passing between two copper electrodes will vaporize a portion and show the copper lines; between iron electrodes, the iron lines, etc. After maps of the spectrum of all known substances have been made out, if, on examination of a complex substance, any new lines should at any time appear in the spectrum, it would indicate the presence of a substance hitherto undiscovered. It was thus that the elements cæsium, rubidium, thallium, and indium were discovered.

FIG. 241

343. General remarks. Gases or vapors, when sufficiently heated to become luminous, emit, under ordinary pressure, color rays which are dispersed into an interrupted spectrum of bright lines; with increasing pressure and density these lines spread into diffuse luminous bands, and finally form a continuous spectrum. Gases are rendered luminous usually by passing electric sparks through glass tubes inclosing them. Substances whose great volatility interferes by causing evaporation before the substance

attains the temperature of incandescence, such as most metalloids, give no flame-emission spectra. These are commonly rendered luminous in glass tubes called Plücker tubes (Fig. 241).

Colored liquids are usually placed in glass cells with flat parallel sides, or of wedge shape, so as to allow the examination of different thicknesses of the liquid. The cells are placed between a bright source of light and the slit (or better in reflected sunlight), and the result is absorption spectra of the coloring matter, consisting of dark bands. Thus blood in its normal or healthy state is readily recognized by the absorption bands of its coloring matters and their modifications by the presence of gases. A solution of fresh blood gives two easily distinguishable dark bands in the green. But blood is capable of existing in different stages of oxidation, which are distinguishable by difference of color and corresponding difference of spectrum. By means of a microspectroscope (a combination of microscope and spectroscope) it is claimed that the thousandth part of a grain of blood is easy of detection, and its presence may be detected in stains that have been kept a very long time; hence this instrument often becomes of great importance in criminal trials.

344. Solar and stellar physics. The spectrum of iron has been mapped to the extent of more than 600 bright lines. Of these, Kirchhoff succeeded in showing the coincidence of 460 with dark lines of the solar spectrum. Can there be any doubt of the existence of iron in the sun? By examination of the reversed spectrum of the sun we are able to determine with certainty the existence there of sodium, calcium, copper, zinc, magnesium, hydrogen, and many other known substances. Again, from our knowledge of the way in which a reversed spectrum can be produced, we may conclude that the sun consists of a luminous solid, a liquid, or an intensely heated and greatly condensed gas (called the *photosphere*), and that this nucleus is surrounded by an atmosphere of cooler vapor (called the *chromosphere*) in which exist at least all the substances just named. The moon and planets that are visible only by reflected sunlight give the same spectrum as

the sun, while those heavenly bodies that are self-luminous give spectra which differ from the solar spectrum to some extent.

345. Effect of motion in the radiating body. If the radiating body be in motion, either to or from the observer, obviously the effect of this motion will be to shorten the wave length in the former case and to lengthen it in the latter. This will tend to move the spectrum lines toward the more refrangible end in the former instance and toward the less refrangible end in the latter. Thus from the displacement of hydrogen lines in the absorption spectrum Young computes that the greatest speed upon the sun, observed by him, is 400 kilometers per second. Pickering (Harvard University) has observed that the k line in the spectrum of β Aurigæ is alternately single and double at intervals of about seventeen hours, thus showing this star to be double, each part revolving about the other in less than four days at a speed of 240 kilometers per second.

The *telespectroscope* (a combination of telescope and spectroscope) has disclosed to us a much-coveted knowledge of the true nature, chemical composition, and physical condition of those points of light called the *fixed stars*, immensely more remote and less bright than the planets. In like manner nebulæ, comets, and meteors have been investigated, and valuable knowledge has been obtained as to their physical constitution.

346. Distribution of energy in the spectrum. The energy of ether waves is capable, as has been before observed, of producing calorific, luminous, or chemical effects, according to the nature of the bodies upon which they fall. When a sensitive thermoscope is passed along the spectrum, heat effects are observed throughout the visible spectrum, and for considerable distances beyond at each extremity. All ether waves are capable of producing heating effects.

It thus appears that the solar spectrum is not limited to the visible spectrum, but extends beyond at each extremity; and spectroscopic analysis, besides sifting the waves of one color from those of another, is able to sift out rays which do not

produce the sensation of light from those which do. Those rays that lie beyond the red are called the *infra-red* rays, while those that lie beyond the violet are called the *ultra-violet* rays. The infra-red rays are of longer vibration period, and the ultra-violet of shorter period, than the luminous waves.

Inasmuch as glass largely absorbs the energy of ether waves of certain lengths, it is customary in studying heat spectrums to use lenses and prisms of rock salt, since this substance transmits waves of all lengths with great freedom.

In a prismatic spectrum obtained by the use of a rock-salt prism the maximum heating effect for the solar spectrum is in the infra-red. Langley finds, however, that in the *normal* spectrum the maximum heating effect in the solar radiation coincides quite closely with the maximum luminous effect which is in the orange-yellow.

Chemical effects are produced by rays of all refrangibilities. A photograph can be taken of all portions of the visible spectrum, and the photographic spectrum may extend far beyond the visible spectrum in the ultra-violet, and even the infra-red rays may be photographed. Ordinary silver salts are decomposed by rays extending from the green upward, while the decomposition of carbon dioxide is aided chiefly by rays of lower refrangibility.

347. Only one kind of radiation. The fact that radiant energy produces three distinct effects, — namely luminous, heating, and chemical, — has given rise to a prevalent idea that there are three distinct kinds of radiation. There is, however, absolutely no proof that these different effects are produced by different kinds of radiation. Science recognizes in radiations no distinctions but periods, wave lengths, and wave forms. *The same radiation that produces vision can generate heat and chemical action.* The fact that the infra-red and ultra-violet rays do not affect the eye does not argue that they are of a different nature from those that do, but it does show that there is a limit to the susceptibility of the eye to receive impressions from radiation. Just as there are sound waves of too long and others of

too short period to affect the ear, so there are ethereal waves, some of too long and others of too short period to affect the eye.

348. Phosphorescence. There is a class of substances such as the sulphides of calcium, strontium, etc., which after several hours' exposure to light waves absorb their energy without becoming hot, and in return emit light waves, which are quite perceptible in a dark room for several hours after the exposure. This property of shining in the dark after having been exposed to light waves is termed *phosphorescence*. A so-called *luminous paint* is prepared and applied to certain parts of bodies that are exposed to sunshine during the day ; at night those parts to which the paint is applied are alone luminous. This paint may be used for a variety of purposes, such as rendering luminous danger signals, door numbers and plates, etc.

349. Fluorescence. There is another class of substances which are acted upon in a somewhat similar and yet somewhat different manner. The vibrations of the extremely small ether atoms may set up in the more ponderous molecules of matter *slower* (forced) vibrations. We have an example of this when light waves are absorbed by a body and it in turn emits only the longer, invisible waves. Much as short, choppy waves acting upon a vessel anchored at sea impart a *slower* pitching and rolling motion to the vessel, and these in turn generate new and *slower* waves in the water, the invisible ultra-violet waves may give rise to disturbances of the interior molecules of a body impinged upon, and to other waves which are not so frequent as to be invisible. In this case the body emits light from within, and in some cases continues to emit light for some seconds after the light is shut off. This phenomenon is known by the name *fluorescence*, from fluorspar, one of the first substances in which it was observed. Among fluorescent substances are esculin, derived from the bark of the horse-chestnut, quinine sulphate, sulphides of barium and calcium, diamond, uranium glass, etc. These substances when illuminated by certain rays of the spectrum in a darkened room shine from within with a luster of their own, each showing its own

special color, — a color not by any means the same as the natural color of the body itself as seen in the open white light of day. This is best accomplished by means of the light of electric discharges. For this purpose solids and liquids are often inclosed in the so-called Geissler tubes and an electric current is passed through them.

350. Calorescence. On the other hand, slow waves may be transformed into more rapid ones. A beam of light passing through a solution of iodine in carbon bisulphide loses all its visible rays and only the long infra-red rays pass through. If these be brought to a focus by means of a lens upon a piece of platinum, the platinum will become luminous, and the light emitted therefrom when examined by means of a prism shows a continuous spectrum. This elevation in the rank of wave length is called *calorescence*. It was studied by Tyndall.

SECTION X

COLOR

351. Color by absorption. Color is a sensation; it has no material existence. The term "yellow light" means, primarily, a particular sensation; secondarily, it means the physical cause of this sensation, i.e. a train of ether waves of a particular frequency. "All objects are black in the dark"; this is equivalent to saying that without light there is no color.

By means of a porte-lumière introduce a beam of sunlight into a dark room. With the slit and prism form a solar spectrum. Between the slit and prism introduce a deep red glass; all the colors of the spectrum except the red are much reduced in intensity.

It thus appears that the color of a colored transparent object, as seen by transmitted light, arises from the unequal absorption of the different colors of white light incident upon it. A red glass absorbs less red light than light of other colors. The color produced by absorption is rarely very pure, the particular hue of the transmitted light being due merely to a predominance of

certain colors, and not to the absence of all others. As the absorbing layer is thicker, the resulting color is purer but less intense.

We have found that borax introduced into a Bunsen flame renders it luminous, and that the light when analyzed with a prism is found to contain only yellow. Expose papers or fabrics of various colors to this light in a darkened room. *No one of them except yellow exhibits its natural color.*

Hold a narrow strip of red paper or ribbon[1] in the red portion of the solar spectrum; it appears red. Slowly move it toward the other end of the spectrum; on leaving the red it becomes darker, and when it reaches the green it is quite black or colorless, and remains so as it passes the other colors of the spectrum. Repeat the experiment, using other colors, and notice that only in light of its own color does each strip of paper show its natural color, while in all other lights it is dark.

These experiments show that the color of a body seen by light reflected from it depends both upon the light incident upon it and upon the nature of the body.

If a thick piece of colored glass *AB* (Fig. 242) be held near a window so as to receive obliquely rays of sunlight, a portion of the light will be reflected by the anterior surface of the glass, and, falling upon the white ceiling, will illuminate it with white light. Another portion of the light will enter the glass and be reflected from the posterior surface; this light, having entered the glass and traveled in it a distance a little greater than twice its thickness, will suffer an unequal absorption of its rays, and after emerging from the glass will, if the glass be blue, illuminate a neighboring portion of the ceiling with blue light. This

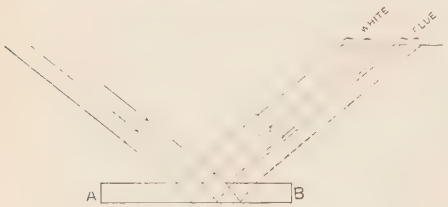


FIG. 242

[1] Care must be exercised to select only pure colors.

illustrates the method by which pigments afford color. Thus the first surface of a water-color drawing reflects the white daylight. Most of the light reflected to the eye has, however, passed through the pigment to the white paper beneath, and, being reflected from this, again passes through the layer of pigment before reaching the eye. With less transparent pigments the light may be reflected merely by particles of pigment beneath the surface. The color of paints and pigments is therefore due to the rays which they absorb least readily. When we paint our houses we do not apply color to them ; we apply substances which have the property of absorbing or subtracting from white light largely all the colors except those which we would have our houses appear. This is technically called *selective absorption.*

The color of bodies thus depends generally upon their molecular structure. Different bodies quench different portions of the complex sunlight. *The unquenched light determines the color of a body.*

The molecular action in the case of absorption is this. The molecules of the substance receiving incident light are capable of vibrating in unison with certain ether waves. The energy of certain of the ether waves is employed in setting up molecular vibrations in the substance, thus raising its temperature, while the energy of waves of different period is propagated through the medium without producing this effect.

352. Opalescence ; sky colors. Dissolve a little white castile soap in a tumbler of water ; or, better, stir into the water a few drops of an alcoholic solution of mastic, enough to render the water slightly turbid. Place a black screen behind the tumbler, and examine the liquid by reflected sunlight. — the liquid appears to be blue ; examine the liquid by transmitted sunshine, — it now appears yellowish red.

Pour some of the turbid liquid into a small test tube, and examine it and the tumbler of liquid by transmitted light ; the former appears almost colorless, while the latter is deeply colored.

When a medium holds fine particles of matter in suspension, the shorter light waves are most abundantly reflected, giving a

blue color. The blue is purer as the particles are smaller. Objects seen *through* such media appear of the complementary hue. This phenomenon is called *opalescence*. It accounts for the blue of watery milk, opalescent glass, smoke, and the sky.

Skylight is reflected light. The minute particles (of water, probably) that pervade the atmosphere, like the fine particles of mastic suspended in the water, reflect blue light, while beyond the atmosphere is a black background of darkness. But we must not conclude from this that the atmosphere is blue; for, unlike blue glass, but like the turbid liquid, it transmits yellow and red rays freely, so that seen by reflected light it is blue, but seen by transmitted light it is yellowish red.

The remarkable "yellow days" of the summer of 1883 are explained in this way. The atmosphere on this continent was very turbid during those days.

When the sun is near the horizon its rays travel a greater distance in the air to reach the earth than when it is in the zenith; consequently there is a greater loss by absorption and reflection in the former case than in the latter. But the yellow and red rays suffer less destruction, proportionally, than the other colors; consequently these colors predominate in the morning and evening.

FIG. 243

353. Mixing colors. A mixture of all the prismatic colors in the proportion found in sunlight produces white. Can white be produced in any other way?

On a black surface *A* (Fig. 243) lay two small rectangular pieces of paper, one yellow and the other blue, about two inches apart. In a vertical position between these papers, and from three inches to six inches above them, hold a slip of plate glass *C*. Looking obliquely down through the glass, you may see the blue paper by transmitted light waves and the yellow paper by reflection; that is, you see the object itself in the former case, and the image of the object in the latter case. By a little manipulation

the image and the object may be made to overlap each other, when both colors will apparently disappear, and in their place the color which is the result of the mixture will appear. In this case it will be white, or *gray*, rather, which *is white of a low degree of luminosity*. If the color be yellowish, lower the glass; if bluish, raise it. If the yellow and blue disk (Fig. 244) be rotated rapidly, the sensation will be also that of gray.

Fig. 245 represents "Newton's disk," which contains the seven prismatic colors arranged in a proper proportion to produce gray when rotated. In like manner you may produce white by mixing purple and green. Again, the three colors red, green, and violet,

FIG. 244

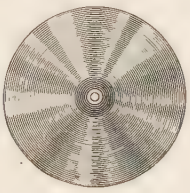


FIG. 245

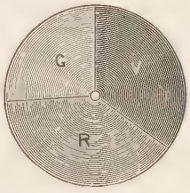


FIG. 246

arranged as in Fig. 246, with rather less surface of the green exposed than of the other colors, will give gray, and violet mixed with red gives purple.

All colors are represented in the spectrum except the purple hues. The latter form the connecting link between the two ends of the spectrum.

354. Mixing pigments. If a little of the two pigments chrome yellow and ultra-marine blue be mixed, a green pigment results.

This shows that mixing certain colors, and mixing pigments of the same name, may produce very different results. In the first experiments you mixed colors; in the last experiment you did not mix colors, and we must seek an explanation of the result obtained. If a glass vessel with parallel sides containing a blue solution of sulphate of copper be interposed in the path of the light waves which form a solar spectrum, it will be found that the red, orange, and yellow waves are cut out of the spectrum, i.e.

the liquid absorbs these waves. And if a yellow solution of bichromate of potash or picric acid be interposed, the blue and violet waves will be absorbed. It is evident that, if both solutions be interposed, all the colors will be destroyed except the green, which alone will be transmitted; thus:

Absorbed by the blue solution,	R̸ O Y G B V.
Absorbed by the yellow solution,	R O Y G B̸ V̸.
Absorbed by both solutions,	R̸ O Y G B̸ V̸.

In a similar manner, when white light strikes a mixture of yellow and blue pigments on the palette, it penetrates to some depth into the mixture; and, during its passage in and out, all the colors except the green are destroyed; so the mixed pigments necessarily appear green. But when a mixture of yellow and blue waves enters the eye, we get, as the result of the *combined* sensations produced by the two colors, the sensation of white; hence a mixture of yellow and blue gives white.

355. Theories of color vision. Brewster (1831) presented the hypothesis that all colors are formed by the union of three primaries, — *red, yellow, and blue,* — which together compose white light, and give, by combinations in twos, the hues orange, green, purple, etc. This hypothesis was based on the mixing pigments. But the actual addition of colors does not give the same result as the mixing of pigments, as has been shown.

The generally accepted theory of color vision is that of Dr. Young (1801–1802), verified and developed by Maxwell and Helmholtz. It supposes the existence of three color sensations, — red, green, and violet. These excited simultaneously, and with proper intensities, produce the sensation of white light. Combined in twos, they produce the remaining color sensations; thus red and green sensations combined give yellow or orange, green and violet give blue, etc. The longer light waves excite the sensation of red; together with those somewhat shorter, they excite both red and green, thus giving yellow, and so on. Strictly speaking, light waves of any length excite all three sensations, but usually either one or two of them greatly predominate.

The relative intensities of the various color sensations throughout the spectrum as obtained from actual measurement by Maxwell are shown in Fig. 247, in which the ordinates indicate intensities and the place in the spectrum is indicated by the Fraunhofer lines *C*, *D*, *E*, etc.

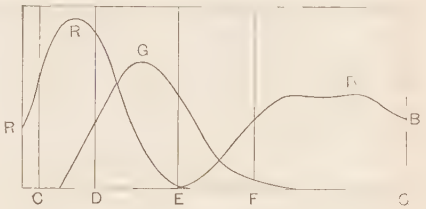


FIG. 247

356. Complementary colors. On a piece of gray paper lay a circular piece of blue paper about 15 millimeters in diameter. Attach one end of a piece of thread to the colored paper and hold the other end in the hand. Place the eyes within about 15 centimeters of the colored paper, and look steadily at the center of the paper for about fifteen seconds; then, without moving the eyes, suddenly pull the colored paper away, and instantly there will appear on the gray paper an image of the colored paper, but the image will appear to be yellow. This is usually called an *after-image*. If yellow paper be used, the color of the after-image will be blue; and if the paper be of any other color, the color of its after-image will be its complementary.

This phenomenon is explained as follows: When we look steadily at blue for a time, the eyes become fatigued by this color, and less susceptible to its influence, while they are fully susceptible to the influence of other colors; so that when they are suddenly brought to look at white, which may be regarded as a compound of yellow and blue, they receive a vivid impression from the former and a feeble impression from the latter; hence the predominant sensation is yellow. Any two colors which together produce white are said to be *complementary* to each other.

The complement of green is purple, a compound color not existing in the spectrum.

The eye gives no direct knowledge that the composition of light produces the sensation of white. Whether this sensation

is produced by the coexistence of all the rays of the visible spectrum, by a combination of light of two complementary colors, or by a mixture of rays of the three primary colors can be determined only by some process of physical analysis.

357. Effect of contrast. When different colors are seen at the same time their appearance differs more or less from that observed when they are seen separately. Thus a red object (e.g. a red rose) appears more brilliant if a green object be seen in juxtaposition with it. Such effects are said to be due to *contrast.*

When any two colors are brought into contrast, as when they are placed next each other, the effect is to move them farther apart in the color scale. For example, if red and orange be brought into contrast, the orange assumes more of a yellowish hue, and the red more of a purplish hue. Colors that are already as far apart as possible, e.g. yellow and blue, do not change their hue, but merely cause each other to appear more brilliant.

358. Color blindness. In this defect in vision one of the three color sensations is either wanting or deficient, usually that of red ; so that the colors perceived are reduced to those furnished by the remaining two sensations, namely green and violet. This causes the red-blind person to confound reds, greens, and grays. In some rare cases the sensation of green or violet is the one deficient.

SECTION XI

INTERFERENCE AND DIFFRACTION

359. We have already studied wave interference in the case of sound, and must recall that two sets of sound waves may neutralize each other and produce silence. As an example we cite the phenomenon of beats, in which the alternate increase and diminution of intensity is due to the interference of two sets of sound waves in the same and opposite phases respectively. If radiation be a wave motion, similar phenomena ought to occur under proper conditions.

360. Young's theory. The earliest authentic experiments on the interference of light waves were made by Dr. Young in 1801. He admitted a beam of sunlight into a dark chamber through a very narrow aperture, and in its path placed a screen having two very small openings quite near together. When the two overlapping pencils from these openings were made to fall on a white screen there appeared a series of bands alternately bright and dark, which disappeared when one of the holes was covered.

Although Young ascribed this phenomenon to interference, and explained it very satisfactorily, with the wave theory as a basis, his views were by no means universally accepted.

Grimaldi, nearly a century and a half before, had noticed light and dark fringes at the edges of shadows of small opaque bodies placed in the path of sunlight admitted through a small hole. This action was called *diffraction* (see § 363), and was afterwards studied by Newton. In view of the emission theory, the phenomenon was explained by assuming that the light particles experience an attractive or repellent force as they come near the edges of bodies. Believers in the latter theory then contended that Young's experiment was simply one of diffraction.

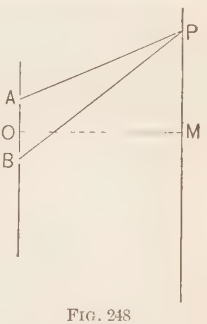


FIG. 248

To remove these objections it was necessary to devise means of producing the same result, but entirely independent of apertures and opaque bodies. This was first done by Fresnel,[1] who contrived two most ingenious experiments for producing the light and dark bands, in which the results could be accounted for only by assuming Young's theory of interference to be correct.

Let us now consider what was observed by Young. A and B (Fig. 248) are two illuminated apertures very near together, and emitting waves in all directions. When these two sets of waves arrive at any point P on the screen, they will be in the same

[1] A full account of Fresnel's experiments is given in Preston's "Theory of Light," §§ 86, 87.

phase only if the distance of P from one of the sources, say B, is one or more complete wave lengths more than the distance PA. In this event the two waves will conspire to increase the illumination at P, and this point will be on a *bright* band.

On the other hand, if $PB - PA$ be equal to a half wave length or any odd number of half wave lengths, the waves from A and B will arrive at P in exactly opposite phases and destroy one another, and P will be a point on a *dark* band. At intermediate positions for P, $PB - PA$ will not equal any whole number of half wave lengths, and hence the waves will meet in neither the same nor opposite phases, and the illumination at P will be intermediate in its intensity between that of the brightest and darkest points. Thus the bands will shade off imperceptibly into one another.

In the above discussion we have assumed the light to be monochromatic, i.e. all of the same wave length. Let OM be a perpendicular to the screens halfway between A and B; then the system of bands evidently is arranged symmetrically about M, this point being on a bright band. It is easy to see that the distance of any given band from M bears a simple relation to the wave length, and hence the bands will be closer together for short waves than for long ones, as shown in Fig. 249.

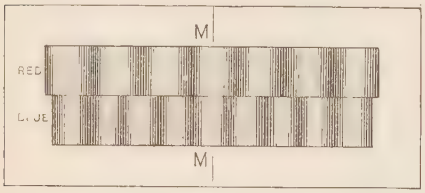


FIG. 249

If composite light be used, we should expect the bands to be colored, which is the case for a few of them near the central band, which is white; the edges nearer M are blue, while the outer edges are red.

As each color gives rise to a separate system of bands, the red ones being broadest and the violet ones narrowest, it will happen that after a few alternations a red band and a violet one will fall at the same place; or the dark spaces of one system will be filled by the bright parts of another. Soon, then, as we recede from M,

the bands become less distinctly marked, and finally merge into one another and fade into uniform illumination.

361. Colors of thin plates. Everybody is familiar with the beautiful color effects produced when ordinary white light falls upon a thin film of a transparent substance, such as a soap bubble or a film of oil on water. This is a case of interference. Some of the light comes to the eye reflected from the front or nearer surface of the film; another portion has entered the medium and been reflected from the back surface. When it emerges it has been retarded an amount depending on the distance it has traveled in the film. We might expect that if the retardation were an *even* number of half wave lengths, the two portions of light would be in a condition to interfere in the *same* phase, and that the effect would be increased; while if the retardation were an *odd* multiple of half wave lengths, the interference would be destructive and darkness would result.

The fact is exactly the reverse of this, since by the act of reflection in the denser medium the phase of the wave is reversed and the result is as if the wave had been retarded another half wave length.

If the eye view such a film in monochromatic light, the portion entering the film will be retarded varying amounts depending on the angle of incidence and on the thickness. The film will therefore be crossed by bright and dark bands.

If, however, composite light be used, coöperative or destructive interference evidently cannot take place for the different colors at the same points, and the familiar iris-colored bands result.

FIG. 250

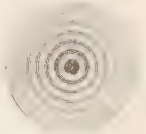


FIG. 251

362. Newton's rings. Newton's method of studying these colors was very simple and effective, and the phenomena exhibited are known as "Newton's rings." If a convex lens of very small curvature be placed firmly upon a piece of plate glass (Fig. 250), the film of air between the two increases in thickness from the

center radially, and hence beautiful circular interference bands are shown, having the point of contact as their center (Fig. 251).

363. Diffraction. This phenomenon, first observed by Grimaldi, and already referred to, occurs when light passes through a very narrow aperture or close to the edge of an opaque body.

Newton's strong reason for rejecting the wave theory was that light, as he supposed, did not go around corners as sound does. Closer examination, however, shows that the cases are similar if consistent conditions be maintained. Light really does bend around corners, while, on the other hand, well-defined sound shadows may be cast by sounds of sufficiently short wave length. In other words, sound bends around corners very much more readily than light, merely because its wave length is so much greater in comparison with the size of the obstacle.

The true explanation of diffraction phenomena was given by Fresnel, who attributed them to the mutual interference of the secondary wavelets which diverge from the primary or main wave front as it meets the obstacle or edges of the aperture, just as the cases of interference previously described are due to the mutual interference of two waves. For a fuller description of the cases of diffraction and their treatment, the student is referred to special works on optics.

364. The diffraction grating.[1] When a distant source of light is viewed through a system of very narrow rectangular openings, a central image is seen, and on either side of it there are several highly colored spectral images, increasing in breadth but diminishing in brightness as they recede from the center. A device like the above is known as a diffraction grating, and may be produced by ruling with a diamond fine lines on a piece of glass.[2] The lines form the opaque portions of the grating, while the spaces between them are the slits through which the light passes. The effect is most marked when the opaque parts and transparent parts are equal.

[1] For a more complete treatment of gratings, see Preston's "Theory of Light."

[2] Many of the fine gratings of Rowland have 20,000 lines to the inch. These, however, are now usually ruled on speculum metal, as glass is apt to injure the diamond point.

A magnified section of a grating perpendicular to the lines is shown in Fig. 252. Let ab represent a line and adjacent slit in such a position with reference to the point P, where the eye is placed, that $Pb - Pa$ equals one wave length. It is clear that the portion of the incident wave (supposed to be homogeneous) corresponding to ab may be divided into two nearly equal parts, which would destructively interfere at P if the grating were not present. The effect of the grating, then, is to intercept one part of these interfering portions and enable the other to become visible. A bright band will then be seen in the direction Pb.

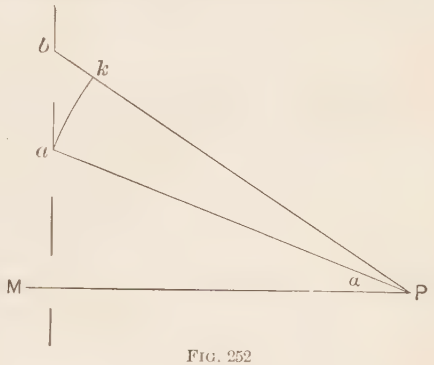


FIG. 252

The same will happen for all similar parts of the grating provided the distances of their extremities from P differ by a whole number of wave lengths. Thus there will appear a succession of bright bands at increasing angular distances from PM. It should be noticed in studying this figure that ab is extremely small compared with PM, and therefore the figure is necessarily distorted for convenience.

If the light incident on the grating be composite, evidently the angle α, indicating the direction from PM of the bright band, will be less for the short waves than for the long ones. Therefore in this case the resulting band will be colored, — the inner edge blue, the outer one red, the portion lying between, yellow and green.

Such a phenomenon is called a diffraction spectrum. It is what is known as a *normal spectrum*, because it exhibits the colors, or Fraunhofer lines which locate them, always in their true order and separated by spaces bearing a simple relation to their wave lengths.

This is not the case in spectra produced by refraction. The rays at the red end are crowded together and condensed out of all proportion to those at the violet end; i.e. a given difference in wave length causes a much greater separation in the more refrangible parts. This is called *irrationality of dispersion.* It is

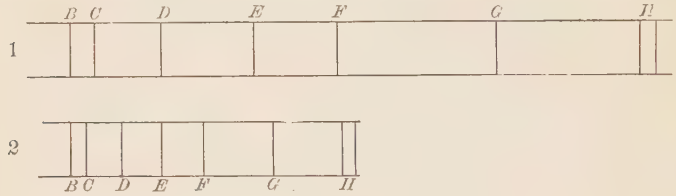


FIG. 253

exhibited variously by different substances. Fig. 253 shows the positions of the principal Fraunhofer lines as given with prisms (1) of flint glass, and (2) of crown glass, having the same refracting angle. The difference between a normal or diffraction spectrum and a prismatic one may be understood by a glance at

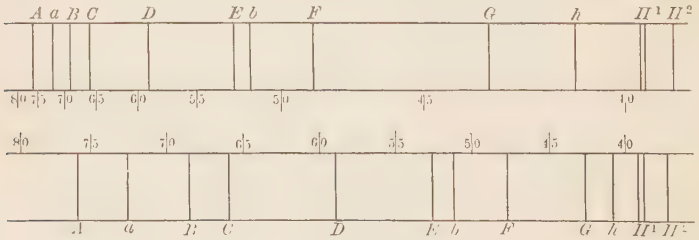


FIG. 254

Fig. 254. The scales between the spectra indicate the wave lengths in millionths of a centimeter.

The grating furnishes a simple means of measuring wave length. Referring again to Fig. 252, in which Pb is the direction of the first bright band, we see that the triangles bak and bPM are very nearly similar (ak being a very short arc practically perpendicular to Pb). Then $\frac{bk}{ab} = \frac{bM}{bP} = \sin \alpha$; or $bk = ab \sin \alpha$. But

bk is one wave length, λ, and ab is $\frac{1}{n}$ where n is the number of lines per unit of length on the grating; hence $\lambda = \frac{1}{n} \sin \alpha$.

365. To measure λ. Introduce a borax bead into the flame of a Bunsen burner, and place them near the wall of a darkened room. From a distance of six or eight meters view the flame through a grating (one having from one to two thousand lines to the centimeter preferred), holding it perpendicular to the line of sight and with the rulings vertical. Note the position on the wall of the first image of the flame, and measure in centimeters its distance from the flame. This divided by the distance from the point of observation to the image will be $\sin \alpha$. (In practice it is found to be sufficiently accurate to measure from the point of observation directly to the burner.) If we divide $\sin \alpha$ by the number of lines per centimeter on the grating, the result will be the wave length in centimeters for sodium light.

SECTION XII

DOUBLE REFRACTION AND POLARIZATION OF LIGHT

366. Double refraction. We have hitherto assumed that a ray of light incident upon a transparent body is refracted according to the *law of sines*. This is the case when the transparent body is isotropic, i.e. having the same properties in all directions. There are numerous transparent substances which fulfill this condition at least approximately, such as fluids, well-annealed glass, etc. On the other hand, there are numerous transparent substances for which the law of sines does not hold. When a ray of light enters such a body it is split into two rays, and this law does not hold for both these resulting rays; sometimes it does not hold for either ray. This gives rise to a group of phenomena known by the term *double refraction*, and the substances which affect light in this manner are called *doubly refracting substances*.

These include various crystals, animal substances such as horn and shells, vegetable substances such as resins and gums, and certain artificial substances such as jellies and unannealed glass.

This property of double refraction was first discovered in the well-known mineral Iceland spar, or calcite, by Bartholinus, who published his results in 1669 at Copenhagen. Further investigations by Huygens and others have shown that all crystallized minerals except those belonging to the cubical system possess the property of double refraction, though in different degrees. In calcite it is exceedingly well marked.

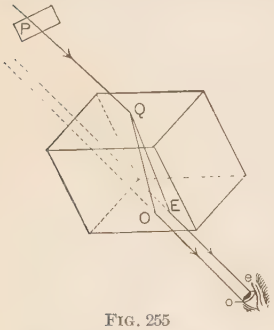


FIG. 255

If a rhomb of this material be brought between the eye and a card (Fig. 255) with a small hole in it, there will appear to be two holes with light shining through each. If now the crystal be rotated in a plane parallel with the card, one of the images will appear comparatively at rest, while the other will revolve around the first.

A ray of light PQ immediately on entering the crystal is divided into two parts, one of which, QO, obeys the regular law of refraction; the other, QE, does not. The former is called the *ordinary ray*: the latter, the *extraordinary ray*. In all crystals which produce this phenomenon there is one direction, and in some there are two, in which an object looked at through the crystal does not appear double. If all the edges of a crystal of Iceland spar (Fig. 256) be equal, and if the line AB, connecting the two opposite obtuse solid angles, be cut near each extremity by a plane perpendicular to it, objects viewed in this line, or in any line parallel with it, do not appear double.

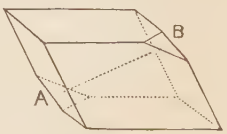


FIG. 256

The line AB is called the *optic axis* of the crystal, and is a line around which the molecules of the crystal appear to be

arranged symmetrically. A crystal is called *uniaxial* when it has only one optic axis, and *biaxial* when it has two such axes. A plane parallel to this axis and perpendicular to one of the rhombic faces of the crystal is called a *principal section*. The two rays travel in the crystal with unequal speeds in all directions except in the direction of the optic axis of the crystal.

The property of double refraction may be imparted permanently or transiently to certain substances which do not naturally possess it. Glass may be given this power by heating different parts unequally, and also by compression and bending.

367. Polarization. Up to the present time, in our discussion of the properties of light waves, it has been assumed that a ray has no relation to space except that dependent on its direction; that, in fact, around this direction its properties and effects were the same on all sides, so that if the ray be revolved around its direction as an axis, the phenomena due to its presence would not change.

Huygens, however, in the course of his studies on double refraction, found that this is not always the case. He found that, while the two rays from a rhomb of Iceland spar were always of equal intensity, if these were directed through a second similar rhomb each was divided again into two, but that in general these components were not of equal intensity; that their relative brightness depended on the position of the second crystal referred to the first, and that there were two positions in which one of the components of each pair disappeared altogether. These phenomena were found to depend on the relative positions of the principal sections of the two rhombs. When these planes are parallel or perpendicular to each other the second rhomb fails to divide either of the beams falling upon it from the first crystal. In all other positions there result four parts differing in brightness in pairs, except when the inclination of the principal sections is 45°; in this case all four components are of the same intensity.

From this it appears that the rays from the first rhomb have taken on properties distinguishing them altogether from ordinary

light. They have acquired "sides"; and the relation of these sides to the surfaces of other transparent media upon which incidence occurs determines the behavior of the rays in the new medium.

This property can be given to light in various ways, as will be seen, and in general is called *polarization.*

368. The Nicol prism. For certain purposes it is best to allow only one of the two rays to leave the prism (that, namely, in the direction of the incident light), and to eliminate the other. The Nicol prism consists of a crystal of Iceland spar divided diagonally, as *ab* (Fig. 257), the two surfaces being cemented together with Canada balsam. All the faces of the prism are painted black except the two end faces. The extraordinary ray, falling upon the transparent balsam at an angle less than the critical angle, passes through it, but the more refracted (or ordinary) ray meets the balsam at an angle greater than the critical angle, and is therefore totally reflected, thrown to one side of the prism, and absorbed by the black paint. By means of such a prism a single beam of polarized light having the same direction as the incident beam can easily be obtained.

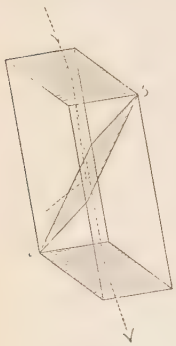


FIG. 257

The unaided eye cannot usually detect any difference between common and polarized light. An instrument which will enable the eye to detect polarized light is called an *analyzer;* thus the first slice of tourmaline serves as a polarizer, and the second slice as an analyzer. A complete polarizing apparatus, called a *polariscope,* used for observing the phenomena of polarized light, consists essentially of a polarizer and an analyzer.

369. Polariscope consisting of two Nicol prisms. When a ray of light becomes split by double refraction, each of the resulting rays is found to be plane polarized, one being polarized in the plane of incidence, the other at right angles thereto.

A and *P* (Fig. 258) represent two corks having Nicol prisms extending through them lengthwise, *P* serving as a polarizer and

A as an analyzer. If the analyzer A be turned so that its principal section $c'd'R$ is parallel to the principal section cdR of the polarizer P, the ray R which enters P as unpolarized light becomes polarized, and the extraordinary ray emerging from this prism will pass freely through the analyzer A. The same happens if A be turned through an angle of 180° so as to bring the same planes parallel again. But if A be adjusted so that its principal section is at right angles to that of the polarizer (as in the lower part of Fig. 258), then the ray is quenched by the prism A. No light leaves the analyzer, and accordingly, to an eye placed at the end of A, the field of vision is dark. The same happens if A be turned through an angle of 180°. In all cases where the principal sections are neither parallel nor at right angles, the polarized light entering the analyzer is separated into ordinary and extraordinary rays, and the light which emerges from the analyzer varies in intensity with the angle at which the principal sections of the two prisms are inclined to each other.

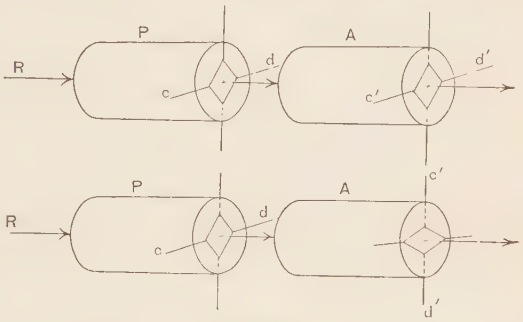


FIG. 258

370. Polarization by reflection. For more than a hundred years the polarization produced by double refraction remained a unique fact in science, and it was not till 1808 that Malus discovered, while looking through a double-image prism at the light reflected from a window in the Luxemburg palace in Paris, that light may be polarized by reflection. The plane of reflection in which polarization occurs is called the *plane of polarization* of the beam.

It was subsequently found that the amount of polarization depends on the incident angle. The angle at which the polarization

is a maximum is called the *angle of polarization by reflection*, which in the case of glass is between 55° and 56°. Then, of course, if a Nicol prism be held in certain positions, the reflected light will pass through it, but at a distance of 90° it refuses to pass. Again, light which has been polarized by reflection from one glass surface A (Fig. 259) will be reflected from another glass surface B, placed so that the plane of incidence of the polarized ray is parallel to the plane of polarization, but refuses to be reflected when the plane of incidence is at right

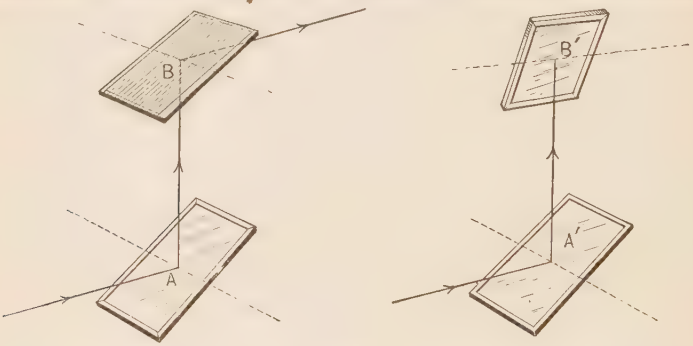


FIG. 259

angles to the plane of polarization, as in B'. So that in polariscopes a plate (better a bundle of plates) of glass may be substituted for either the polarizer or the analyzer.

Light reflected obliquely from nonmetallic smooth surfaces, such as water, polished furniture, oil paintings, etc., is found on examination to be partially polarized.

371. Polarization by refraction. Not only the reflected portion of the light is polarized, but also the part that enters the medium and is transmitted. For example, if A (Fig. 259) be a transparent glass plate, a part of the incident light will of course pass through, and this part will be found to be partially polarized, but in a plane *perpendicular* to the plane of incidence. Further, Arago found that the reflected beam and the transmitted beam contain the same amount of polarized light. If the transmitted beam be

examined by making it incident upon a second transparent surface, it will be reflected only if the plane of incidence is made parallel to the plane of polarization of the light.

372. Polarization by absorption. Slices of crystal of the mineral tourmaline, cut in planes parallel with their axes, are prepared and sold for optical experiments. If two of these slices similarly situated, as in Fig. 260, be placed between the eye and a card pierced by a hole, the hole will be plainly visible. But if one of the slices be slowly rotated in a plane at right angles with the beam of light, the hole will grow dimmer until the slice has passed through a quarter of a revolution (as represented in Fig. 261), when it disappears. If the rotation be continued, the hole reappears, faint at first, but reaching its maximum brightness at the end of another quarter revolution. Thus, at successive quarter revolutions it is alternately extinguished and restored.

FIG. 260

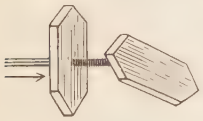


FIG. 261

It appears, then, that light which has passed through one transparent slice of tourmaline differs so much from common light that a second similar slice may act like an opaque body, and stop the light altogether. The action of the tourmaline may be compared to that of a grating (A, Fig. 262) formed of parallel vertical rods, which will allow all vertical planes (as aa') to pass, but stops the planes (as cc') that are at right angles to these rods. Any plane that has succeeded in passing one grating will readily pass a second similarly placed.

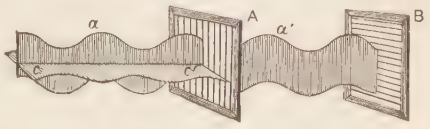


FIG. 262

But if the second grating B be turned so that its rods are at right angles to the first, the plane that has succeeded in passing through the first grating will be stopped by the second, thus illustrating the action of a pair of tourmalines.

This result is explained by supposing that one of the two polarized beams into which the crystal separates the incident light has been *absorbed* by the tourmaline; thus only one is transmitted and that is polarized in a plane parallel to the axis of the crystal.

In order to understand this phenomenon, it is necessary to know more of the undulatory theory of light. This theory supposes that the undulations in ether which constitute light are much like undulations in a cord when one end is shaken by a hand, as seen in Fig. 263. The vibratory motion which constitutes light must be transverse to the direction of the ray; it cannot be in the direction of the ray, for then there would be no difference between the different sides of the ray, and the phenomena of polarization would be unexplainable on this hypothesis. If the hand move vertically, all the undulations will lie in a vertical plane; if the movements of the hand be horizontal or oblique, the undulations lie in corresponding planes. So we can produce these waves on the rope in any plane passing through the rope, and can change rapidly from one plane to another. These waves appear differently when viewed from different sides. It is believed that if we could look endwise at a ray of light for an instant, we should see the ether vibrations, as in the figure of the rope, in one plane; but in only a thousandth of a second so many million waves reach the eye that there is time for the vibrating particles, which, like the hand, start the waves, to vibrate in many planes. In an ordinary beam of light, as it reaches the eye, there are therefore undulations in all possible planes, as is partially represented by the cross section .1 (Fig. 264). But

Fig. 263

Fig. 264

rectilinear motion may be considered as the resultant of two similar motions at right angles to each other. So here, for many practical purposes, the vibrations may be regarded as taking place in only two sets of planes at right angles to each other, as represented by *B* of the same figure. Now, when a ray of light consisting, according to supposition, of undulations, as in either *A* or *B*, strikes a slice of tourmaline, its molecular structure resolves all the motions into two, one parallel to and the other perpendicular to its axis. The former of these is transmitted and the other is absorbed. By this means the undulations are reduced to those in parallel planes only, as represented in *C*.

373. Circular and elliptical polarization. In polarization thus far treated we have assumed the vibrations to be in straight lines and all confined to one plane. It can be shown, however, that if two such waves, one a quarter of a period in advance of the other, be compounded, with their planes perpendicular to each other, the result will be a wave in which the ether particles move in circles or ellipses, according as the amplitudes of the components are equal or not. Such a wave is circularly or elliptically polarized, and since (as in all wave motion) the particles move successively, the wave has the form of a helix. For fuller discussion of this form of polarization and the methods of producing and detecting it, the student is referred to special works on light.

374. Chromatic phenomena. When crystalline substances of various kinds are examined between an analyzer and a polarizer, in many cases very beautiful colored effects are produced, which are due in general to the interference of the polarized light which is transmitted by the crystals. The method is frequently employed in connection with a microscope, since by means of polarized light the structural detail of many specimens can be far better studied than by the ordinary methods. In general all doubly refracting substances, when examined by polarized light through an analyzer, present colored phenomena which are more or less characteristic of the substance. For a

SECTION XIII

ABSORPTION OF RADIATION

378. Thermal effects. We have learned that heat may travel *through* matter (by conduction) and *with* matter (by convection), and it is sometimes stated that there is a third method by which it travels, namely "radiation." Heat itself is not transferred by radiation at all; heat generates radiation (i.e. ether waves) at one place, and radiation absorbed produces heat at another; it is *radiation* which travels, not heat. It does not exist as heat in the intervening space, and therefore does not necessarily heat the substance filling that space. Heat can flow only one way, namely from a given point to a point that is colder; radiation travels in all directions. The sun sends us no heat, but it sends radiations which the earth transforms into heat; but it should be borne in mind that while it is radiation it is not heat, and vice versa. Temperature is a condition of bodies, not of radiations; wave lengths belong to radiations, not to heat which produces them.

379. Diathermancy and athermancy. What becomes of radiations which strike a body depends largely upon the character of the body. If the nature of the body be such that its molecules can accept the motion of the ether, the vibrations of ether are said to be *absorbed* by the body and the body is thereby heated, i.e. the undulations of ether are transformed into molecular energy or *heat*. Glass, for instance, allows the sun's radiation to pass very freely through it, and very little is transformed into heat. But if the glass be covered with the soot of a candle flame, the soot will absorb the radiations and the glass become heated. Observe how cold window glass may remain while radiations pour through it and heat objects in the room. *Only those radiations that a body absorbs heat it; those that pass through it do not affect its temperature.*

Bodies that transmit radiations freely are said to be *diathermanous*, while those that absorb them largely are called *athermanous*. These terms bear the same relation to the transmission of radiant energy of any and all wave lengths as do transparency and opacity to the transmission of light or visible radiations. The most diathermanous substance known is rock salt. A solution of iodine in carbon bisulphide absorbs almost completely the rays of the visible spectrum, but transmits almost completely all of longer wave length than the red end of the spectrum. A plate of alum acts in the reverse manner, transmitting the visible and absorbing the invisible. Among liquids carbon bisulphide is exceptionally transparent to all forms of radiation; while water, transparent to short waves, absorbs the longer waves, and is thus quite athermanous.

Bring the bulb of an air thermometer into the focus of a burning glass exposed to the sun's rays. The radiation concentrated on the inclosed air scarcely affects this delicate instrument. If, however, the outside of the bulb of the air thermometer be covered with lampblack, the lampblack absorbs the radiant energy, and the heat conducted through the glass to the inclosed air raises its temperature and causes it to expand and rapidly push the liquid out of the stem.

Dry air is almost perfectly diathermanous. All of the sun's radiations that reach the earth pass through the atmosphere, which contains a vast amount of aqueous vapor. This vapor, like water, is comparatively opaque to long waves; hence it modifies very much the character of the radiations which reach the earth. This fact, together with what we have learned from the above experiments, enables us to understand the method by which our atmosphere becomes heated. First, that portion of the radiant energy which comes to us from the sun in the form of relatively long waves is stopped by the watery vapor in the air, which is thereby heated. The portion that comes to us in short waves escaping this absorption heats the earth by falling upon it. The warmed earth loses its heat, — partly by conduction

to the air, still more largely by radiation outward. The form of radiation, however, has been greatly changed; for now, coming from a body at a low temperature, it is chiefly in long waves that the energy is transmitted; while, as we have seen, it was largely in the form of short waves that the earth received its heat. But it is exactly these long waves which are most readily stopped by the atmosphere; hence the atmosphere, or rather the aqueous vapor of the atmosphere, acts as a sort of trap for the energy which comes to us from the sun.

Remove the watery vapor (which serves as a blanket to the earth) from our atmosphere, and the chill resulting from the rapid escape of heat by radiation would probably put an end to all animal and vegetable life. Glass does not screen us from the sun's heat, but it can very effectually screen us from the heat radiated from a stove or any other terrestrial object. Glass is diathermanous to the sun's radiations (simply because they have already lost most of the very long waves by atmospheric absorption), but quite athermanous to other radiations. This is well illustrated in the case of hotbeds and greenhouses. The sun's rays pass through the glass of these inclosures almost unobstructed, and heat the earth; but the radiations given out in turn by the earth are such as cannot pass out through the glass, and hence the heat is retained within the inclosures.

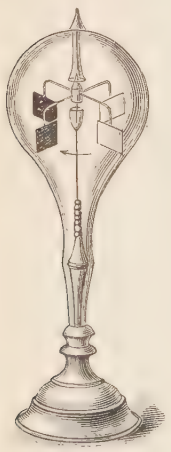


FIG. 266

380. The radiometer. Fig. 266 represents an instrument called a *radiometer*. The moving part is a small vane resting on the point of a needle. It is so nicely poised on this pivot that it rotates with the greatest freedom. To the extremities of each of the four arms of the vane are attached disks of aluminum or mica which are light on one side and black on the other. The whole is inclosed in a glass bulb from which the air is nearly exhausted. Air must be exhausted from the bulb till the mean free path of the

air particles is greater than the distance of the glass from the surface of the vane, so that particles after impinging on the disks do not as a rule collide with other particles before reaching the glass.

Exposed to the radiations of the sun, a candle flame, or even the radiations from the human body, the vane will rotate with the unblackened faces in advance. The blackened faces absorb the radiant energy and become heated, and, by striking against them, the air particles remaining in the bulb have their speed increased, thus causing an increased pressure upon the blackened surfaces, and a more or less rapid revolution of the arms.

It can be shown that the glass bulb tends to rotate in the opposite direction to that of the vane. This is a proof that the rotation is due to action and reaction between the vane and the glass, and not, as it might appear, between the vane and the source of radiation. The radiometer thus serves indirectly to transform radiant energy into mechanical work, and may be used to measure the mechanical effects of radiant energy. For example, the radiations from two candle flames produce twice the effect of that from one; and when the distance from the source of radiations is doubled the effect is one fourth as great. Hence the radiometer may be used to verify in a very direct and simple manner the law of inverse squares as applied to radiant energy. It is also used in a variety of experiments to illustrate the mechanical effects of the rapidly moving molecules of gases.

If the opening at the top be kept open and connected with a pump, so that the exhaustion can be regulated at will, then after a certain degree of exhaustion has been attained the black disks exposed to radiation are repelled and rotation ensues. If the exhaustion be gradually increased, a maximum speed is reached; further exhaustion diminishes the speed, and ultimately rotation ceases.

381. Solar radiation; pyrheliometer. Fig. 267 represents an instrument, called a *pyrheliometer*, used to determine solar radiation. It consists of a shallow cylindrical vessel A of thin metal. The upper surface is covered with lampblack. The bulb of a

thermometer is inclosed in this cylinder and its stem in a tube B; the remaining space in cylinder and tube is filled with water. The blackened surface is turned toward the sun, and to insure that the rays are normal to this surface the shadow of the cylinder is made to cover exactly and coincide with the disk C.

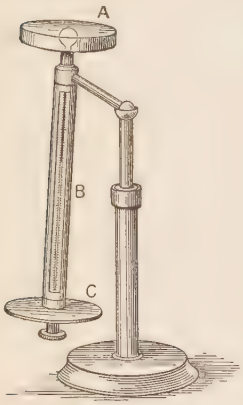


FIG. 267

First the instrument, sheltered from the sun, is permitted to radiate its heat into the clear sky for, say, five minutes. Let the fall in temperature be $r°$.

Next it is turned to the sun for five minutes. Let the rise in temperature be $R°$.

Finally it is allowed (at its increased temperature) to radiate into the clear sky as before for five minutes. Let the fall of temperature be $r'°$.

Now since r denotes the change of temperature during radiation into clear sky before heating, and r' the same after heating, the change of temperature during the heating, due to radiation, conduction, etc., will be very nearly a mean between the two, or $\frac{r + r'}{2}$; but it is evident that this cooling effect takes place even when the instrument is receiving the sun's radiations, and tends to diminish the heating effect produced by these radiations. Hence, therefore, the whole heating effect will be

$$R + \tfrac{1}{2}(r + r').$$

This is the number of degrees which the sun's radiant energy is able to raise the temperature of n kilograms of water (suitable allowance being made for the water equivalent of the vessel and thermometer) when it falls upon a units of area of lampblack for m units of time. Then the radiant energy received per unit area during one unit of time is equivalent to

$$n\left(R + \frac{r + r'}{2}\right) \div (a \times m)$$

calories of heat. By Joule's equivalent we are able to calculate the energy in mechanical units.

It is estimated that if the whole radiation received by the earth from the sun were employed in melting ice, it would in a year melt a layer of ice 137 feet in thickness all round the earth.[1] Only a very small fraction of the sun's radiation is intercepted by the earth. Lord Kelvin estimates that the total energy emitted by the sun is at the rate of 7000 horse power per square foot of radiating surface.

382. All bodies radiate heat. Hot bodies *usually* part with their heat much more rapidly by radiation than by all other processes combined. But cold bodies, like ice, emit radiations even when surrounded by warm bodies. This must be so from the nature of the case, for the molecules of the coldest bodies possess some motion, and, being surrounded by ether, they cannot move without imparting some of their motion to the ether, and to that extent becoming colder themselves.

383. Prévost's theory of exchanges. Let us suppose that we have two bodies, A and B, at different temperatures, — A warmer than B. Radiation takes place not only from A to B but from B to A; but, in consequence of A's excess of temperature, more radiation passes from A to B than from B to A, and this continues until both bodies acquire the same temperature. At this point radiation by no means ceases, but each now gives as much as it receives, and thus equilibrium is kept up. This is known as "Prévost's theory of exchanges."

384. Good absorbers, good radiators. Select two small tin boxes of equal capacity; one should be bright outside, while the other should be covered thinly with soot from a candle flame. Cut a hole in the cover of each box large enough to admit the bulb of a thermometer. Fill both boxes with hot water, and introduce into each a thermometer. They will register the same temperature at first. Set both in a cool place, and in half an hour you will find that the thermometer in the blackened box registers several

[1] Young, "Elements of Astronomy," pp. 147–153.

degrees lower than the other. Then fill both with cold water, and set them in front of a fire or in the sunshine, and it will be found that the temperature in the blackened box rises more rapidly.

As bodies differ widely in their absorbing power, so they do in their radiating power, and it is found to be universally true that *good absorbers are good radiators, and bad absorbers are bad radiators.* In both cases much depends upon the character of the surface as well as of the substance. Bright, polished surfaces are poor absorbers and poor radiators; while tarnished, dark, and roughened surfaces absorb and radiate rapidly. Dark clothing absorbs and radiates more rapidly than light clothing.

385. Dew. It requires no elaborate experiments to show that some bodies radiate more rapidly than others. All nature testifies to this every still, cloudless summer night. During the day objects on the earth's surface gain more heat by radiation than they lose, but as soon as the sun has set this is reversed. Then everything begins to cool by radiation into space. Objects becoming cool, the air in contact with them becomes chilled, its water vapor condensing and collecting in tiny drops on their surfaces. But these dewdrops collect much more abundantly on certain things, such as grasses and leaves, than on others, such as stones and earth. The reason that dew does not collect on the latter so freely is because of their poor radiating power; they do not get cool as rapidly.

QUESTIONS AND PROBLEMS

1. What objections can you raise to the term "radiant heat"?
2. Explain why the temperature of a hotbed is above that of the surrounding air.
3. How could you separate the dark radiation of an electric arc lamp from the luminous radiation?
4. How can you demonstrate the existence of ether waves of greater length than the light-giving waves?
5. Ice appears to radiate cold. Explain the phenomenon by Prévost's theory.
6. A radiometer placed near ice rotates with the blackened sides of its disks in advance. Explain.

7. What parts of the spectrum are invisible to the eye?

8. How can you prove the existence of invisible solar rays?

9. On what does the color of bodies primarily depend?

10. What agency does a body perform in determining its own color when illuminated with white light?

11. (*a*) Why is grass green? (*b*) snow white? (*c*) soot black?

12. How does a spectrum produced by a crown-glass prism differ from a spectrum produced by a flint-glass prism?

13. State some phenomenon which the undulatory theory alone is competent to explain.

14. Describe the appearance which an iridescent soap bubble would present in a monochromatic light.

15. Objects seen across the top of a hot stove appear unsteady and indistinct. Explain.

16. State how, with a pair of tourmaline tongs, you may distinguish a glass spectacle lens from a quartz lens.

17. State how the sensation of purple is produced.

18. What is meant by the artist's three primary pigments?

19. Describe the surface which a hot-water vessel should have in order to retain its heat well.

20. Suppose beams of sunlight enter a dark room through two apertures in a shutter, and a blue glass be placed in the path of one beam and a yellow glass in the path of the other. (*a*) What color will that portion of a white wall appear where the two images of the sun overlap? (*b*) State the result if one of the apertures be closed and the beam from the other be made to pass through both colored glasses.

21. When red and green sensations coexist, what is the resulting sensation?

22. What phenomenon shows that ether waves do not traverse all substances with equal speed?

23. State how light may "turn a corner."

24. What utility is there in keeping certain parts of a steam engine very bright?

SECTION XIV

SOME OPTICAL INSTRUMENTS

386. Compound microscope. When it is desired to magnify an object more than can be done conveniently and with distinctness by a single lens, two convex lenses are used, — one O (Fig. 268), called the *objective*, to form a magnified real image $a'b'$ of the object ab; and the other E, called the *eyepiece*, to magnify this

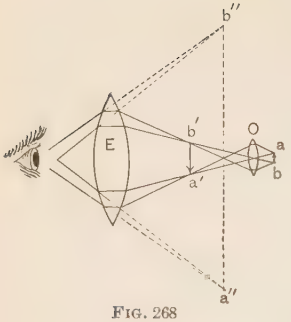


FIG. 268

image so that the image $a'b'$ appear of the size $a''b''$. Instead of looking at the object, as when we use a simple lens, we look at the real inverted image $a'b'$ of the object.

This represents the simplest possible form of the *compound microscope.* In practice, however, the construction is more complicated.

Fig. 269 represents a perspective and a sectional view of a simple form of a modern compound microscope. The body of the instrument

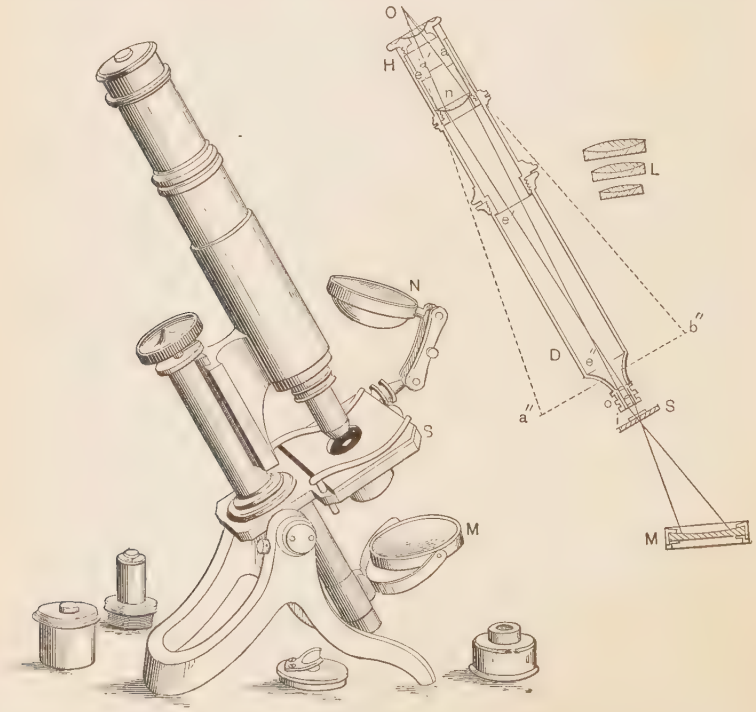


FIG. 269

consists of a series of brass tubes movable one within another. In the upper end H is the *ocular* or *eyepiece*. It consists of two plano-convex lenses O and n, the former called the *eye lens*, the latter called the *field lens*. The advantages derived from the use of two lenses in the eyepiece are as follows:

(1) *The combination diminishes spherical aberration and thereby increases the flatness of the field.* The images $a'b'$ and $a''b''$ (Fig. 268) are in reality curved in consequence of the spherical aberration caused by the objective. The effect of the field lens is to correct this curvature in a measure.

(2) *The combination increases the field of view*, so that a larger area of the object is made visible at the same view.

(3) *The combination diminishes chromatic aberration.*

All microscopes, however, should be furnished with an achromatic objective. This consists of from two to four achromatic lenses (the achromatic triplet, the most common form, is represented on an enlarged scale at L), combined so as to act as a single lens of short focus. By the use of several lenses the aberrations can be better corrected than with a single lens.

The object to be examined is placed on a stage S, and, if the object be transparent, it is strongly illuminated by focusing light upon it by means of a concave mirror M. If the object be opaque, it is illuminated by light directed upon it obliquely from above by the converging lens n.

387. Oculars. The *negative* (or Huygenian) ocular consists of two convex lenses of crown glass, F and E (Fig. 270), the convex surfaces being turned toward the object glass. A pencil of rays from the object glass converging towards a focus a is brought to a focus a' halfway between the two lenses.

FIG. 270

This ocular is called negative because it is adapted to rays already converging. The focal length of F is three times that of E, and the distance between the lenses is one half the sum of the focal lengths.

The *positive* (or Ramsden) ocular consists of two plano-convex lenses, E and E' (Fig. 271), with the convex surfaces turned towards each other. These lenses are of equal focal length, and the distance between them is two thirds the focal length of one of them. This combination is not achromatic. It is used when spider lines are placed in the focus of the field lens for purposes of exact measurement.

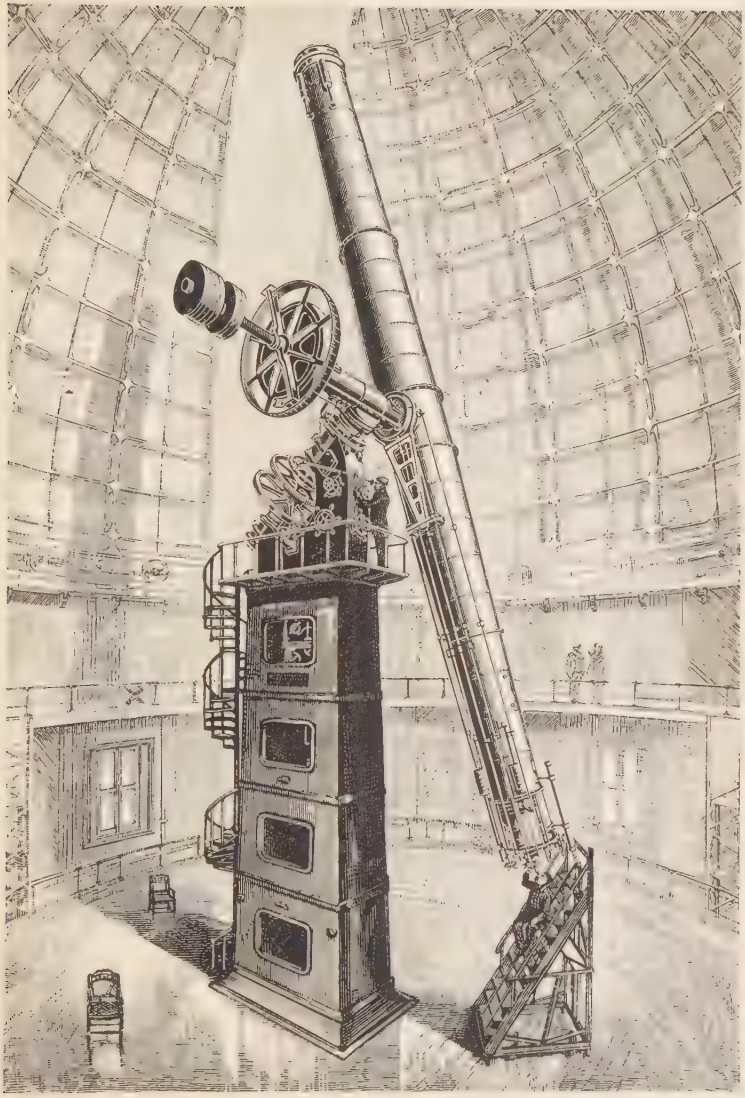
FIG. 271

In obtaining high magnifying power it is generally best to use objectives of short focal length rather than oculars of high power, as the latter magnify the imperfections of the former.

388. Magnifying power. The magnifying power of a compound microscope is the product of the respective magnifying powers of the object glass and the eyepiece; that is, if the first magnify 20 times and the other 10 times, the total magnifying power is 200. The magnifying power is determined experimentally by means of a micrometer scale, for a description of which the student is referred to technical works on microscopy.

389. Telescopes. Telescopes are used to view (scope) objects afar off (tele). They are classified as astronomical or terrestrial, according as they are designed to be used in viewing heavenly bodies or terrestrial objects; *reflecting* or *refracting*, according as the objective is a concave mirror or a converging lens. The terrestrial telescope differs from the astronomical in producing images in their true position without inversion. This is effected by means of an extra object lens, which corrects the inversion of the main object lens. The matter of inversion is of little or no consequence in viewing heavenly bodies.

The refracting astronomical telescope consists essentially, like the compound microscope, of two lenses. The object glass O (Fig. 272) forms a real diminished image ab of the object AB; this image, seen through the eyeglass E, appears magnified and of the size cd. The object glass is of large diameter, in order



The Telescope in the Lick Observatory in California

to collect as much light as possible from a distant object for a better illumination of the image.

The telescope at the Lick Observatory in California is represented in Plate I. It is 57 feet long; its object glass is 36 inches in diameter, and its total weight 40 tons. It is exceeded in size only by the Yerkes instrument in Chicago, which has an objective of 40 inches aperture.

This telescope is analogous to the microscope, but the two instruments differ in this respect: in the microscope, the object being very near the object glass, the image is formed much beyond the principal focus, and is greatly magnified, so that both the object glass and the eyepiece magnify; while in the telescope, the heavenly body being at a great distance, the incident rays

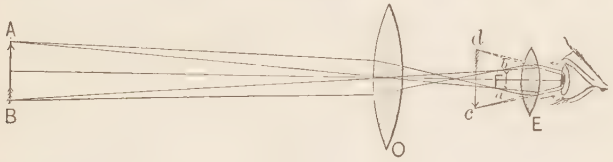


FIG. 272

are practically parallel, and the image formed by the object glass is much smaller than the object. The only magnification which can occur is produced by the eyepiece, which ought therefore to be of high power. The magnifying power of this instrument[1] equals approximately *the focal length of the object glass divided by the focal length of the eyepiece.*

390. The Newtonian reflecting telescope. For an instrument of moderate cost, specially adapted to school and college use owing to the ease of manipulation and the comfort with which the observer may view any part of the sky, the Newtonian reflecting telescope meets with much favor. It also possesses the great advantage of giving a colorless image of bright objects, which

[1] The student may ascertain the magnifying power of a terrestrial telescope by viewing a scale directly with one eye, and its magnified image, as seen through the telescope, with the other eye.

cannot be obtained in a refractor.[1] Fig. 273 represents a horizontal sectional view of this instrument. Incident rays are reflected from the parabolic mirror M; striking the rectangular prism mn, they undergo total reflection and form at ab a small

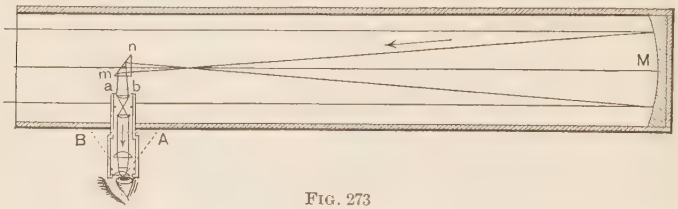


FIG. 273

image of the heavenly body. The image is viewed through an eyepiece inserted in the side of the telescope. The reflector serves as an object glass, and is of course free from chromatic aberration, while spherical aberration is corrected by the shape given the mirror.

391. Photographic camera. The *photographic camera* or *camera obscura*, of which AB (Fig. 274) represents a diagram, consists of a dark box painted black on the interior. A screen of ground glass S forms a partition in the box. A sliding tube T contains

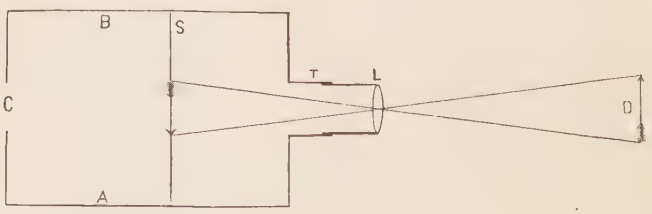


FIG. 274

a convex lens L. If an object D be placed some distance in front, and the distance of the lens from the screen be suitably

[1] On the whole, however, if the matter of expense be disregarded, the balance of advantage is generally considered to lie with the refracting telescope. Briefly, its chief advantages are: (1) it gives a brighter image than a reflector of the same size; (2) it gives a better definition; (3) the lens does not deteriorate with age as does the speculum in a reflecting telescope.

adjusted, a distinct, real, and inverted image can be seen upon the screen by looking through the aperture C. When the image is properly focused the photographer replaces the ground-glass screen by a sensitized plate, and by their chemical power the light waves imprint a latent image of the object on this plate.

The length of exposure should be such that the various opacities of the developed picture shall be proportional to the light intensities from the corresponding parts of the object.

392. Objective. The modern photographic objective has attained a high degree of perfection, and the elimination of the various aberrations, astigmatism, etc., has been made possible in recent years by the skillful combination of several varieties of Jena glass.

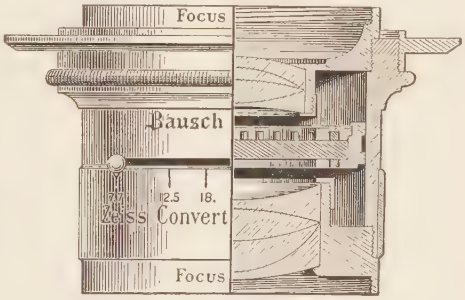


FIG. 275

The most perfect objectives now usually consist of a combination of two separated similar elements, each consisting of from two to four different glasses cemented together in optical contact. Fig. 275 represents one of these highly corrected modern objectives. It is composed of two combinations, each one made up of four single lenses, the glass and shape both being specially selected for its purpose. The two elements are mounted at either end of a short brass or aluminum tube. Between them and appropriately placed is an adjustable iris diaphragm for regulating the amount of light that enters the camera.

393. The sensitive plate. The photographic dry plate has been much improved during the last few years. The old plates failed to represent correctly the relative luminosity of various objects of different colors. A photograph of objects of red, yellow, and blue, all equally bright, invariably represented the blue much brighter than either of the others, the red producing almost no effect at all.

By the *orthochromatic* method all this has been changed. It was discovered over thirty years ago that certain yellow or red dyes in the bromide of silver emulsion served to render it more sensitive to the yellow and green of the spectrum. A year or two later eosin was found to be especially valuable in rendering the emulsion sensitive to the lower colors of the spectrum, even to the red, and from this the orthochromatic photography of the present day has developed.

With a "technically perfect negative," therefore, made possible by this method, a positive print can easily be made that shall correctly represent relatively all the various values of light intensity which the different parts of the scene send to the eye or lens, and this quite independent of the particular colors involved. According to Hurter and Driffield a technically perfect negative is "one in which the opacities of its gradations are proportional to the light emitted by those parts of the original object which they represent."

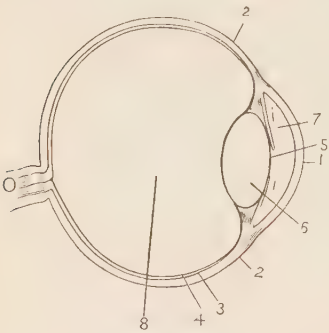


FIG. 276

394. The human eye. Fig. 276 represents a horizontal section of this wonderful organ. Covering the front of the eye, like a watch crystal, is a transparent coat, 1, called the *cornea*. A tough membrane, 2, of which the *cornea* is a continuation, forms the outer wall of the eye, and is called the *sclerotic coat*, or "white of the eye." This coat is lined on the interior with a delicate membrane, 3, called the *choroid coat*: the latter consists of a black pigment, which prevents internal reflection. The inmost coat, 4, called the *retina*, is formed by expansion of the optic nerve *O*. The muscular tissue *ii* is called the *iris*: its color determines the so-called "color of the eye." In the center of the iris is a circular opening, 5, called the *pupil*, whose function is to regulate, by involuntary enlargement and

contraction, the quantity of light waves admitted to the posterior chamber of the eye. Just back of the iris is a tough, elastic, and transparent body, 6, called the *crystalline lens*. This lens divides the eye into two chambers; the anterior chamber, 7, is filled with a limpid liquid, called *aqueous humor;* the posterior chamber, 8, is filled with a jellylike substance, called the *vitreous humor*. The lens and the two humors constitute the refracting apparatus.

The eye is a camera obscura, in which the retina serves as a screen. Images of outside objects are projected by means of the crystalline lens, assisted by the refraction of the humors, upon this screen, and the impressions thereby made on this delicate network of nerve filaments are conveyed by the optic nerve to the brain. If the two outer coatings be removed from the back part of the eye of an ox recently killed, so as to render the back somewhat transparent, true images of whole landscapes may be seen formed upon the retina of the eye, when it is viewed from behind.

With the ordinary camera the distance of the lens from the screen must be regulated to adapt itself to the varying distances of outside objects, in order that the images may be properly focused on the screen. In the eye this is accomplished by changing the convexity of the lens. We can almost instantly and unconsciously change the lens of the eye, so as to form on the retina a distinct image of an object miles away or only a few inches distant. The nearest limit at which an object can be placed so as to form a distinct image on the retina is about five inches. On the other hand, the normal eye in a passive state is adjusted for objects at an infinite distance.

The retina, on careful examination, is found to be composed in part of little elements in its back portion, which have received, from their appearance, the names of *rods* and *cones*. It is thought that these rods and cones receive and respond to the vibrations of ether; in other words, that they covibrate with the undulations of the ether, and thereby we get our sensation of light.

The eye is not free from spherical or chromatic aberration, though these are very much reduced by the action of the iris,

which acts as the diaphragm of an ordinary objective to cut off all except the central rays.

395. Defects of vision. *Myopia* (short-sightedness) is caused by the excessive length of the globe from front to back, so that the images of all but near objects are formed in front of the retina. A remedy may be found in the use of diverging lenses. *Hypermetropia* (long-sightedness) occurs when the axis of the globe is so short that the image of an object is back of the retina unless the object is held at an inconvenient distance, in which case it tends to become indistinct. The remedy for this is converging lenses. *Presbyopia* is due to loss of accommodation power, so that while vision for distant objects remains clear, that for near objects is indistinct. This defect is incident to advancing years, and is due to progressive loss of elasticity of the crystalline lens. The remedy is converging lenses. *Astigmatism* is caused by an inequality in the curvature of the cornea in different meridians, so that when, for example, a diagram like Fig. 277 is held at a distance, vertical lines will be in focus and horizontal lines will be out of focus and will appear blurred and indistinct, or vice versa. Converging lenses will correct this trouble. But for this, as well as for all other defects or troubles of the eyes, consult a skilled oculist, and the earlier the better.

FIG. 277

Advice *to all:* Do not overstrain or overtax the eyes, or use them in insufficient or excessive light, in flickering light such as that of a gas jet, or in unsteady light such as that in a moving vehicle; and avoid so far as practicable sudden changes of light, such as lightning flashes, etc.

396. Stereopticon or projecting lantern. This instrument is extensively employed in the lecture room for producing on a screen magnified images of small, transparent pictures on glass, called *slides;* also for rendering a certain class of experiments

visible to a large audience by projecting them on a screen.[1] The electric light is preferred for most classes of projections. The light from the arc A (Fig. 278) is condensed by means of a convex lens C, called the *condensing lens* (usually two plano-convex lenses

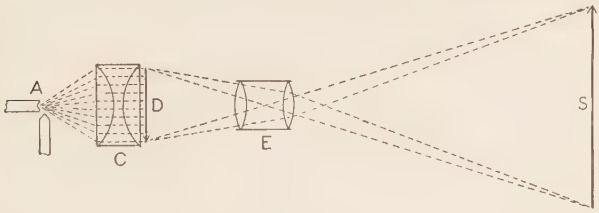


FIG. 278

are used), so that a larger quantity will pass through the convex lens E, called the *projecting lens*. The latter lens produces (or projects) a real, inverted, and magnified image of the picture D on the screen S. The mounted lens E may slide back and forth, so as to focus the image properly.

[1] For useful information relating to the operation of projection, especially for scientific illustrations, see Wright's "Light" and Dolbear's "Art of Projecting."

ENERGY OF ETHER STRAIN: ELECTROSTATICS

SECTION I

INTRODUCTION

397. Electrification. It has long been known that certain bodies, when the conditions are suitable, acquire by contact and subsequent separation, or more readily by friction, because in the act of rubbing more points are brought in contact, the property of attracting light bodies such as feathers, pieces of tissue paper, etc. For example, glass rubbed with silk, and sealing wax or ebonite with woolen cloth, manifest this property by picking up scraps of paper, etc. Because this property was first discovered in amber, the Greek for which is ἤλεκτρον, bodies in this state are said to be *electrified* or *charged with electricity.*

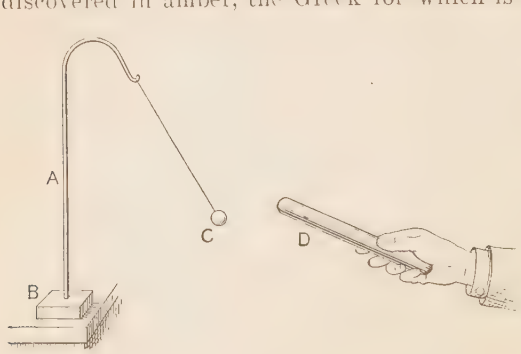


FIG. 279

398. Two kinds of electrification. Suspend a ball of elder pith C (Fig. 279) by a silk thread. Electrify a glass rod D with a silk handkerchief and present it to the ball; attraction at first occurs, followed by repulsion soon after contact. Next excite a stick of sealing wax or a rubber comb with a woolen cloth and present it to the

ball which is repelled by the electrified glass; it is attracted by the electrified wax or rubber.

Suspend in two stirrups two glass rods that have each been rubbed with silk (Fig. 280), and present them to each other; they repel each other. Suspend two sticks of sealing wax that have been rubbed with flannel in the same manner; the same result follows. Now, in a like manner, present one of the glass rods and one of the sticks of sealing wax to each other; they attract each other.

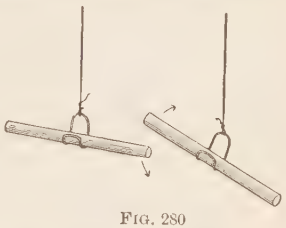


FIG. 280

It is evident (1) that *there are two kinds or conditions of electrification;* or, for convenience, we sometimes say *two kinds of electricity:* (2) that *bodies similarly electrified repel one another, and bodies oppositely electrified attract one another.*

Glass rubbed with silk is said to receive a charge of *vitreous* electrification; the wax, after being rubbed with woolen cloth, on the other hand, is charged with *resinous* electrification. Vitreous and resinous electrifications bear to each other somewhat the same relation as positive and negative quantities in algebra; and by arbitrary convention vitreous charges are said to be *positive* and resinous *negative.*

Once more electrify a stick of sealing wax with woolen cloth and present it to the pith ball; after the ball is repelled bring near it the surface of the flannel which electrified the rod; the ball is attracted by it, showing that the rubber is also electrified, and with the opposite kind to that which the sealing wax possesses.

One kind of electrification is never developed alone; when two substances are rubbed together, and one becomes electrified, electrification of the opposite kind is always developed in the other.

399. A phenomenon of ether strain. When small pieces of glass and silk are rubbed together it is found that after they are pulled apart they attract each other with a definite and measurable force, and that this force varies inversely as the square of

the distance between them. When two bodies are pulled apart, energy is expended upon them which will be restored when they are allowed to approach each other. This phenomenon is explainable on the hypothesis that in the work of separation the ether between or around them is strained; and that the tendency of the two bodies to approach each other is the tendency of the elastic ether to recover its normal condition. The phenomena of electric attraction and repulsion are most satisfactorily explained on the hypothesis of ether strain. By whatever hypothesis explained, it is certain that *electrification is the result of work done, and is a form of potential energy.*

400. What is electricity? The student naturally has already begun to ask the never-answered question, What is electricity? and to inquire, What is the function of electricity in these operations? What the ultimate nature of electricity is, whether it be the ether itself or a constituent of the ether, "as water is a constituent of jelly"; whether it be a fluid (it certainly possesses the property of fluidity); whether, according to Franklin, a positive charge is an excess and a negative charge a deficit in a certain standard quantity of the fluid which all bodies are supposed to possess in their unexcited state; or whether (more probably) positive and negative electricities are distinct entities (whose relations to each other are more like those of sodium and chlorine than like those of heat and cold), such that when combined in any body they neutralize each other so that the body possesses none of the properties of electrification, but when they by any means become separated two separate portions of matter always become oppositely charged — are questions too recondite for discussion in any general work on physics. Electricity is *not* a form of energy. It is quite true that electricity *under pressure* or *in motion* possesses energy; in the same sense do water and air under like conditions possess energy, but we do not therefore deny them to be forms of matter. Electricity, rather, in many respects possesses the nature of matter. Like matter it can neither be created nor annihilated, and like matter it can be

moved and put under stress. For present purposes, then, electrification may be regarded as a state of strain in some associated medium, produced by a transfer of electricity from one body to another and the subsequent separation of the two bodies. Electrification is the result of work done, and is most certainly a form of energy.

401. Quantity of electricity. When we do not know what a thing is, it is difficult to conceive a definite quantity of it. But our knowledge of electricity, like that of force, is derived from its effects. From the measurement of its effects, therefore, we may define a unit quantity of electricity. For purposes of calculation at least, electricity of either kind may be treated precisely as if it were a *material incompressible fluid*, and any increase or decrease of electrification may be considered to be produced by the addition or taking away of a quantity of electricity. Quantities of electricity are added and subtracted by the usual rules of algebra, the kinds of electrification being denoted by the + and − signs.

402. Law of attraction. When two equally electrified bodies attract or repel each other with a force of one dyne at a distance of one centimeter in air, each is charged with a certain definite quantity which may be taken as a unit quantity of electricity. If two bodies repel or attract each other with some other intensity, the quantities with which they are charged are easily determined. For example, suppose that a body charged with three units is attracted at a distance of one centimeter by one charged with six units. The total attraction of the six units of the second body for each one of the other three is obviously expressed by 6, giving a total attraction of $6 \times 3 = 18$ dynes. It is evident that if any two of these three quantities be known, the third can be determined.

Again, suppose that the attraction (or repulsion) be at some other distance; the force, being a radiant one, varies inversely as the square of the distance. Consequently, to determine this force the product of the two quantities must be divided by the

square of the distance between them. Now if we substitute quantities of electricity q and q' for masses m and m' in the formula for attraction of gravitation, we shall have the formula for electrical attraction (or repulsion) f; namely $f = \frac{qq'}{kd^2}$, in which k is the dielectric constant. That is, the electrical attraction between two charged bodies (provided the areas of the bodies are small, so as to keep them under the law of radiant force) varies as the products of their charges and inversely as the square of the distance between them.

403. Electroscope. This is an instrument used to detect the presence of electrification in a body, and to determine its kind. It usually consists of two strips of gold foil, A and B (Fig. 281), suspended from a brass rod within a glass jar. To the upper end of the rod is fixed a metal disk C. On the opposite sides of the interior of the jar are two strips of metal foil D and E, of sufficient height to be touched by the strips A and B on their extreme divergence.

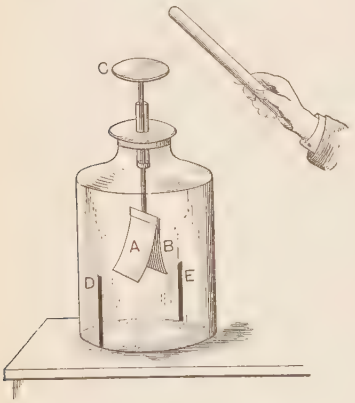


FIG. 281

(1) If an unelectrified body be brought near the disk C, no change takes place in the two strips of foil A and B, but if an electrified body be brought near the disk, the strips diverge, thus indicating the existence of a charge of electricity in the body.

(2) If the electroscope be charged by contact with an excited body, the strips will remain in a divergent position. While in this condition, if a body similarly charged be brought near the disk, the strips will diverge *more;* but if an unexcited body or a body oppositely electrified be brought near the disk, the strips will collapse.

404. Conduction. Rub a brass tube, held in the hand, with warm silk. Bring it near the disk of the electroscope; the leaves are unaffected. Wrap a piece of sheet rubber around one end of the tube and hold this end in the hand and rub as before. Bring it near the disk of the electroscope; notice that now the leaves diverge. Repeat the last operation, but touch the tube with the finger before bringing it near the disk. The leaves no longer show signs of electrification.

In the first and last operations electricity escaped through the hand and body to the earth; in the second it was prevented from escaping by the intervening sheet rubber. Substances which allow electricity to spread over them, i.e. substances which offer little resistance to the flow of electricity, are called *conductors*. Those which offer great resistance to its passage are called *non-conductors, insulators,* or *dielectrics.*

Some of the best insulating substances are *dry air, ebonite, shellac, resins, glass* (free from lead, e.g. common bottle glass), *silks,* and *furs.* On the other hand, metals are, as a class, exceedingly good conductors. Moisture injures the insulation of bodies; hence experiments succeed best on dry, cold days of winter, when moisture of the air is least liable to be condensed on the surfaces of apparatus, *especially if it be kept warm.*

Water cannot be retained in a reservoir unless its walls be of sufficient strength; so a body, in order to become charged and to retain the charge, must be surrounded by something that will offer sufficient resistance to the escape of electricity. As regards a specific body, there is no limit to the quantity of electricity with which it can be charged, provided the charge can be retained. This entity, which represents the walls of the reservoir, is termed the *dielectric.* It may be the air[1] or any of the so-called non-conductors of electricity. Even the ether may be considered a dielectric. A body thus surrounded is said to be *insulated.*

[1] If the air were a conductor of electricity, a body could not be charged in it; there could be no thunderstorms, and man would probably never have known of the existence of electricity.

SECTION II

INDUCTION

405. Electricity acts across a dielectric. Fig. 282 represents a ping-pong ball covered with tin foil to make it a good conductor. It is suspended from a glass rod by a silk thread. (1) Electrify a glass rod and bring it near the ball, which moves towards it. (2) Next introduce a glass plate between the rod and ball. The ball approaches the rod as before.

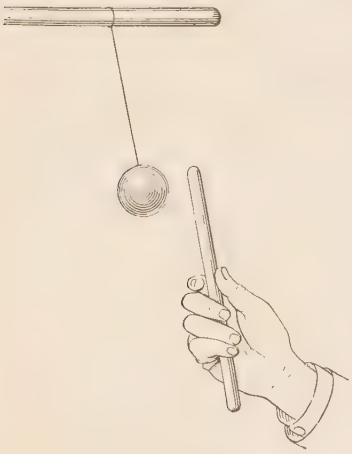


FIG. 282

The chief lesson we learn from this experiment is that *electricity acts across a dielectric.*[1] In the first case the dielectric was air; in the second, air and glass.

406. To examine the influence of a charge on an insulated conductor. (1) Suspend, as above, two balls, *A* and *B* (Fig. 283), so as just to touch each other, thus making practically one conductor. Bring near to one end of the pair a sealing-wax rod *D* excited negatively. While the rod is in this position carry a thin strip of tissue paper *C* around and near the balls. It is attracted to them,

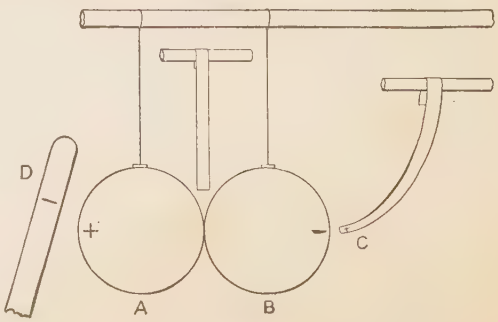


FIG. 283

[1] Insulators across which electric action takes place are called dielectrics, from the Greek *διά*, across.

but most strongly at the ends. In the middle of the conductor, where A and B touch, there is little if any electrification.

(2) While the rod D is in position separate B from A, and then remove D. Test each ball with the tissue paper; both are found to be excited.

(3) Charge an electroscope positively, and then bring A near it; the leaves diverge, showing that A is charged positively. Bring B near the electroscope; the leaves collapse, showing that B is charged negatively.

(4) Finally bring the two balls near each other; they attract each other. Allow them to come in contact, and then test each with the tissue paper or the electroscope; it will be found that both have become discharged.

From the above operations we learn that when an electrified body is brought near but not in contact with an insulated conductor, it acts across the dielectric upon the conductor, repelling electricity of the same kind to the remote side of the conductor and attracting the opposite kind to the side near to it. Such electrical action is called *induction*. The electrified body which produces the action is called the inducing body; the charge of electricity thus produced is called *induced electricity*.

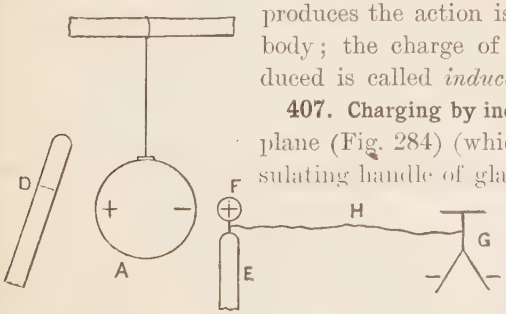


FIG. 284

407. Charging by induction. Take a proof plane (Fig. 284) (which consists of an insulating handle of glass or gutta-percha E, terminating at one end with a thin metal disk F about the size of a 5-cent piece) and connect it with an electroscope G by a fine wire H. Bring a stick of sealing wax electrified negatively as before near the eggshell conductor. Holding the proof plane by the insulating handle, bring the disk near the end of the conductor charged by induction negatively. The negative charge will act inductively upon the

continuous conductor consisting of disk, wire, and electroscope, charging the end nearest itself (i.e. the disk) positively and the remote end (i.e. the leaves) negatively. The leaves of the electroscope show the presence of a charge by their divergence.

Now, while everything is in the position indicated by the cut, touch with the finger any part of the continuous conductor; the leaves of the electroscope instantly collapse. The negative charge with which the leaves had been charged, being *free*, is discharged through your body; but the positive charge concentrated on the disk of the proof plane is *bound* by the attraction of the negative charge on the end of the ball nearest it, and cannot escape. Remove the finger from the electroscope and the proof plane from the influence of the ball; the leaves again diverge.

The last phenomenon is explained as follows: After the negative charge had been discharged from the continuous conductor, there was left an excess of positive electricity; but this excess was all concentrated in the disk F so long as it remained near the negative charge of the ball. But as soon as F was removed from the influence of the ball the charge spread itself over the entire conductor, and the leaves, which received a portion of the charge, diverged. The conductor is said to be charged by *induction*.

To electrify the ball by induction, bring the excited wax near it, touch the ball with a finger, remove the finger, and finally remove the rod. The proof plane being connected with the electroscope and being charged negatively, bring F near to the ball A; the leaves collapse, showing that the ball is charged positively, which draws the negative charge away from the leaves.

Observe that when a body becomes charged by induction the charge which it receives is *opposite in kind to that of the inducing body*.

408. Charging by conduction. Disconnect the proof plane from the electroscope. Charge the electroscope negatively and the ball positively; touch the ball with the disk of the proof plane, then hold the disk near the electroscope; the divergent leaves collapse, showing that the disk bears a positive charge which

it received by *conduction* from the ball when they were brought in contact. Of course the charge is of the same kind as that of the body which communicated it.

409. Induction precedes attraction. When a pith ball is brought near an electrified glass rod the positive charge on the rod A (Fig. 285) induces a negative charge on the side of the ball B nearest A and a positive charge on the farther side. The positive charge of A and the negative charge of B therefore attract each other; likewise the positive charge of A and the positive charge of B repel each other; but since the former charges are nearer each other than the latter are, the attraction exceeds the repulsion.

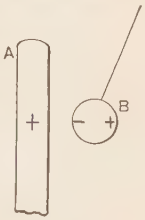


FIG. 285

410. Electrification confined to the outside surface of a conductor. Metal screens. Place a tin cup A (Fig. 286) on a glass tumbler coated with shellac and charge it heavily with electricity from an electrical machine. Introduce a proof plane into the cup and touch the interior surface of the cup. Remove the proof plane and place it near the electroscope; *the leaves of the electroscope are unaffected.*

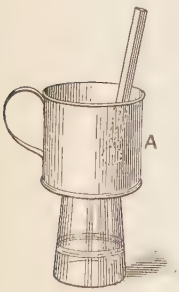


FIG. 286

If a conducting ball A (Fig. 287), suspended by an insulating thread, be electrified and then covered with two hemispherical metallic cups, B and C, having insulating handles, and the cups be afterwards removed, the ball when tested with the electroscope will be found to have lost all its charge, while the cups will be found to be charged. It does not make the slightest difference to the result whether an insulated conductor be solid or hollow. Wood covered with tin foil answers the purpose as well as any other body.

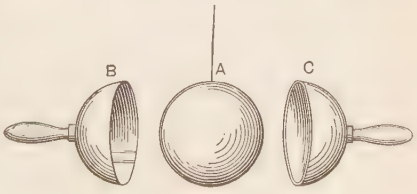


FIG. 287

If a hollow conductor be charged, however highly, with electricity, the whole of the charge is found upon the outside surface. If the electroscope in the last experiment were placed inside the tin cup, or if it be set inside a vessel of wire gauze (e.g. a bird cage) and the vessel be charged with electricity, or a heavily charged body be brought near the vessel, the electroscope will be unaffected. This interesting and important fact shows that *a metallic shell, however thin, entirely screens bodies inside it from external electrification, however great.*

To test this still further, Faraday built a cubical cage 12 feet on an edge, of copper wire, and lined the interior with paper covered with tin foil. This chamber was insulated and put in connection with a powerful electrical machine while working. He says: "I went into the cube and lived in it, using electrometers and all other tests of electrical states; I could not find the least influence upon them, though all the time the outside of the cube was powerfully charged, and large sparks and brushes were darting from every part of its outer surface."

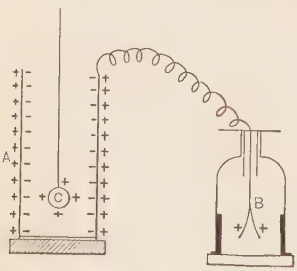


FIG. 288

411. Faraday's "ice-pail experiment." (1) Insulate well a tin pail (Fig. 288) and connect it with an electroscope. Heavily charge a conducting ball suspended by an insulating thread with, say, a positive charge. Lower the ball within the pail; the pail becomes charged by induction, the inside with a negative charge and the outside (together with the electroscope) with a positive charge. The leaves of the electroscope diverge.

(2) Touch the outside of the pail with a finger; the free positive charge escapes to the earth, and the leaves of the electroscope collapse. Remove the finger and then withdraw the ball slowly from the pail; the leaves of the electroscope slowly diverge, and remain practically in the same diverged condition after the ball is removed.

(3) The ball is still charged positively and the pail negatively. Bring the ball in contact with the pail; the leaves of the electroscope completely collapse, showing that the positive charge and the negative charge have combined, neutralizing each other and leaving no excess of either. Hence we conclude that the positively charged ball, when lowered into the pail, must have induced an equal negative charge in the pail. This is generally called the "ice-pail experiment," because Faraday in the original experiment used an ice pail.

The amount of opposite electricity induced on surrounding conductors by an electrified body is equal to the body's own charge. One body cannot be charged with a quantity of positive electricity without an equal charge of negative electricity being established somewhere else, and vice versa. The student will bear in mind that whenever in his experiments he charges any body with electricity, an equal complementary charge always exists distributed over neighboring objects or on the walls of the room. When a thunder-cloud is charged it has its equal complementary charge in the part of the earth nearest it. The sum total of all the positive and negative electricity in existence is zero.

412. Definition of electrostatic induction. We are now in a position to understand the following definition: *Electrostatic induction is the action whereby a charged body surrounded by a dielectric evokes an equal and opposite charge on the inner surface of the inclosure containing the body and the dielectric.*

413. Inductive capacity. The power of transmitting induction varies with different substances. Across glass, sulphur, and shellac the effect produced by an electrified body is different from that across air, the distance being the same. The power of a dielectric substance to receive and transmit that electric strain which we call induction depends on *the specific inductive capacity*, or the dielectric constant, of the substance.

When we electrify a body a certain quantity of energy is expended, and this is regarded as the energy of the electric charge, and may be recovered by discharging the body. The energy is, however, stored in the ether around the body said to be charged.

SECTION III

DISTRIBUTION OF ELECTRICITY

414. Electric density. The following experiments by no means give exact quantitative measurements, but they are suitable for our purpose.

Suppose a well-insulated metal sphere charged positively. Touch some point on the surface with a proof plane and bring it in contact with an uncharged electroscope. Notice the amount of divergence. Discharge the proof plane and electroscope and touch a different point on the sphere with the proof plane; then touch the electroscope as before. Notice that *there is equal divergence of the leaves.*

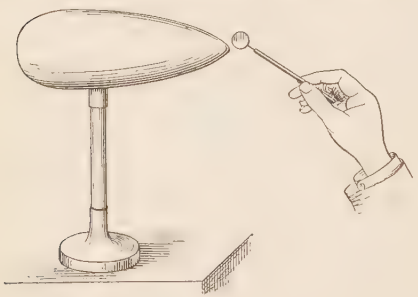


FIG. 289

Again, (1) electrify an insulated pear-shaped conductor (Fig. 289). Touch the larger end of the conductor with a proof plane and bring it in contact with an electroscope. Notice the amount of divergence of the leaves.

(2) Discharge the proof plane and electroscope. Touch the pointed end of the conductor; notice that on bringing the proof plane in contact with the electroscope the divergence of the leaves is much greater than before. In this case, therefore, the charge is not uniformly distributed.

We conclude that the electricity is *equally* distributed on the sphere, but *unequally* on the pear-shaped conductor. *Electric density at any point on a conductor is defined as the quantity of electricity per unit area around that point.*

It follows, from this definition, that if the surface be increased, the quantity of electricity being the same, the density is diminished, and vice versa. The relation between density and area is

readily shown in the following manner: Suspend a sheet of tin foil from a glass rod (Fig. 290), connect the lower end of the foil with an electroscope, and charge the foil lightly. Roll the foil up on the rod, and as the surface becomes reduced the leaves diverge more widely.

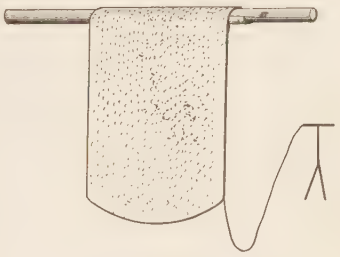


FIG. 290

Distribution of electrification, or the electric density at different points on a conductor, depends on its shape. In Fig. 291 the distances between the surfaces of the bodies and the dotted lines are intended to represent approximately the relative densities at different parts of each body.

415. Effect of points. As bodies become pointed the electric density increases at the pointed end until it becomes so great that the electricity is discharged. The particles of air surrounding the point become heavily charged and are repelled; other particles rush in to take their place, and they in turn are electrified and repelled. A current of air proceeding from the point,

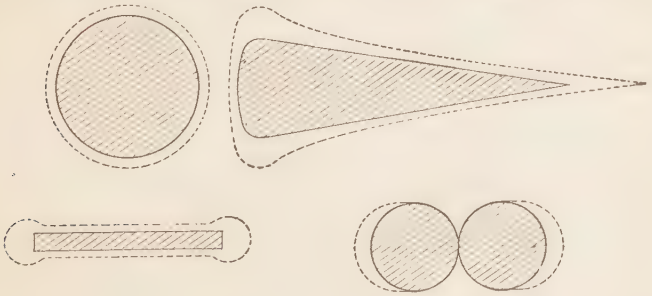


FIG. 291

called the "electric wind," is thus produced, and the conductor becomes discharged by a process somewhat analogous to convection of heat. Points or edges cause leak, and so are usually avoided.

SECTION IV

ELECTRICAL POTENTIAL

416. Electrostatics and electro-kinetics. Electricity may be at rest, as in a charged body, or it may be in motion, as in the case of a charged body connected by a conductor with the earth, when it is discharged through the conductor to the earth. It will be shown later on that as long as a flow of electricity continues the conductor along which it flows has properties different from those of a simple electrified body. That branch of electrical science which treats of the properties of simple electrified bodies is called *electrostatics*, because in them electricity is supposed to be *at rest;* and that branch which treats of electricity in motion is called *electro-kinetics*.

417. Potential. The fundamental fact of electricity is that we are able to place bodies in different electrical conditions. A charge of electricity, which implies an abnormal electrical condition, is the foundation of all electrical phenomena. We are now to discuss in a very simple manner the meaning and use of the very important term *potential* with reference to electricity.

(1) When a charged conductor is connected with the earth a transfer of electricity takes place between the body and the earth.

(2) If the body be charged positively, we say arbitrarily that electricity passes to the earth; but if the body be charged negatively, electricity passes *from* the earth *to* the body.

(3) If two insulated charged conductors be connected with each other, electricity may or may not pass from one to the other. Now whether electricity passes from one to the other, and in what direction it passes, if at all, depends upon the so-called *potentials* of the conductors.

(4) If two bodies are of the same potential, no transfer of electricity takes place between them when they are connected by a conductor; but if the two bodies are of different potentials, there

will be a transfer, and the body *from* which the electricity flows is said to be at a higher potential than the one *to* which it flows.

418. Definition of potential. The potential of a conductor may therefore be defined provisionally as the electrical condition of that conductor which determines the direction of the transfer of electricity.

The term *potential* is relative; i.e. we compare the potential of one body with that of another.

It is important to have a standard of reference whose potential is considered to be zero, just as it is convenient in stating the elevations and depressions of the earth's surface to give the distances above or below sea level, which is taken as the zero of height. For experimental purposes the earth is usually assumed to be at zero potential. A body charged positively is understood to be one that is at a higher potential than that of the earth, and a body charged negatively is one that is at a lower potential than that of the earth.

419. Analogies. Potential is analogous in many respects to (1) temperature, and (2) liquid level.

(1) When we say that the temperature of air is 20° or − 10° C. we mean that its temperature is 20° above or 10° below the standard temperature of reference, namely that of melting ice. If two bodies at different temperatures be placed in thermal communication, heat will pass from the body at a higher temperature to the one at a lower, and will continue to do so until both are at the same temperature.

(2) If two vessels containing water at different levels be put in communication at their bottoms by a pipe, water will flow from the one at a higher level to the one at a lower until the water is at the same level in both vessels.

Temperature is not heat; level is not water; and potential is not electricity, but merely the state of the conductor which determines the direction of transfer of electricity.

All points of a conductor, when the electricity upon it is at rest, are at the same potential, regardless of any difference of density

which may exist at different points. If it were not so, there would be a continual flow of electricity from the higher to the lower until equilibrium was established, i.e. until all points were at the same potential.

Charge a pear-shaped body A (Fig. 292) with electricity. *Connect* a proof plane with an electroscope and touch the charged conductor with the proof plane at different points; the leaves diverge just the same at all points touched, thus showing that the potential at all points is the same, although the density at different points varies.

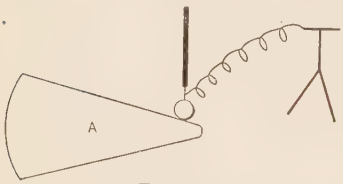


FIG. 292

Observe that it is difference of potential, or simply potential, and not quantity or density, which determines a flow of electricity. Water does not flow from a larger or deeper pond into a smaller or shallower one unless there is a difference of level.

420. Electrical capacity. If two conductors of the same shape, and surrounded by the same dielectric, be charged, it will be found that the larger one requires a larger charge than the smaller one to electrify it to the same potential; i.e. the larger one has a greater *electrical capacity* than the smaller one. Hence the potential of a conductor depends upon its charge and its capacity. If C = the capacity of a conductor, Q = the quantity or charge of electricity, and V = the potential, then

$$C = \frac{Q}{V}.$$

From this we see that *the capacity of a conductor is equal to the charge necessary to raise its potential from zero to unity.*

The capacities of spheres are found to be proportional to their radii. Thus if a sphere charged with 20 units of electricity, and having a radius of 4 inches, be brought in contact with an uncharged sphere having a radius of 1 inch, and these be afterwards separated, the quantity on the large one will be 16 units, and that on the small one 4 units.

SECTION V

INDUCTION. ELECTRICAL MACHINES

421. Electrophorus. This apparatus is used to produce electrification by induction. It consists of a shallow metal dish A (Fig. 293) filled with a good dielectric like hard rubber or sealing wax. At the center of the dish is a conducting protuberance B, which extends just through the wax. A flat metal disk C, a little smaller than A, has a glass insulating handle.

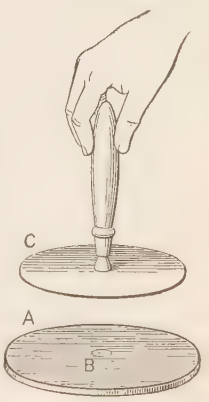


Fig. 293

If now the wax be struck a few times with a cat's fur, or rubbed with a dry flannel, it becomes electrified negatively. Place the disk C upon it. The positive charge of the disk is *bound* by the negative charge of the wax, but the negative charge of the disk is repelled by the negative charge of the wax and passes through the protuberance B to the dish below, and thence to the earth. Consequently when the disk C is raised by the insulating handle from the wax, it is charged positively, and the charge can be transferred to any body, e.g. a Leyden jar (see § 424), and then the disk can be recharged by replacing it on the wax. This may be repeated many times without sensibly decreasing the negative charge on the wax.

422. Continuous electrophorus. Töpler-Holtz machine. Charging by means of the electrophorus like that described above is necessarily intermittent. The various modern induction electrical machines are devices by which the principle of the electrophorus is continuously applied. A single one of these, the Holtz machine, improved by Töpler (and hence known as the Töpler-Holtz type), will suffice for our study.

Fig. 294 represents this machine in perspective, and Fig. 295 is a diagram of its essential parts. On the back of a stationary

glass plate *FF*, called the *field plate*, are pasted two quadrants of varnished paper *I* and *I*, called the *inductors*. In front of the field plate is a revolving glass plate upon which are pasted equidistant tin-foil *carriers* a, a, a', a', n, n', having a flat metal button on the center of each. Two brushes of tinsel, C' and C', connected with the inductors, are so supported as to touch the buttons as they pass, and thus a connection is established between the buttons in temporary contact with the brushes and the inductors.

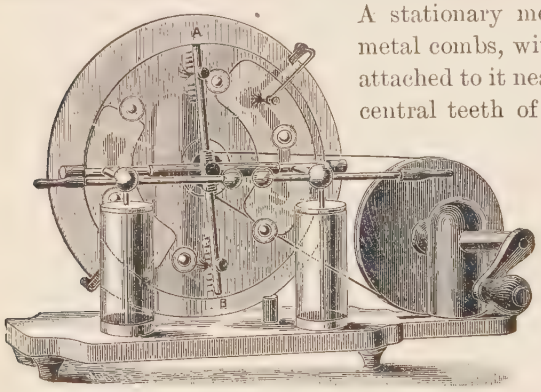


FIG. 294

A stationary metal rod *AB* has metal combs, with pointed teeth, attached to it near each end. The central teeth of these combs are removed and replaced by tinsel brushes. This rod serves as a conductor between the two buttons on the same diameter, and may be called the *neutralizing conductor*. A second pair of combs, C'' and C'', are connected with the separable discharging conductor KK'. Connected with each part of this conductor is a Leyden jar or condenser.

The mere contact between unlike substances is sufficient to produce a very small incipient charge which, as the plate revolves, rapidly increases to a maximum. In starting this action the two parts of the discharging conductor are usually brought in contact. As the two parts K and K' become oppositely charged they may be separated farther and farther apart, and discharges between the two extremities, in the form of sparks and brushes, occur at intervals, which increase in duration with an increase of distance. By the addition of Leyden jars, which also become oppositely charged, the amount of charge previous to each discharge

is increased, and consequently the energy of the discharge and the brilliancy of the spark are increased, though the discharges are less frequent.

As the plate rotates the two inductors are kept constantly and oppositely charged, and as two opposite carriers (n and n', for example) are about to leave the inductors the following takes place: At n the positively charged inductor acts through the glass upon the carrier and comb, attracting and binding the negative electricity and repelling the positive. Similar action takes place at n', but with opposite signs. These repelled charges unite through the conductor AB and neutralize each other, leaving the carriers n and n' charged respectively with negative and positive electricity. As n and n' move away from the inductors their charges become free, and on reaching the brushes C and C' they communicate a portion of their charges to the brushes to make good any losses by leakage or otherwise which the inductors may sustain. We are now able to see how the charges of the inductors are received and maintained.

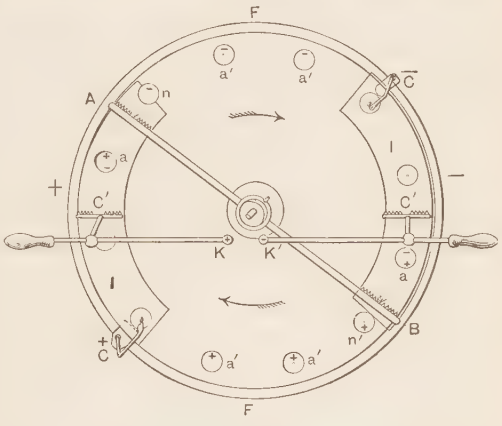


FIG. 295

We now turn our attention to the discharging conductor. The two inductors act inductively upon the two parts K and K' of the conductor, charging K positively and K' negatively. The work required to keep up the motion of the revolving plate increases as the charges rise, as there is a constant pulling apart, at different points, of bodies oppositely charged. Thus mechanical energy becomes transformed into electric energy, or the energy of ether strain.

After a Töpler-Holtz machine has charged a battery of Leyden jars, i.e. stored up in the jars electrical energy, the belt may be slipped from the machine (to reduce friction), when the battery will drive the machine, reconverting the energy into mechanical energy.

423. Condenser. A very important adjunct to an electrical machine is a *condenser* of some kind, by means of which a large quantity of electricity can be collected on a small surface.

As we have seen, the ratio of the charge of a conductor to its potential is its capacity, or

$$C = \frac{Q}{V}.$$

If now by some means the conductor could be made to take a larger charge without increasing the potential, its capacity would be greater. This can actually be done by bringing near to an insulated conductor another conductor connected to the earth.

FIG. 296

Let A (Fig. 296) represent a conductor charged positively, and let its potential be V. Its capacity will then be $\frac{Q}{V}$. If we can lower the potential without decreasing Q, C will, of course, be increased thereby. Suppose B, joined to earth, be moved up near to A; a negative charge will be induced on B. Now it is shown in special works on electricity that the potential of a conductor is equal to the amount of work that must be done to move up to it from the earth (or place of 0 potential) a unit positive charge. It is clear, then, that with B near A, less work would be required to move a plus charge up to A than before, because B by its attraction will assist by neutralizing some of the repulsion exerted by A. Hence A's potential has been lowered by the presence of B, and its capacity increased. It will therefore require a larger charge to raise it to the same potential as it was before B was introduced. Such an A-B combination as has been described is known as a *condenser*. Since $C = \frac{Q}{V}$, it is evident that the increase in capacity

of the conductor is not due to an increase of potential. An electrical condenser may be regarded as an appliance for increasing the charge without increasing the potential.

424. Leyden jar. A convenient form of condenser for some purposes is the Leyden jar.[1] It consists of a glass jar (Fig. 297) coated with tin foil both inside and outside to about two thirds its height. A brass rod passes inside through a varnished wooden stopper, touches the inner foil, and terminates in a brass knob on the outside.

Fig. 297

The jar may be charged by connecting one of its coatings with the conductor of an electrical machine and the other with the earth; or it may be charged by connecting the outside coating with one of the discharging conductors of the Holtz machine, and bringing the other pole near to the ball leading from the inner coating. To discharge the jar, connect the outer coating with the knob of the jar. To avoid a shock in so doing, a *discharger* is used (Fig. 298), which consists of a bent wire terminating at each end with metal balls. The wire is held by a glass insulating handle.

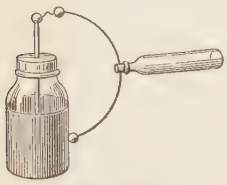


Fig. 298

425. Capacity of a condenser. The capacity of a condenser is proportional (1) to the area of the metallic conductor; (2) to the specific inductive capacity of the dielectric; and (3) is inversely proportional to the thickness of the dielectric. The low inductive capacity of some kinds of glass renders it entirely unsuitable for this purpose.

To secure greater capacity than a single jar of ordinary size will afford, several jars, constituting a "battery" of jars, are placed upon a sheet of tin foil so as to connect all the outer coatings, while the inner coatings are connected by a wire joining their projecting rods. By this means the several jars are practically converted into one large jar.

[1] So called because one of the first jars was constructed (by Cuneus) at Leyden, Holland (1746).

426. Condition of the dielectric; seat of charge. That inductive action is attributable to the dielectric, and not to the conductor, is shown by the Leyden jar with movable coatings (*A*, Fig. 299). *B* is the dielectric; *C* is the outer and *D* the inner conductor. The several parts being put together, the jar is charged in the usual manner and placed upon an insulator. Then the inner conductor *D* is raised out of the jar by a glass rod, and afterwards the glass vessel *B* is removed from the outer coating. The several parts are now tested with an electroscope. The coatings produce

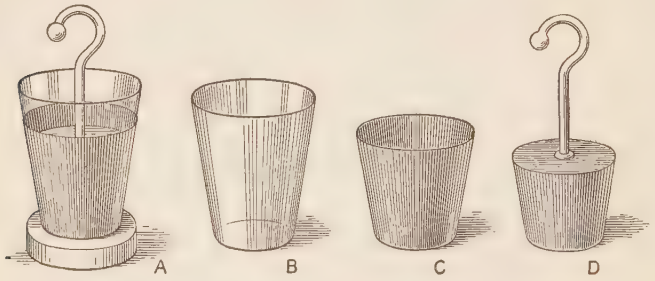


FIG. 299

little or no disturbance of the leaves; the glass, however, causes a divergence of the leaves. On putting the parts together again and discharging in the usual way, there will be nearly as brilliant a spark as if the charged jar had not been dissected.

This experiment demonstrates that (1) the seat of the charges is on the surface of the glass and not on the coatings; (2) the coatings serve merely the purpose of conductors, — to spread electricity at the time of charging, and to allow its escape from all parts of the electrified surfaces at the time of discharge; (3) a charge is not an electrification of the conductors, but rather of the dielectric, or, as we shall say later on, of the "field" itself, the extent of the conductor determining the limits of the field.

427. Limit of the charge. There is a limit beyond which a condenser cannot be charged. When electrification takes place the stress produced by the opposite charges causes a strain in the

dielectric. When the strain becomes too great a discharge occurs across the dielectric, either through the air over the top of the jar, or, if the dielectric be thin enough, by puncturing it.

SECTION VI

ELECTROSTATIC FIELD OF FORCE

428. Contact action. Bring two bodies of dissimilar nature (e.g. a stick of sealing wax and a woolen cloth) in contact — best by rubbing to secure better contact — and separate them, and they exhibit strong attraction for each other, ordinarily vastly greater than that of gravitation. The contact serves to establish bonds of attraction, i.e. the ether between them is supposed to operate like india-rubber bands, pulling the two bodies together. The bodies are thus said to be electrically excited. To separate the excited bodies requires work, and the bodies when separated possess energy of electrical separation.

429. Lines of force; field of force. The space or dielectric between and, to a limited extent, around the excited bodies is assumed by Faraday to be full of what he called *lines of force*, the positive direction of a line at any point in this space being the direction in which a positively electrified particle tends to move under the influence of the electrical field. The space thus occupied by lines of force is called the *field of force*. Faraday remarks that the stress is as if these lines were stretched elastic threads endowed with the property of shortening themselves and also with the property of repelling one another as well. In other words, there is a tension along these lines and a pressure at right angles to them. When bodies oppositely excited are brought near together the lines are almost straight from one to the other (except near the edges) of the facing areas, as shown in Fig. 300. As they are more separated the lines curve outward, always tending to separate from one another and from the common axis of the two bodies,

FIG. 300

some even curling round to the back of the bodies, as represented in Fig. 301. The expression "lines of force" must be regarded as purely a matter of convenience. They have no more and no less existence than have "rays of light."

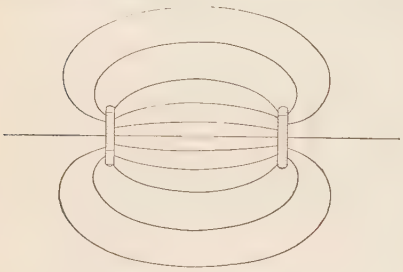


FIG. 301

SECTION VII

ATMOSPHERIC ELECTRICITY

430. Lightning. Franklin, by a series of historic experiments, proved the exact similarity of lightning and thunder to the light and crackling of the electric spark. Certain clouds which have formed very rapidly are highly charged, usually with positive electricity, but sometimes with negative. The surface of the earth and objects thereon immediately beneath the cloud are, of course, charged inductively with the opposite kind of electricity. The cloud and the earth correspond to the coatings, and the intervening air to the dielectric, of an immense condenser. The opposite charges on the earth and on the cloud hold each other prisoners by their mutual attraction.

As condensation progresses in the cloud its capacity decreases and its potential rises (since $CV = Q$). This process continues till the difference of potential between the cloud and the earth becomes great enough to produce a discharge through the air.

It is the accumulation of induced charges on elevated objects, such as buildings, trees, etc., that offers an intensified attraction for the opposite electricity of the cloud in consequence of the greater proximity of these objects, and renders them especially liable to be struck by lightning.

The clouds gather electricity from the atmosphere. Our knowledge of the method by which the atmosphere becomes charged is very limited.

431. Lightning rods. A *good* lightning conductor offers a peaceful means of communication between the earth and a cloud; it leads the electricity of the earth gently up toward the cloud, and allows it to combine with its opposite without disturbance, thereby so far discharging the cloud as possibly to prevent a lightning stroke ; or, if the stress be too great to be thus quietly disposed of, the flash strikes downward, and is led harmlessly to the earth by the conductor. *An ill-constructed lightning rod may be worse than none.* A good rod should be made of good conducting material, so large that it will not be melted, and should be free from loose joints. The lower end should be buried in earth that is always moist, and the upper end should terminate in several sharp points. Insulators should not be used, and all large masses of metal should be joined to the rod system. Ground the wires if possible by joining to the gas and water pipes. Maxwell suggests that the best form for a lightning protector is one which approximates a network covering the entire house.

432. The aurora. This is a luminous phenomenon caused (as experiments performed by Lemström in Lapland seem to indicate) by currents of electricity passing from the higher and rarefied regions of the atmosphere to the earth. In the Arctic regions the aurora borealis (northern lights) is of almost daily occurrence. It sometimes forms an arch, and sometimes illuminates the whole sky.

ENERGY OF ELECTRICITY IN MOTION: ELECTRO-KINETICS

SECTION I

433. In the sections just concluded electric charges or quantities of *electricity at rest* have been considered somewhat in detail, and it will be our duty now to study in general the effects produced by charges in motion. A charge may move either because the body on which it is located moves and carries it, or it may move over or through the material medium with which it is associated. In the latter case the medium is a conductor, as before defined. Other things being equal, the result is the same however the charge is made to move.

It has been seen that a charge tends to move from a place of higher to one of lower potential, so that if we have any two bodies at different potentials joined by a conductor (a copper wire, for example), a charge on the body at higher potential will pass through the conducting path to the other body. If now the charge on the body at higher potential be *continuously* renewed, there will be a continuous flow of electricity from the higher to the lower potential.

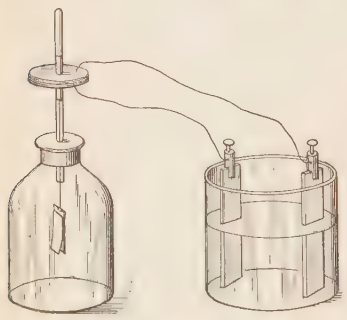


FIG. 302

434. Volta's contact theory. By means of a condensing electroscope (Fig. 302) Volta proved that when two different metals are brought into contact and then separated, they are oppositely

charged with equal quantities. For example, if zinc and copper be used, the zinc will be positive and the copper negative. In air nothing further happens. The air being a nonconductor, the ether is simply strained slightly by the opposite charges on the two metals.

If, however, the metals be placed in a liquid of a class which we shall term an *electrolyte* (i.e. one which is capable of being decomposed by a current of electricity), there is a difference of electrical condition produced between them so that the metal becomes either of higher or of lower potential than the liquid, according to the nature of the metal and liquid.

Under these conditions electricity tends to flow from the one whose potential is higher to that whose potential is lower; if, therefore, two dissimilar metals be placed in the same electrolytic liquid, and we show by actual experiment that the free end of the wire in connection with one plate is charged positively and the free end of the other negatively, we conclude that if the two oppositely charged bodies be brought in contact, a current of electricity will flow from the positively charged plate to the negatively charged one. A current therefore flows through the connecting wire from the copper (which is called the *positive electrode*) to the wire leading from the zinc (which is called the *negative electrode*), when they are connected.

That difference in quality, in virtue of which zinc and copper placed in an electrolyte, e.g. water acidulated with sulphuric acid, can give rise to an electric current, is called their *electro-chemical difference*, and the zinc is said to be electro-positive to the copper in the liquid. There is a perplexing nomenclature in use by which the zinc plate is called the electro-positive element or plate, though it is called the negative *pole* of the combination.

SECTION II

VOLTAIC BATTERIES. ELECTRIC CIRCUITS

435. Voltaic cell. Two electro-chemically different solids (of which zinc is almost invariably one) placed in an electrolytic liquid constitute what is called a *galvanic* or *voltaic*[1] *cell* (or *pair*). One of these plates must be more actively attacked by the liquid than the other; the plate most acted upon is called the electro-positive plate, and the other the electro-negative one.

The greater the disparity between the two solid elements with reference to the action of the liquid on them, the greater the difference in potential; hence the greater the current.

In the following *electro-chemical series* the substances are so arranged that the most electro-positive, or those most affected by dilute sulphuric acid, are at the beginning, while those most electro-negative, or those least affected by the acid, are at the end. The arrow indicates the direction of the current through the liquid.

It will be seen that zinc and carbon are the two substances best adapted to give a strong current.

When the wires from the two plates are joined the discharge of the two plates would produce electrical equilibrium, were there not some means of maintaining a difference of potential between the two plates. This is accomplished by the chemical action between the liquid and the electro-positive plate, and at the expense of the chemical potential energy of the electrolyte and plate. *A voltaic cell is, therefore, a contrivance which converts chemical*

[1] The original Volta's cell consisted of a plate of copper and a plate of zinc immersed in dilute sulphuric acid. It is a very inefficient battery, yet with such an appliance Sir Humphry Davy performed his classic experiments upon the metals of the alkalis, and produced the first voltaic arc.

energy into electrical energy.[1] It should be remembered that it is the rôle of the battery to maintain a difference of potential between the two plates, or, what is the same thing, between the battery terminals or poles, and not to "generate electricity."

436. Circuit. This term is applied to the entire path along which electricity flows, and it comprises the battery itself and the wire or other conductor connecting the battery plates. Bringing the two extremities of the wire in contact and separating them are called respectively *closing and opening*, or *making and breaking, the circuit.* Opening a circuit at any point and filling in the gap with an instrument of any kind, so that the current is obliged to traverse it, is called *introducing the instrument into the circuit.*

437. Ground circuit. It was an early discovery in telegraphic history that a complete metallic circuit is not necessary, but that, in common parlance, the earth can be used as a "return circuit." This type of circuit is represented by a battery with a wire leading from one plate to any convenient point of the earth, and a second wire leading from the other plate to any other point of the earth, which may be many miles distant from the first point. No one can assert that the current in such a case really goes through the earth from one of these points to the other. The earth may be regarded as a great reservoir, rather than as a conductor. The battery acts like a pump, raising electricity at one end from the earth and discharging it at the other end to the earth. It is obvious that a pump might be kept in action by pumping from an ocean and back into the same ocean without disturbing its level, and yet there would be a continuous flow through the water conductors or pipes. This would represent what is known as a *ground circuit.*

438. Theory of the voltaic cell. The following is a brief statement of Grotthus' theory of electrolytic action, somewhat modified. The small ovals in Figs. 303 and 304 represent molecules of the

[1] A single voltaic couple is usually termed a cell; a combination of cells, a battery.

electrolyte, which in this case we suppose to be water,[1] each containing two atoms of hydrogen (H_2) and one atom of oxygen (O). The series of molecules *a* illustrates the condition of the molecules before the metals are introduced; the series *b* represents their condition after the introduction of the metals and before the circuit is closed. Fig. 304 represents the condition of things at the instant the circuit is closed or at the instant of discharge. The molecules are polarized and like sides turned in the same direction. At this instant there is a redistribution of electricity, an electric flow, an equalization of potential. At this instant, chemically speaking, there is, as it were, an interchange of partners, the O next the zinc combining with it to form ZnO, its H_2 combining with the O of the next molecule, and so on through the whole row of molecules, until finally the H_2 appears at the surface of the copper, where it is set free. Immediately following the discharge and equalization of potential a repolarization (*b*) takes place followed by a discharge (*c*). The discharges follow one another so rapidly as to furnish practically a continuous flow of electricity.

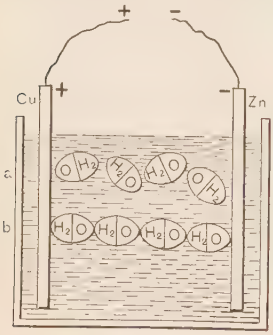


FIG. 303

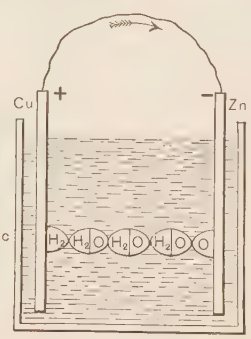


FIG. 304

439. Electrolytic conduction. As rapidly as ZnO (zinc oxide) is formed it combines with the sulphuric acid and forms zinc sulphate. The hydrogen escapes in bubbles from the electro-negative plate. Thus oxygen keeps disappearing by combination with the zinc at one end of the electrolyte; the hydrogen, by evolution as a gas, disappears at the other. Meantime the molecules keep

[1] It is probably not the water (H_2O) but the acid (H_2SO_4) which is the *real* electrolyte. For our purpose it does not matter.

exchanging atoms, so that a constant traveling of the H and O atoms through the electrolyte is kept up, each conveying its own peculiar charge. In this way what is virtually a current traverses the liquid. The current, however, does not traverse the liquid as it does a wire, but electricity is transmitted by electrolytic action. The electricity flows not *through* but *with* the atoms of matter. It is conveyed through the liquid by a procession of charged *ions*, and the process of transmission is more nearly that of convection than of conduction. This process is called *electrolytic conduction.*

SECTION III

SOME DEFECTS OF CELLS

440. Amalgamation of the zinc. All commercial zinc contains impurities, such as carbon, iron, etc. Fig. 305 represents a zinc element having on its surface a particle of carbon *a*, purposely magnified. If such a plate be immersed in dilute sulphuric acid, the particles of carbon will form with the zinc numerous voltaic circuits, and a transfer of electricity along the surface will take place. This coasting trade, as it were, between the zinc and the impurities on its surface diverts so much from the regular current, and thereby weakens it. In addition to this, it occasions a great waste of materials, because, when the regular circuit is broken, this *local action*, as it is called, still continues. If pure zinc were used, no local action would occur at any time, and there would be no consumption of material except when the circuit was closed. If mercury be rubbed over the surface of the zinc after the latter has been dipped into acid to clean its surface, the mercury dissolves a portion of the zinc, forming with it a semi-liquid amalgam, which covers up its impurities, and the amalgamated zinc then comports itself like pure zinc.

FIG. 305

441. Polarization of the negative element. Construct a voltaic cell composed of dilute sulphuric acid and plates of copper and zinc. Introduce into the circuit a galvanoscope (§ 451), and note the deflection of the needle when the circuit is first closed. Watch the needle for a time. Little by little this deflection will decrease, and as it decreases bubbles of gas collect on the copper plate. This accumulation of gas is called *polarization*[1] *of the negative element or plate.*

We already understand that a difference of potential is the indispensable prerequisite to a flow of electricity. Accompanying a difference of potential there seems to be something analogous to a force which is said to cause the flow of, or to urge, the electricity through the circuit. The film of gas on the copper reduces the electro-chemical difference between it and the zinc plate, upon which the generation of this force depends, and thereby diminishes the efficiency of the battery. Instead of a copper-zinc pair we soon have a hydrogen-zinc pair. A single-fluid battery cannot, therefore, yield a current of constant strength unless some means is used to remove the hydrogen.

To overcome this defect some arrangement must be adopted to prevent this deposit of hydrogen upon the negative element. The usual method is to employ in addition to the dilute sulphuric acid, which we will term the *exciting liquid*, some other substance (usually a liquid) which is a strong oxidizing agent, i.e. which can combine with and remove the hydrogen as soon as it is liberated at the negative plate. A substance used for this purpose is termed a *depolarizer*. A mixture of a solution of crystals of bichromate of potassium in water with a suitable quantity of dilute sulphuric acid forms a depolarizer such as is used in the so-called *bichromate batteries*. Other depolarizing substances in common use are bichromate of sodium, nitric acid (an excellent depolarizer but very objectionable on account of its corrosive and unwholesome fumes), chromic acid, peroxide of manganese, copper sulphate, etc.

[1] The term *polarization* in common use is a most senseless term as here applied. Polarization in the electrical world is made to cover a multitude of sins.

442. Grenet cell. This is a bichromate of potassium battery in which two carbon plates, C' and C' (Fig. 306), electrically connected, and a zinc plate Z, suspended between them by a brass rod a, are immersed in the mixed liquid referred to above.

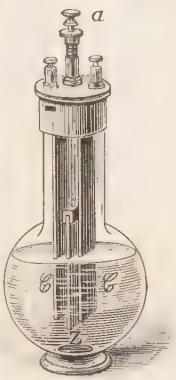


FIG. 306

This combination furnishes a much more energetic and constant current than would be furnished if only dilute sulphuric acid were used. But although polarization of the negative element is diminished, another detrimental action is substituted. The layer of solution next to the carbon plate is soon (say, after a constant use of half an hour to an hour) deprived of its active oxidizing powers, and then polarization of the negative plate and consequent weakening of the current set in. This difficulty may be partially remedied by occasionally agitating the liquid or allowing it to rest, during which time the various portions of it become homogeneous by diffusion, and the liquid near the carbons becomes more active.

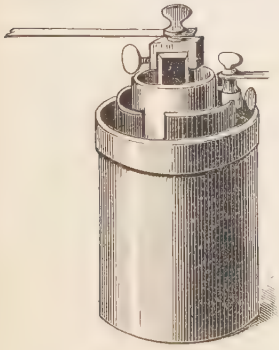


FIG. 307

443. Bunsen cell. A plan generally adopted to keep the oxidizing liquid away from the zinc plate, where it is not wanted and only does harm, is to place the carbon plate in an unglazed, porous, earthen cup and to surround it with the oxidizing substance. This arrangement, called a two-fluid cell, is that adopted by Bunsen (Fig. 307), Grove, Fuller, and others.

In any form of two-fluid cell yet devised the oxidizing fluid sooner or later diffuses through the porous cup and reaches the zinc. Destructive action on the zinc then begins, which amalgamation cannot prevent, and a portion of zinc is uselessly consumed without

effecting anything in the way of generation of electric current. An attempt to prevent this trouble is sometimes made by using a *solid* depolarizer.

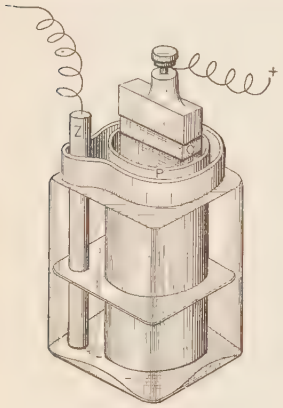


FIG. 308

444. Leclanché cell. There is a class of galvanic cells in which the negative element is protected from polarization by means of metallic oxides. Of these the best known is the Leclanché cell (Fig. 308). In this cell the carbon plate C is contained in a porous cup P, and packed round with fragments of gas-retort coke and manganese peroxide. The manganese compound has a strong affinity for the hydrogen. But the chemical action of solids is sluggish and they quickly polarize when in action. They need periodical rest to recover their normal condition. Such are called *open-circuit batteries*, since they are suited for work only on lines kept open or disconnected most of the time, such as in telephone and bell-ringing circuits. The zinc element Z, which is a rod of zinc, is immersed in a solution of ammonium chloride, which is the exciting liquid.

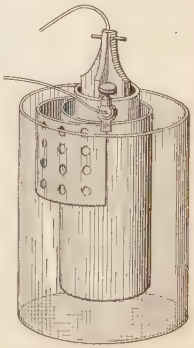


FIG. 309

445. Daniell cell. Leaving the hydrogen-generating batteries, we will examine briefly another form incapable of this species of polarization. The Daniell cell (Fig. 309) uses a solution which, instead of depositing hydrogen, deposits copper upon a copper negative plate, and hence is free from hydrogen polarization. It contains a copper negative and a zinc positive plate. The copper plate is immersed in a solution of copper sulphate, the zinc in a solution of zinc sulphate or dilute sulphuric acid, and a porous cup separates the two liquids. By the electrolytic action the zinc combines with the sulphuric acid

(H_2SO_4), forming zinc sulphate ($ZnSO_4$), and thereby setting hydrogen free. This hydrogen, while on its way to the negative element of the copper plate, meets the copper sulphate solution ($CuSO_4$), which it decomposes, forming sulphuric acid again (H_2SO_4) and setting free the copper, the latter being deposited on the copper plate.

In some copper sulphate batteries the porous cup is not employed, the difference in the specific gravity of the solutions being relied upon to keep them separate, as in the so-called *gravity cell* (Fig. 310). This form of cell is commonly used on *closed circuits*. The chief merits of these cells are the complete absence of polarization of the negative plate (consequently, the constancy in the potential difference of the two elements) and the uniformity of current which is yielded.

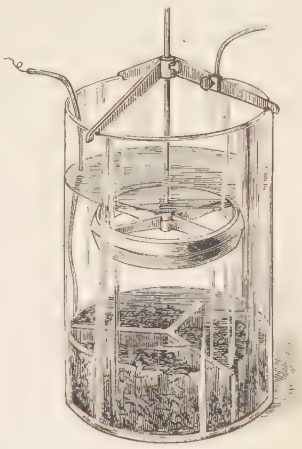


FIG. 310

The kinds of cells that have been devised are numberless. The voltaic battery will probably long continue to be of scientific interest, but for commercial uses it has already become well-nigh obsolete, being replaced by other cheaper and more efficient appliances for generating electric currents, to be hereafter described. The battery must, therefore, command in the future relatively less attention in our text-books than formerly.

QUESTIONS AND PROBLEMS

1. (*a*) What is an amalgamated zinc plate? (*b*) a voltaic cell or pair? (*c*) an electrode?

2. (*a*) How may it be shown that a plate of copper and a plate of zinc become electrified when placed in dilute sulphuric acid? (*b*) Which will be positively electrified? (*c*) If wires leading from these plates be joined, what

change will occur? (*d*) What purpose does the connecting wire serve? (*e*) Why ought not the plates of a voltaic cell to touch each other? (*f*) Why ought not the wires of certain voltaic cells to touch each other when not in use?

3. (*a*) What do you understand by an electric current? (*b*) What is the function of a battery? (*c*) With what propriety is the zinc plate of a voltaic cell called the *positive plate* and the *negative electrode* of a voltaic system?

4. How does electricity pass from plate to plate within a voltaic cell?

5. (*a*) What is meant by local currents? (*b*) How may they be prevented?

6. (*a*) Give an example of a counter-electro-motive force. (*b*) By what other name is it generally known? (*c*) What harm does it do? (*d*) How may it be prevented?

7. If you have the two leading wires of a battery, and the battery be concealed, how can you tell which of the wires is connected with the zinc plate of the battery?

SECTION IV

EFFECTS PRODUCED BY THE CURRENT

446. Summary of effects. The several effects producible by an electric current are as follows:

(1) Certain compounds in solution can be decomposed by causing the current to pass through the solution. This operation is called *electrolysis.*

(2) A magnetic needle suspended on a vertical pivot will be deflected if a wire through which a current is flowing be brought near and parallel to it.

(3) If a wire carrying a current be wound round a rod of soft iron, the iron becomes a temporary magnet; the magnet is called an *electro-magnet.*

(4) Heat is generated in a wire through which a current flows.

(5) If the temperature be raised sufficiently at any point of the circuit, the conductor at that point becomes luminous, as, for example, in the production of the electric light.

(6) Various physiological effects are produced by a current, such as "shocks," a peculiar taste when the poles are applied to the tongue, the sensation of light, etc.

The effects may be classified as *electrolytic, magnetic,* including (2) and (3); *thermal,* including (4) and (5); and *physiological.*

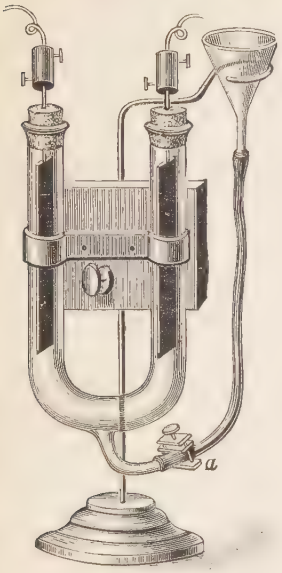


FIG. 311

447. (1) **Electrolysis.** Fig. 311 represents a simple device for the separation of oxygen and hydrogen by the electric current. Into the funnel is poured dilute sulphuric acid (1 part to 20 by volume) so as to fill the U-shaped tube when the stoppers are removed. Through the stoppers, when in place, wires pass, connecting with strips of platinum foil which serve as the electrodes.

When this arrangement is placed in circuit with a battery of two or three volts or more, instantly bubbles of gas arise from both electrodes, accumulating in the upper part of the tube and forcing the liquid back into the funnel. The gas that has collected at the + electrode will be found on examination to be oxygen, while that in the other limb of the tube is hydrogen. Platinum electrodes are used because this metal does not unite with either gas.

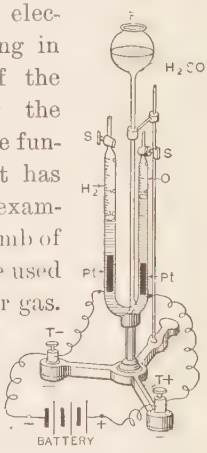


FIG. 312

If the experiment be performed with a Hoffman *voltameter*[1] (Fig. 312), the gas given off at each electrode may be measured by the graduations on the arms of the tube. It will be found that the volume of hydrogen given off is just double that of the oxygen liberated in the same time. Though a little out of place at this point, it is worth while to

[1] Any vessel employed for performing and measuring electrolysis is called a *voltameter.*

mention another important use to which this apparatus is put. The quantity of gas given off, divided by the number of seconds the current has been flowing, is a direct measure of the *mean strength* of the current, i.e. the number of units of electricity which flow through the circuit in one second. Hence this apparatus serves to measure current strength.

The electrode by which the current enters the electrolyte is called the *anode;* and that by which the current leaves, the *cathode*. The elements into which the electrolyte is broken up are called the *ions*. The ion appearing at the anode is the *anion;* and that at the cathode, the *cation*.

448. Relation of the electrodes and battery elements. It will be observed that in both the battery and the voltameter the hydrogen is liberated at the plate *toward* which the current is flowing, as shown in the diagram (Fig. 313). Hydrogen and all metals appear on the plate toward which the current flows, whether in the decomposing cell or the battery; for example, silver appears at the cathode, and the copper in the Daniell cell is deposited on the electro-negative plate. And, in general, *when a chemical salt is electrolyzed the base appears at the cathode and the acid at the anode.*

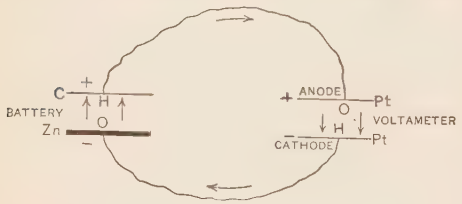


FIG. 313

449. (2) **Magnetic action and magnetic field due to a straight current. Magnetic lines of force.** If a wire bearing a strong current be passed vertically through the center of a board on which have been sifted some fine iron filings, the filings will arrange themselves in circular lines round the current-carrying wire (Fig. 314), thus furnishing a graphic representation of the magnetic field set up by a current. If a small pocket compass be carried around and near the wire, the needle will, at every point, take a position tangent to these circular lines of filings, whichever way the current

passes. If the current be reversed, however, the position of the N and S poles of the needle will be reversed. This clearly indicates that there is a difference of direction of these circular lines according as the current flows in one direction or in the other. These circular lines represent the so-called *magnetic lines of force* which occupy a space or *field* round a current-bearing wire.

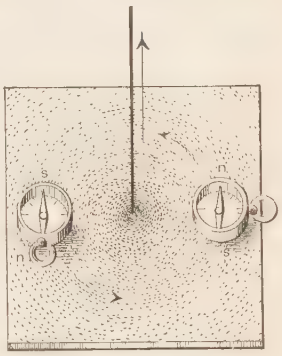


FIG. 314

450. Deflection of the magnetic needle by a current. (1) Place the apparatus (Fig. 315) so that the magnetic needle, which points (nearly) north and south, shall be parallel with the wires W_1 and W_2. Join the + electrode of a battery to screw cup T_2, and the − electrode to screw cup T_1, and pass a current through the upper wire. At the instant the circuit is closed the needle swings on its axis, and after a few oscillations comes to rest in a position which forms an angle with the wire bearing the current.

(2) Break the circuit by removing one of the wires from the screw cup. The needle, under the influence of the magnetic action of the earth, returns to its original position.

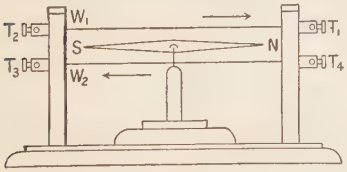


FIG. 315

(3) Reverse the current by connecting the + electrode of the battery to the screw cup T_1, and the − electrode to the screw cup T_2. Again there is a deflection of the needle, but the direction of the deflection is reversed; that is, the north-pointing pole (N pole), which before turned to the west, is now deflected toward the east.

(4) Place your *right hand* above the wire with the palm *towards* the wire, and with the fingers pointing in the same direction as that in which the current is flowing, and extend your thumb at

right angles to the direction of the current (Fig. 316). You observe that your thumb points in the same direction as the *N* pole of the needle *under* the current-bearing wire.

(5) Reverse the current again (so that it will flow northward), and place your right hand as before (namely, with the palm towards the wire, and with the fingers pointing in the same direction as the current); your outstretched thumb still points in the *same* direction as the *N* pole of the needle.

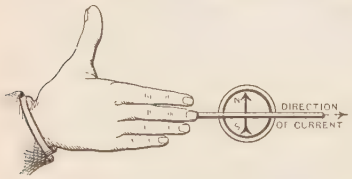


FIG. 316. Right hand above the wire; needle below it

(6) Join the + electrode of the battery to screw cup T_3 and the − electrode to screw cup T_4 so that the current will flow northward *under* the needle. Place the right hand as directed before, except that it must be *under* the wire, so that the wire shall be between the hand and the needle; the thumb will point in the same direction as the *N* pole (Fig. 317). Reverse the direction of the current in this wire and apply the same test; the same rule holds.

The rule for determining the direction of the deflection of the *N* poles of a needle when the direction of the current is known is this: *Place the outstretched right hand over or under the wire so that the wire shall be between the hand and the needle, with the palm towards the needle, the fingers pointing in the direction of the current and the thumb extended laterally at right angles to the direction of the current; then the extended thumb will point in the direction of the deflection of the N pole.*

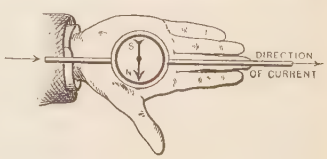


FIG. 317. Right hand below the wire; needle above it

Conversely, the direction of the current may be determined by ascertaining the direction of the deflection it produces, as follows: *Place the outstretched right hand over or under the wire* (always allowing the wire to come between the hand and the needle) *with the palm toward the needle and the extended thumb*

in the same direction as the N pole of the needle is deflected; then the fingers will point in the direction the current is flowing.

It will be observed that a deflection is reversed either by reversing the current or by changing the relative positions of the wire and needle, e.g. by carrying the needle from above the wire to a position below it.

The force exerted by the current upon the needle in deflecting it is called an *electro-magnetic force.*

451. Simple galvanoscope or current detector. Join the + electrode of the battery to screw cup T_2 (Fig. 315) and the − electrode to screw cup T_3, so that the current will pass above the wire in one direction and below it in the opposite direction, as indicated by the arrows. *A larger deflection is obtained than when the current passes the needle only once.*

If the right-hand test be applied it will be seen that the tendency of the current, both when passing the needle in one direction above and in the opposite direction below, is to produce a deflection in the same direction, and consequently the two parts of the current assist each other in producing a greater deflection.

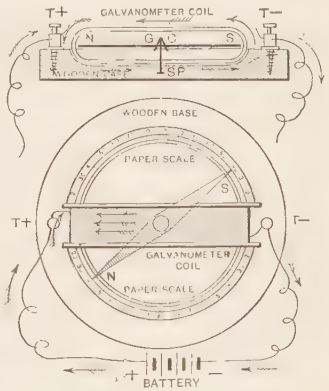


FIG. 318

If a more sensitive instrument, i.e. one which will produce considerable deflections with weak currents, be required, then it will be necessary to pass the current through an insulated wire wound many times around the needle, as shown in the sectional elevation and plan (Fig. 318). Such an instrument is called a *galvanoscope* or *current detector*, since one of its important uses is to detect the presence of a current. A graduated card divided off like that of the mariner's compass is placed beneath the needle so that the number of degrees of deflection may be read from it. From the number

of degrees of deflection an approximate idea of the strength of the current is obtained.

452. (3) **Magnetizing effect of an electric current. Electro-magnets.** (*a*) Wind an insulated copper wire in the form of a spiral round a rod of soft iron (Fig. 319). Pass a current of electricity through the spiral, and hold an iron nail near the end of the rod. Observe, from its attraction for the nail, that the rod is magnetized. A magnet may be provisionally defined as a body which attracts iron. (*b*) Break the circuit; the rod loses its magnetism and the nail drops.

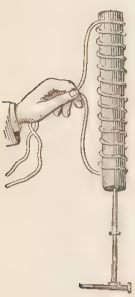


Fig. 319

The iron rod is called a *core*, the coil of wire a *helix*, and both together are called an *electro-magnet*. In order to take advantage of the attraction of both ends or *poles* of the magnet, the rod is most frequently bent into a U-shape (*A*, Fig. 320). More frequently two iron rods are used, connected by a rectangular piece of iron, as *a* in *B* of Fig. 321. The method of winding is such that if the iron core of the U-magnet were straightened, or the two spools were placed together end to end, one would appear as a continuation of the other. A piece of soft iron *b*, placed across the ends and attracted by them, is called an *armature*. The piece of iron *a* is called a *yoke*.

Fig. 320

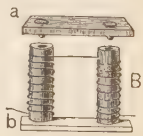


Fig. 321

453. (4) (5) **Heating and luminous effects.** Construct a low-resistance battery of four to six cells, and introduce into the circuit a platinum wire, No. 30, about $\frac{1}{4}$ inch long. The wire very quickly becomes white hot, i.e. it emits white light, which indicates a temperature of approximately 1900° C.

This experiment illustrates the conversion of the energy of an electric current into heat energy. In this case the energy of the current is said to be used in overcoming the *resistance* which the conductor or the circuit offers to its passage. Heat is developed

by a current in every part of the circuit, because all substances offer some resistance to a current; in other words, there are no perfect conductors. The small platinum wire offers much greater resistance than an equal length of a larger copper wire; whence the greater quantity of heat generated in this part of the circuit. All of the energy of any electric current that is not consumed in doing other kinds of work is changed into heat.

Fig. 322 represents a calorimeter of simple construction. An inverted wide-mouthed bottle has its stopper pierced by two stout copper wires, which are united within the bottle by a coil of fine platinum wire. Through a hole bored in the bottom of the bottle is introduced a thermometer *T*. The bottle contains a known quantity of water. Now if a current be passed through the coil during a given time, it is clear that the amount of heat generated can easily be determined by multiplying the known mass of water by its temperature change, as indicated by the thermometer. This heat, however, is equivalent to a definite quantity of electric energy transformed in the wire.

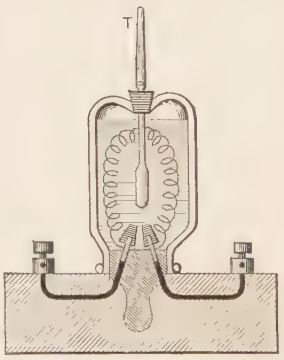


FIG. 322

By means of apparatus of this kind *Joule's law* was established, namely: *The number of units of heat generated in a conductor varies as* (1) *its resistance*, (2) *the square of the strength of the current, and* (3) *the time the current flows.*

454. (6) **Physiological effects.** Besides the several objective effects already described, the electric current and the resulting magnetic field are capable of producing certain physiological effects. These are numerous and very important, but are treated in detail under the title of "electro-therapeutics," in another department of science. As the resistance of the body is very high (in the neighborhood of 10,000 ohms from hand to hand across the chest), a very high E.M.F. is necessary.

SECTION V

ELECTRICAL QUANTITIES AND UNITS

455. Importance of electrical measurements. Less than half a century ago the experimental sciences of electricity and magnetism were in a great measure collections of isolated qualitative results. Now, happily, all this has been changed. The introduction of the absolute system of units has been largely instrumental in changing experimental electricity and magnetism into sciences of which the most delicate and exact measurement is the very essence.

The wonderful developments which have been made in recent years in electrical science, and which have led to the employment of electric energy in connection with a great diversity of industrial arts, are almost wholly due to a better understanding of what electrical measurements can be made, and how to make them. Indeed, little of a *practical nature* can be done without some acquaintance with the methods of making these measurements.

456. Electro-motive force. The volt. Water flows from one place to another in virtue of a difference of pressure between the two places, and the flow takes place *from* the place of high pressure *to* the place of low pressure. For instance, when water flows from a reservoir or cistern the pressure at any point in the pipe is due to the "head" of water above it. If it be set flowing by a force pump, we might say the flow of water was due to a water-motive force which could be expressed as equal to a "head" of a certain number of feet of water.

Similarly, electricity flows in a conductor only when there is a difference of what may be termed *electrical pressure* between its ends. If such be maintained between two points connected by a conductor, it obviously represents a kind of *current-producing force*, one which can keep electricity in motion against resistance. It is for this reason called *electro-motive force* (E.M.F.). Electro-motive force is that which maintains or tends to maintain a current of electricity through a conductor.

Difference in electrical pressure we have hitherto assumed to be due to difference of potential. It is this difference of electrical pressure which sets up a current in the conductor. Potential difference may be due to contact of dissimilar substances, as in the voltaic cell, or to the movement of a part of the conductor in a magnetic field, as in the dynamo.

The *volt* is the name chosen for the practical unit of E.M.F. and difference of potential. For purposes where great accuracy is not required, it will answer to consider a volt as the E.M.F. of a Daniell's cell; i.e. it is *about* the difference of potential between the zinc and copper of this cell, the E.M.F. of an ordinary Daniell cell being about 1.07 volts.

457. Resistance. The ohm. It is clear that under a given pressure the quantity of water flowing through a pipe per unit of time is dependent upon both the size of the pipe and the condition of its interior surface, or collectively upon what we might term its *conductivity*. So the quantity of electricity per second flowing in a wire depends on the conductivity of the wire, which is directly proportional to its cross section and is influenced by the material of which it is composed, silver and copper being excellent conductors. The converse of conductivity is called *resistance*, and this term and the idea it conveys are in constant use in the discussions that follow.

The practical unit of resistance is called the *ohm*, and is equal to the resistance of a column of pure mercury at 0°, 106.3 centimeters long, and 1 square millimeter in cross section. It is about the resistance of 9.3 feet of No. 30 (American gauge) copper wire (0.01 inch diameter) at ordinary temperature.

458. Current strength. The ampere. The expression *current strength* means the *rate of flow of electricity*. The "size" of a stream of water, or the "rate of flow" might be indicated by stating the number of gallons which flow past a given point in a second. We have not adopted in hydraulics any particular name for a *gallon per second*, but there is a necessity in electricity for a term to denote the corresponding idea; in other words,

there is a necessity for a unit for measuring rate of flow or current strength.

The practical unit of current is called the *ampere* and is that current which would flow through a conductor of one ohm resistance when the E.M.F. is one volt.

The quantity of electricity passing any cross section of the conductor in one second when the current is an ampere is one *coulomb*.

459. Electrical work and electrical activity. The joule and the watt. The volt is of such a magnitude that one coulomb of electricity conveyed against an E.M.F. of one volt requires an expenditure of one *joule* = one *volt coulomb* of energy. The volt coulomb is analogous to the foot pound and kilogrammeter. Hence if a coulomb of electricity flow between two points in a conductor whose difference of potential is one volt, then one joule of work is done thereby.

If a conductor be traversed by a current of one ampere, i.e. a coulomb per second, and we find two points whose difference of electrical level is one volt, then the rate at which work is being done in that portion is one *watt* = one joule per second. The joule and the watt are units of electrical work and electrical activity, respectively.[1]

460. Ohm's law. The three factors, current (C), electro-motive force (E), and resistance (R), are evidently interdependent. Their relations to one another are stated in the well-known Ohm's law thus: *The current is equal to the electro-motive force divided by the resistance;* or

$$C = \frac{E}{R}.$$

Hence the strength of a current is directly proportional to the E.M.F. and inversely proportional to the resistance. This famous law is taken as the basis of a large portion of electrical measurements, and some of its applications will be developed as we proceed.

[1] The several quantities discussed in the last few sections have not been introduced in their logical order from a scientific point of view, but the order adopted is much easier since the subject is to be treated only in its elements.

SECTION VI

MUTUAL RELATIONS OF ELECTRICAL QUANTITIES

461. Activity. As seen above, the unit of electric activity (or rate of doing work) is the *watt. A watt is the activity of a current of one ampere maintained by a difference of potential of one volt.* 1 volt × 1 ampere = 1 volt ampere or watt. Hence

(1) A (watts) = C (amperes) × E (volts).

(2) The watt = 1 joule or 10^7 ergs per second, or $\frac{1}{746}$ horse power. Hence $\frac{CE}{746}$ = activity in horse power.

For example, to find the rate at which energy is transformed in an electric lamp, measure the whole current in amperes; measure the difference of potential (with a voltmeter) between the terminals of the lamp, in volts; multiply together the quantities thus obtained and divide by 746; the result will be the horse power absorbed in the lamp. That is, a current of C amperes falling E volts will perform, in passing through the instrument, work at the rate of $\frac{CE}{746}$ horse power.

a. Substituting in equation (1) the value of C as given in Ohm's formula, $C = \frac{E}{R}$, we have $A = \frac{E^2}{R}$; i.e. *the activity is equal to the square of the E.M.F. divided by the resistance.*

b. $E = CR$ (Ohm's formula). If this value of E be substituted, formula (1) becomes $A = C^2R$; i.e. *the activity is directly proportional both to the square of the current strength when R is constant and to the resistance when C is constant.*

(3) *The amount of chemical decomposition produced by a current in a given time varies as the strength of the current.* On this principle is constructed the voltameter, which measures the strength of a current by the amount of chemical action it effects in a given time.

(4) *The mass in grams of an element deposited by electrolysis is found by multiplying its electro-chemical equivalent* (i.e. the mass in grams of the element deposited by one ampere in one second) *by the strength of the current in amperes, and this product by the time in seconds during which the current electrolyzes.*

(5) *The number of units of heat developed in a conductor is proportional* (1) *to its resistance,* (2) *to the square of the strength of the current, and* (3) *to the time the current is flowing.*

A current of one ampere flowing through a resistance of one ohm develops therein 0.00024 calorie of heat per second. Hence H (calories) $= C^2$ (amperes) $\times R$ (ohms) $\times t$ (seconds) $\times$ 0.00024.

Whenever the current heats a wire, produces decomposition, or performs work of any kind, each of these acts is accomplished at the expense of the energy of the generator. If the current operate an electric motor which pumps water, or lifts a hammer, the battery or other generator loses energy proportional to the work required for each of these mechanical acts.

SECTION VII

INSTRUMENTS FOR ELECTRICAL MEASUREMENTS

Our attention is next directed to the consideration of instruments for measuring the quantities which, we have seen, are required to be known. First we consider the instrument for measuring *quantity* of electricity, properly called a coulomb-meter.

462. Coulomb-meter. Voltameter. The simplest quantity meter is based on the electrolytic effect of the current, and is called a *voltameter*. If, for instance, using the Hoffman voltameter (Fig. 312), we measure the hydrogen generated during a given time, the mass or volume of this hydrogen under constant pressure and temperature is exactly proportional to the number of coulombs of electricity which have passed through the liquid. The mass in grams of any constituent of an electrolyte liberated by the

passage of one coulomb of electricity is called its *electro-chemical equivalent*. For commercial purposes, instead of the coulomb as the unit of quantity, a larger unit, the *ampere hour* (= 3600 coulombs), is frequently used.

The following are the electro-chemical equivalents[1] per coulomb and per ampere hour for a few substances:

	Electro-chemical equivalent per coulomb.		Equivalent per ampere hour.	
Hydrogen	.000010354	grams	0.03727	grams
Silver	.00111800	"	4.0248	"
Copper	.0003284	"	1.1822	"
Zinc	.00033696	"	1.223056	"
Lead	.00107160	"	3.85776	"

The electrolytic action in the voltaic cell itself is proportional to the strength of the current while it passes. One coulomb of electricity in passing through a Daniell's cell dissolves 0.00033696 gram of zinc and deposits 0.0003284 gram of copper.

A coulomb-meter will serve as an *amperemeter* (abbreviated, *ammeter*) when the current is very nearly constant, but not otherwise. For the quantity of electricity measured in coulombs which has passed through the circuit in a given time, divided by the number of seconds, obviously gives the coulombs per second, or the mean ampere strength of the current.

463. Galvanometer. This is an instrument for measuring current strength by means of the deflection of a magnetic needle or of a coil of wire when placed in the field of the current. It is so constructed that either the deflection angle itself, or some function of it, is proportional to the current strength.

464. Tangent galvanometer. A tangent galvanometer is one so constructed that the current passing through it is proportional to the tangent of the angle of deflection produced. To this end it is necessary that the needle be very short (not more than $\frac{1}{30}$) in comparison with the diameter of the coil.

[1] The student in chemistry will understand that the electro-chemical equivalents of different metallic elements are proportional to their combining equivalents.

In its simplest form it consists of a large vertical coil (better two coils, one on each side of the needle, Fig. 323, so placed that the needle is at the center of the common axis), in the center of which is either a small compass needle or a needle suspended by a silk fiber.

A needle thus placed in the field of a current is acted on by a couple tending to place it at right angles to the plane of the coil, and it is deflected until this couple is balanced by that due to the earth's magnetism. The value of the earth's magnetic field in a horizontal plane is denoted by H.

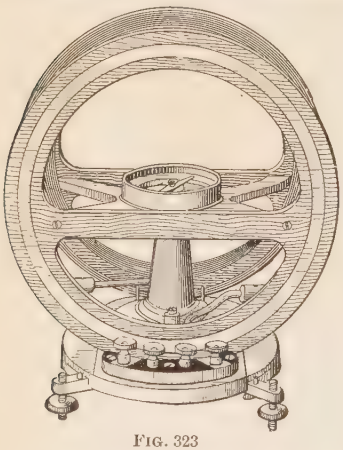


FIG. 323

The formula for the tangent galvanometer is $C = A \tan \theta$ in which C is the current strength, θ the angle of deflection, and A a value depending partly on H and partly on the construction of the instrument.

With the *proper* value of A determined for an instrument in any definite location, the current may be calculated directly in amperes.

If the strengths of two currents are to be compared, it is only necessary to obtain deflections with each current separately, and compare the tangents of the angles.

465. Ammeter. Now if the value in amperes of each division of the scale be found, and these results be placed upon the scale in place of degree numbers, we shall have a direct-reading amperemeter (ammeter). There is a great variety of ammeters in use, for a description of which the student is referred to technical works on the subject.

We shall consider only two other forms, one called the (Kohlrausch) solenoid ammeter, selecting this because of its simplicity. In Fig. 324 a is a helix of thick wire, b is a soft-iron tube which

serves as a core, suspended by a light spring c. The core carries a marker j; f is merely a wooden guide rod for the tubular core. The action of this ammeter depends upon the principle that when an insulated wire is wound into a helix (called also a solenoid), and a current is passed through the wire, an iron rod or tube placed at the opening will be drawn into the helix with a force increasing with the strength of the current. This force against the elastic force of the spring may be measured in the same manner as weight by a spring balance. Now as the current strength bears a definite relation to the force, the instrument can easily be calibrated in amperes.

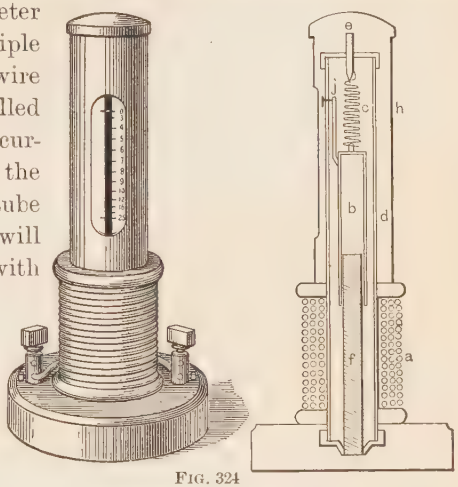


FIG. 324

466. The D'Arsonval Galvanometer. We shall now describe an instrument illustrating a type of galvanometer noted for its extreme sensitiveness. The design is due to D'Arsonval. In this type the magnet is fixed and the coil is the movable part, the amplitude of its deflection indicating the strength of the current. The arrangement is shown in Fig. 325. NS are the poles of a strong permanent magnet and between these, with its plane parallel to the field, is suspended the coil C by means of a delicate torsion conductor, a strip of phosphor-bronze by means of which the current is conveniently led to the coil. To the other and lower

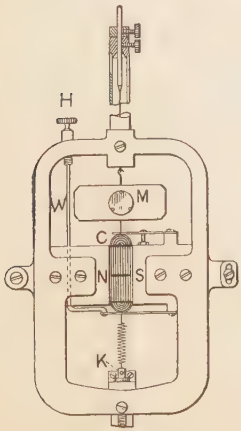


FIG. 325

end of the coil is attached a spiral spring which runs vertically downward to the bottom of the instrument, and which is drawn just tight enough to hold the coil in place and yet allow it to rotate freely about a vertical axis. This is done by adjusting a screw K. A mirror M is fixed to the suspension just above the coil and a telescope and scale are used to read the deflections of the coil. When a current is passed through the coil it develops a field of its own, which tends to set itself parallel to the field of the magnet. This is opposed by the torsion of the suspension. When not in use the coil is raised a little to relieve the suspension by means of the screw H through the wire W. For small angles the current is very approximately proportional to the deflection it produces. This form of instrument has the great advantage of not being appreciably affected by outside influences. Iron or steel in the vicinity has little or no effect on it, and it is practically dead-beat, — i.e. its time for coming to rest after being disturbed is very brief. This is due to the reaction induction currents set up in the coil due to its motion in the magnetic field, the direction of all such currents being always such as to oppose the motion.

SECTION VIII

RESISTANCE OF CONDUCTORS

467. External and internal resistance. For convenience the resistance of an electric circuit is divided into two parts, the *external* and the *internal*. External resistance includes all the resistance of a circuit except that of the generator, while the latter is termed internal resistance.

When the external resistance in a circuit is considered separately from the internal, Ohm's formula must be converted thus (calling the former R, and the latter r):

$$C = \frac{E}{R + r}.$$

If a cell have $E = 1$ volt, and $r = 1$ ohm, and the connecting wire be short and stout, so that R may be disregarded, then the cell

yields a current of one ampere. If by any means the internal resistance of this cell can be decreased one half, it will then be capable of yielding a two-ampere current under the same conditions.

468. External resistance. Introduce into a circuit a galvanometer, and note the number of degrees the needle is deflected. Then introduce into the same circuit the wire on the spool numbered 4 on the platform[1] S (Fig. 326). (The wire on any one of the five

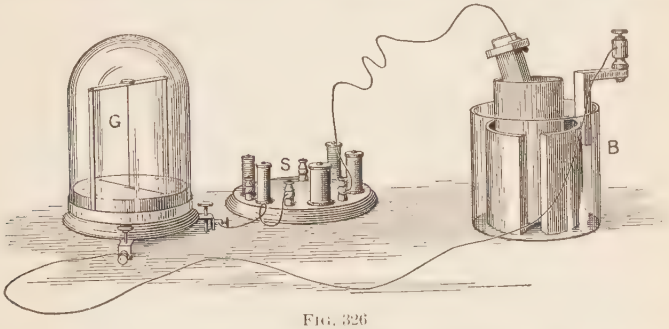


FIG. 326

spools on this platform can at any time be introduced into a circuit by connecting the battery wires with the binding screws on each side of the spool to be introduced.)

The deflection is now less than before. The copper wire on this spool is 16 yards in length; its size is No. 30 of the Brown and Sharpe wire gauge. When this spool is in circuit the latter is 16 yards longer than when the spool is out. The effect of lengthening the circuit is to weaken the current, as shown by the diminished deflection.

Next substitute spool 2 for spool 4. This contains 32 yards of the same kind of wire as that on spool 4. The deflection is still smaller.

The weakening of the current by introducing these wires is caused by the resistance which the wires offer to the current,

[1] The platform of spools containing wire of different (known) sizes, lengths, and material, so arranged that any one, two, or more can be introduced into the circuit for the purpose of measurement of resistance, is an instrument of great convenience in a laboratory.

much as the friction between water and the interior of a pipe impedes, to some extent, the flow of water through it. The longer the pipe the greater is the resistance to the flow.

If the wire on the spools had been the only resistance in the circuit, then, when spool 2 was in the circuit the resistance would have been double what it was when spool 4 was in the circuit, and the current, with double the resistance, would have been half as strong.

(1) *Other things being equal, the resistance of a conductor varies as its length.*

Next substitute spool 1 for spool 2. This spool contains 32 yards of No. 23 copper wire, — a thicker wire than that on spool 2, but the length of the wire is the same. The deflection is now greater than it was when spool 2 was in circuit. This indicates that the larger wire offers less resistance.

Careful experiments show that (2) *the resistance of all conductors varies inversely as the areas of their cross sections. If the conductors be cylindrical, it varies inversely as the square of their diameters.*

Substitute spool 5 for spool 1 and compare the deflection with that obtained when spool 4 was in the circuit. The deflection is smaller than when spool 4 was in circuit. The wire on these two spools is of the same length and size, but the wire of spool 5 is German silver. It thus appears that German silver offers more resistance than copper.

(3) *In obtaining the resistance of a conductor, the specific resistance of the substance must enter into the calculation.*

The resistance of metal conductors increases slowly with the temperature of the conductor. The resistance of German silver is affected less by changes of temperature than that of most metals; hence its frequent use for resistance coils.

469. Internal resistance. Connect with the galvanometer a copper and a zinc strip and introduce the strips into a tumbler nearly full of acidulated water. Note the deflection. Then raise the strips, keeping them the same distance apart, so that less and

less of the strips will be submerged. As the strips are raised, the deflection becomes smaller. This is caused by the increase of resistance in the liquid part of the circuit, as the cross section of the liquid lying between the two strips becomes smaller.

(4) *The internal resistance of a circuit, other things being equal, varies inversely as the area of the cross section of the liquid between the two elements.*

In a large cell the area of the cross section of the liquid between the elements is larger than in a small cell, consequently the internal resistance is less. This is the only way in which the size of the cell affects the current.

Obviously the resistance of the battery would be increased by any increase of the distance between the elements, since this increases the length of the liquid conductor, but as this distance is usually made as small as convenient, and is kept invariable, it demands little of our attention.

SECTION IX

MEASUREMENT OF RESISTANCE

470. Description of the resistance box. Fig. 327 represents a wooden box containing what is equivalent to a series of coils of German silver wire, whose resistance ranges from 0.1 ohm to 100 ohms.[1] Each of these coils is connected with a brass stud on the top of the box.

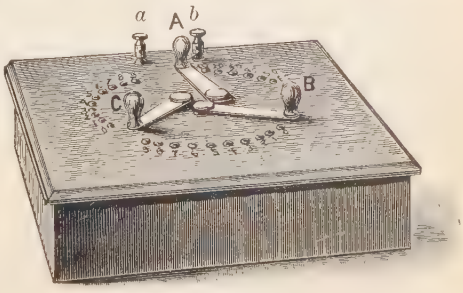


FIG. 327

Three switches, *A*, *B*, and *C*, so connect the coils with the binding

[1] Each additional switch with its corresponding coils increases the range about tenfold, so that the range of the instrument may be very much increased.

screws a and b that a current can be sent through any three coils at the same time by moving the switches on to the proper studs. The resistance in ohms of each coil is marked on the box near its stud. When the three switches rest upon studs marked 0 the current meets with no appreciable resistance in passing through the box, but any desired resistance within the range of the instrument can be introduced by moving the switches on to the studs, the sum of whose resistances is the resistance required. This instrument we shall call a *resistance box* or *rheostat.*

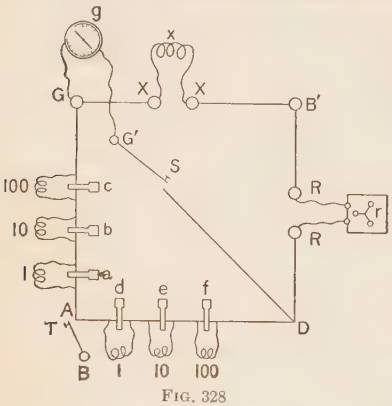


FIG. 328

471. Wheatstone bridge. Fig. 328 represents a diagram of the essential electrical connections of the Wheatstone bridge. The battery wires are connected with the bridge at the binding screws BB'. A galvanometer g is connected at GG', a resistance box r at RR, and the conductor x, whose resistance is sought, at XX.

When the circuit is closed by means of the key T the current, we will suppose, enters at B; on reaching the point A it divides, one part flowing via the branch AGB', and the other via the branch ADB'. If points D and G in the two branches be at different potentials and a connection be made between them through the galvanometer g by closing the key S, there will be a current through this wire and through the galvanometer, and a deflection of the needle will be produced. But if the points D and G be at the same potential, there will be no cross current through the bridge wire and no deflection. Now it can be demonstrated that points D and G will be at the same potential when R (the resistance) of $AD : R$ of $DB' :: R$ of $AG : R$ (the unknown resistance) of GB'. Between A and D and A and G there are three coils of wire having resistances respectively of 1,

10, and 100 ohms. One or more of these coils are introduced into the circuit by removing the corresponding plugs *a*, *b*, *c*, *d*, *e*, and *f*. As the other connections between *A* and *D* and *A* and *G* have no appreciable resistance, being for the most part short brass bars, the only practical resistance between these points is that introduced at will through the coils. Similarly between points *D* and *B'* the only practical resistance is that introduced at will through the resistance box, and between the points *G* and *B'* the resistance is the resistance (x) sought.

It is apparent, then, that in using the bridge after the connections are properly made through the several instruments and certain known resistances are introduced between *A* and *D* and *A* and *G*, we have simply to regulate the resistance through the resistance box so that there will be no deflection in the galvanometer; then we are sure that the above proportion is true. The first three terms of the proportion being known, the fourth term, which is the resistance sought, is computable.

The accuracy of the results obtained largely depends upon so choosing resistances of the bridge as to make the arrangement have maximum sensibility, and upon the sensitiveness of the galvanometer. In using the bridge the following directions should be observed: (1) Always close the circuit at *T* before closing the bridge at *S*, and in breaking the circuit reverse this order. (2) Introduce between *A* and *D* and *A* and *G* resistance as nearly equal to the resistance sought (x) as practicable. If you have no conception what the unknown resistance is, it is best to begin by using high resistances. (3) Use a sensitive galvanometer, e.g. a mirror galvanometer, or the galvanometer shown in Fig. 326, substituting the astatic needle for the tangent needle.

If the same resistance be introduced between points *A* and *G* as between *A* and *D*, it is evident that the resistance in the resistance box r must be made equal to the unknown resistance x in order that there may be no deflection in the galvanometer. Consequently when this result is obtained the resistance of x may be read from the resistance box.

472. Measurement of galvanometer resistance. Lord Kelvin's method. The bridge may be used for measuring the resistance of the galvanometer actually in use. The arrangement is as in Fig. 329. The resistance in the resistance box R is then varied until the deflection of G does not change when the key S is closed; then

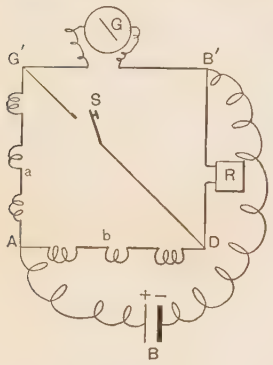


FIG. 329

$$r = R\frac{a}{b},$$

in which r is the resistance of the galvanometer, R is the resistance in the resistance box, and a and b are the resistances in the arms AG' and AD respectively. If $a = b$, then $r = R$.

473. Battery resistance. Mance's method. No definite meaning can be attached to the expression "battery resistance," since this resistance is complicated by variation in the polarization, and this in turn is dependent upon the strength of the current, external resistance, etc. Hence the numerous methods depending on Ohm's law that have been devised for deducing battery resistance are of little value in many cases.

The method known as *Mance's method* is one of the best for measuring battery resistance, since it requires the battery to be constant only during the short time the key is closed. A Wheatstone bridge is arranged as shown in Fig. 330. The resistance in the resistance box R is adjusted until, on pressing down the key A, the deflection of the galvanometer does not change; then

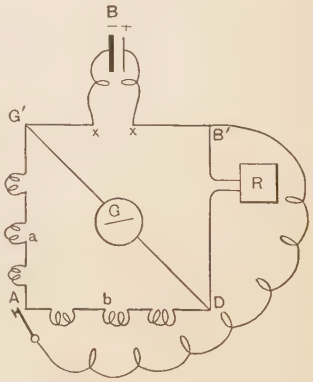


FIG. 330

$$r = R\frac{a}{b},$$

in which r is the battery resistance, R is the resistance in the resistance box ; a and b are the resistances respectively in the arms AG' and AD. If $a = b$, then $r = R$ as before.

The information generally required in practice, however, is what available difference of potentials can be obtained with a certain working resistance in the external circuit. This can be obtained by connecting the terminals of the battery by the body offering this resistance, and measuring the difference of potentials between these points by means of a potential galvanometer. If we call this difference of potentials V, and the E.M.F. of the battery when on open circuit E, and R = external resistance, then we may write

$$\frac{E}{R+r} = \frac{V}{R} = C,$$

where r is such a quantity as satisfies this equation. In other words, this quantity may be taken as the resistance of the battery for the current C.

SECTION X

E.M.F. OF DIFFERENT CELLS. DIVIDED CIRCUITS. METHODS OF COMBINING VOLTAIC CELLS

474. Electro-motive force of different cells. If a galvanometer be introduced into a circuit with different battery cells, e.g. Bunsen, Daniell, Grenet, etc., very different deflections will be obtained, showing that the different cells yield currents of different strengths. This may be in some measure due to a difference in their internal resistance, but it is chiefly due to the difference in their electro-motive forces. We have learned that difference of electro-motive force is due to the difference of the chemical action on the two plates used, and this depends upon the nature of the substances used. It is wholly independent of the size of the plates; hence the electro-motive force of a large battery cell is no greater than that of a small one of the same kind. Consequently any difference in strength of current

yielded by battery cells of the same kind, but of different sizes, is due wholly to a difference in their internal resistances.

The electro-motive forces of the Bunsen, Daniell, and Grenet cells are respectively about 1.8, 1, and 2 volts.

475. Divided circuits; shunts. Make a divided circuit as in Fig. 331 (using double connections a and b). Insert a galvanometer G in one branch and a resistance box R in the other. When the current reaches a it divides, a portion traversing one branch through the galvanometer, and the remainder passing through the other branch and the resistance box. The branch aRb is called a *shunt* or *derived circuit.* Increase gradually the resistance in the resistance box. The result is that it throws more of the current through the galvanometer, as shown by the increase of deflection.

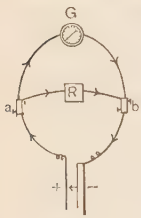


FIG. 331

In a divided circuit the current divides between the paths inversely as their resistances. For example, if the resistance of the resistance box above be 4 ohms, and the resistance in the galvanometer be 1 ohm, then four fifths of the current will traverse the latter and one fifth the former.

Suppose that the resistance box and galvanometer be removed from the shunts, and that the shunts be of the same length, size, and kind of wire, and consequently have equal resistances. Using the two wires instead of one to connect a and b is equivalent to doubling the size of this portion of the conductor; consequently the resistance of this portion is reduced one half.

Generally, *the joint resistance of two branches of a circuit is the product of their respective resistances divided by their sum.*

476. Kirchhoff's law. The following discussion will make the above law evident. First it should be understood that when a conductor conveys a constant current the strength of current across all cross sections of the conductor, as A, C, E F, D, and B (Fig. 332), is the same. Hence the current arriving at C or D of the main circuit is equal to the sum of the currents which flow by the branches r_1, r_2, and r_3. This is known as Kirchhoff's first law.

By Ohm's law if two points, a and b, between which a difference of potential V is maintained, be connected by two wires having resistances r_1 and r_2, the current in that of resistance r_1 will be $\frac{V}{r_1}$, and in the other $\frac{V}{r_2}$. If C be the whole current flowing in the circuit, we have by Kirchhoff's law

$$C = \frac{V}{r_1} + \frac{V}{r_2} = \frac{V}{R},$$

where R is the resistance of a wire which might be substituted for the divided conductor between a and b without affecting the current. Hence

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{R}, \text{ and } R = \frac{r_1 r_2}{r_1 + r_2}.$$

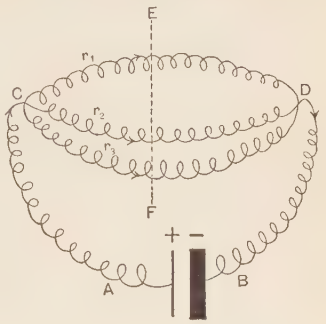


FIG. 332

If there be three separate wires of resistance, as in Fig. 332, we shall have in a similar manner

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}.$$

The reciprocal of the resistance R of a wire, i.e. $\frac{1}{R}$, is called its conductivity. We may say, therefore, in general, when two points in a circuit are connected by a *multiple arc* (a term in common use to denote a divided circuit between any two points) consisting of n branches, *the conductivity of the multiple arc is equal to the sum of the conductivities of the n branches; in other words, the reciprocal of its resistance is equal to the sum of the reciprocals of the resistances of its branches.*

477. Shunted galvanometer. When a current is so strong as to produce too violent an impulse upon the needle of a galvanometer, its terminals may be shunted through a resistance box, so that any known fraction of the current may be deflected through the shunt. For example, if the shunt have a resistance $\frac{1}{9}$ as great as that of the galvanometer, then the current through the latter will be $\frac{1}{9}$ that through the shunt, or $\frac{1}{10}$ of the total current.

478. Methods of combining cells. Take two Bunsen cells, and connect the two zinc plates by a wire. Then connect each of the carbon plates with a galvanometer. The E.M.F. of each cell would tend to send a current opposite to that of the other cell. But you find that there is either no deflection in the galvanometer, or at most a very small one, and this shows either that there is no current or that the current is very weak. The reason is evident. You have connected, through the galvanometer, two carbons which have the same potential; consequently there should be no current between them. The cells are said to be *connected in opposition.*

A very simple way of showing that a large cell has no greater electro-motive force than a small one is to connect two such cells in opposition through a galvanometer, or, what answers the same purpose, to raise the zinc of one of two cells of the same size, connected in opposition, nearly out of the liquid. The absence of a current shows that the two carbons have the same potential, and consequently their electro-motive force is the same.

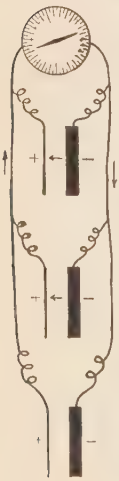


FIG. 333

A number of cells connected in such a manner that the currents generated by all have the same direction constitutes *a voltaic battery.*

The object of combining cells is to get a stronger current than one cell will afford. We learn from Ohm's law that there are two, and only two, ways of increasing the strength of a current. It must be done either by increasing the E.M.F. or by decreasing the resistance. So we combine cells into batteries, either to secure greater E.M.F. or to diminish the internal resistance. Unfortunately both purposes cannot be accomplished by the same method.

479. Batteries of low internal resistance. Fig. 333 represents three cells having all the positive (+) terminals electrically connected with one another, and all the negative (−) terminals connected with one another, and the triplet positives are connected with the triplet negatives by the leading-out wires through a galvanometer *G.*

It is easy to see that through the battery the circuit is divided into three parts, and consequently the conductivity in this part of the circuit, according to the principle stated in § 478, must be increased threefold; in other words, the internal resistance of the three cells is one third of that of a single cell. This is called connecting cells "in multiple arc," and the battery is called a "battery of low internal resistance." The resistance of the battery is decreased as many times as there are cells connected in multiple arc, but the E.M.F. is that of one cell only.

The formula for the current strength in this case is written thus:

$$C = \frac{E}{R + \frac{r}{n}},$$

in which n represents the number of cells. It is evident from this formula that when R is so great that $\frac{r}{n}$ is a small part of the whole resistance of the circuit, little is added to the value of C by increasing the number of cells in multiple arc.

480. Batteries of high internal resistance and great E.M.F. Fig. 334 represents four cells having the positive terminal of one connected with the negative of the next, and the + terminal at one end of the series connected by leading-out wires through a galvanometer with the — terminal at the other end of the series. It is evident that the current in this series traverses the liquid four times, which is equivalent to lengthening the liquid conductor four times, and of course increasing the internal resistance fourfold. But, while the internal resistance is increased, *the E.M.F. of the battery is increased as many times as there are cells in series.* The gain by increasing the E.M.F. more than offsets, in many cases (always when the internal resistance is a small

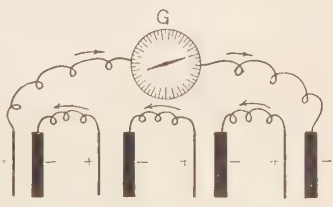


FIG. 334

part of the whole resistance of the circuit), the loss occasioned by increased resistance.

The formula for current strength in this case becomes

$$C = \frac{n\,E}{R + nr}.$$

It is evident that C is increased most by adding cells in series when nr is smallest in comparison with R.

481. Best arrangement of cells. Hence (1) *when the external resistance is large, connect cells in series;* (2) *when the external resistance is less than the internal resistance, connect cells in multiple arc.*

The two systems may be combined in one battery. Thus one pair in series may be placed in multiple arc with another pair in series. This combination would give double the E.M.F. of a single cell, but the resistance of only one cell.

The formula for the current strength in this case becomes

$$C = \frac{x\,E}{R + \dfrac{xr}{y}},$$

in which x is the number of cells in series and y the number in multiple arc. Also $xy = n$, the number of cells.

With a given number of cells and a given external resistance the maximum current is generated when the external and internal resistances are equal. It is seldom possible in practice so to join the cells as to fulfill this condition; but if the strongest possible current be required it should be fulfilled as nearly as possible. The fallacy, however, of introducing resistance into any part of the circuit for the purpose of making these resistances equal must be carefully avoided, for resistance wherever introduced can tend only to weaken the current. Nor must it be supposed that of two batteries of equal E.M.F., but one having a high, the other a low, resistance, the former is better adapted for working through a high resistance. It should be borne in mind that the *rôle* of a battery in general is to maintain a difference of

potential between its poles, and the element of resistance that it introduces into the circuit is a necessary evil, not to be voluntarily increased.

Electro-magnets and galvanometers must be adapted to the circuits in which they are to be placed. In connection with the above discussion it seems proper to introduce, somewhat parenthetically, a few facts pertaining to the use of electro-magnets and galvanometers.

In order to produce the greatest effect, the resistance of the helix of an electro-magnet should be equal to that of the portion of the circuit not included in the helix, i.e. to the rest of the circuit. When several electro-magnets are used in the same circuit, the sum of the resistances of all the helices should be equal to the resistance of the rest of the circuit.

The same general rule applies to galvanometers and in fact to all devices operated by the current. High-resistance galvanometers are most suitable for high-resistance circuits, and low-resistance galvanometers are most suitable for low-resistance circuits. In other words, all electrical devices should be adapted to the resistance of the circuit in which they are to be used.

This must not be confused with the law of *maximum efficiency.*

EXERCISES

1. What E.M.F. is required to maintain a current of one ampere through a resistance of one ohm?

2. Through what resistance will an E.M.F. of ten volts maintain a current of 5 amperes?

3. What current ought an E.M.F. of 20 volts to maintain through a resistance of 5 ohms?

4. A voltmeter applied each side of an electric lamp shows a difference of potential of 40 volts; what current flows through the lamp, if it have a resistance of 10 ohms?

5. The resistance between two points in a circuit is 10 ohms. An ammeter shows that there is a current strength in the circuit of 0.5 ampere; what is the difference in potential between the points?

6. What current will a Bunsen cell furnish when $r = 0.9$ ohm (about the resistance of a quart cell), $E = 1.8$ volts, and $R = 0.01$ ohm (about the resistance of 3 ft. of No. 16 wire)?

[In the following exercises, whenever a Bunsen cell is mentioned it may be understood to be a quart cell, having a resistance of about 0.9 ohm. Its E.M.F. is about 1.8 volts.]

7. (*a*) When is a large cell considerably better than a small one? (*b*) When does the size of the cell make little difference in the current?

8. If you have a dozen quart cells, how can you make them equivalent to one 3-gallon cell?

9. If a battery of 10 cells have an E.M.F. ten times greater than that of a single cell, why will not the battery yield a current ten times as strong?

10. (*a*) The internal resistance of ten cells connected in multiple arc is what part of that of a single cell? (*b*) If the cells were connected in series, how would the resistance of the battery compare with that of one of its cells? (*c*) How would the E.M.F. of the latter battery compare with that of a single cell?

11. What current will a single Bunsen cell furnish through an external resistance of 10 ohms?

12. What current will 8 Bunsen cells in series furnish through the same resistance?

13. What current will 8 Bunsen cells in multiple arc furnish through the same external resistance?

14. A coil of wire having a resistance of 10 ohms carries a current of 1.5 amperes. Required the difference of potential at its ends.

15. (*a*) The resistance between two points, A and B, of a conductor is 2.5 ohms; the resistance of a shunt between the same points is 1.5 ohms; what is the joint resistance between these points? (*b*) If a current of 10 amperes be maintained between these points, what will be the strength of current in each branch? (*c*) How would the strength of current between these points be affected if the shunt be removed and the same fall of potential be preserved? Why?

16. (*a*) Points A and B in a circuit are connected in multiple arc by three branches whose respective resistances are 2, 3, and 4 ohms. State in order their relative conductivities. (*b*) State the joint conductivity of the multiple arc. (*c*) State the joint resistance of the multiple arc.

17. Four conductors in multiple arc have resistances of 100, 50, 27, and 19 ohms. What is their combined resistance?

18. If the terminals of a galvanometer be shunted with a resistance of $\frac{1}{99}$ that of the galvanometer, what part of the total current will the galvanometer measure?

19. An electric lamp has a resistance of 50 ohms; it is connected to a street main by leads of 2.5 ohms resistance. The heat developed in the leads is wasted. What portion of the entire heat developed in the derived circuit is wasted?

20. (*a*) The internal resistance of a voltaic circuit is 2 ohms, and the external resistance is 16 ohms; what portion of the entire heat developed

in the circuit is generated in the battery ? (*b*) If the external resistance of this circuit be reduced to 0.5 ohm, the heat generated in the battery will be what part of the total heat developed in the circuit ?

21. What is the strength of a current which deposits 0.02 gram of copper per minute ?

22. Suppose that there are a number of cells joined in series but the circuit is completed by short, thick, copper leading wires of practically no resistance, would any advantage be gained by adding thereto more cells in series ? Explain.

23. A battery of 20 cells is divided into four groups. Each group consists of five cells connected in series, and the four groups are connected in multiple arc. Compare the E.M.F. and the resistance of this battery with that of a single cell.

24. What E.M.F. of a dynamo generator will be necessary to maintain a 12-ampere current for 100 arc lamps in series, each of which has a resistance of 5 ohms, the resistance of the line wire being 20 ohms, and the dynamo resistance being 25 ohms ?

SECTION XI

VERIFICATION OF OHM'S LAW

482. Relation of current, resistance, and potential difference. This relation is best understood by the aid of the hydraulic analogue. *B* (Fig. 335) represents a tank in which water is maintained at a fixed level by means of a pump (battery), while the tank discharges through a pipe (conductor) *NO*. At equal intervals glass tubes, serving as manometers (potential meters), rise from the discharge tube. The height to which the water rises in each tube indicates the pressure at that point. The pressure falls off uniformly toward *O*, as indicated by the dotted line *as*. The pressure of the column *a'a*

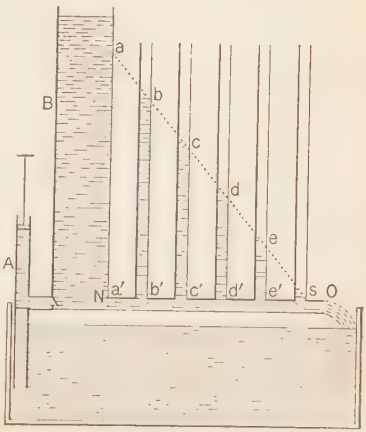


FIG. 335

is required to force the current against the resistance of the pipe NO. The force urging the water from a' to b' along the pipe is the difference of pressure at a' and b'. This might be called the water-motive force between a' and b'. If $a'e'$ be four times $d'e'$, the resistance will be four times as great and the water-motive force between a' and e' will be four times as great as that between d' and e'. The fall of pressure is the same for each unit of resistance. These facts in hydraulics have their exact parallel in the case of electric flow, and the analogues are too apparent to require rehearsal. The fact to be established is that every current is due to a determinable E.M.F. in the circuit, and fractional parts of the circuit require fractional parts of the corresponding difference of potential. The portion of the total E.M.F. exerted in forcing the current through any section of the circuit is in exact proportion to the relative resistance of such section. For example, a battery of ten units resistance may supply an outer circuit of ten units resistance; then one half of the E.M.F. will be exerted against battery resistance, and one half against external resistance. If the same battery supply a line of 1000 units resistance, the energy expended in the outer circuit is about $\frac{99}{100}$ of the total energy. The electrical efficiency of an electric generator is the resistance of the outer circuit divided by the total resistance. In the example above, the efficiency of the battery is

$$\frac{1000}{1000+10}, \text{ or about } 99\%.$$

483. Expenditure of energy. The relative resistances of conductors carrying constant currents define the relative expenditure of energy upon such conductors. The energy may appear as heat, as mechanical work, or as that of chemical decomposition. The work done is proportional to the quantity of electricity and to a fall of potential along the conductor, and the fall is determined by the relative resistances of the several conductors considered.

This subject may be illustrated graphically thus: Let the line AB (Fig. 336) represent the length of a circuit (say 1000 feet), and the line AC the total fall of potential; then obviously the slope of the line CB will represent the average rate of fall of potential throughout the circuit. But suppose that the line for equal lengths of the conductor varies in resistance. Thus assume that one tenth the resistance and consequently one tenth the fall of potential (Cd) is included in the first quarter (Aa), or 250 feet; then that the next 250 feet (ab, being very fine wire, perchance) represents one half the total resistance; the next 250 feet (bc) represents one fourth the total resistance; and the remaining resistance, fifteen hundredths, is in the last section (cB) of 250 feet. Then the lines Ca',

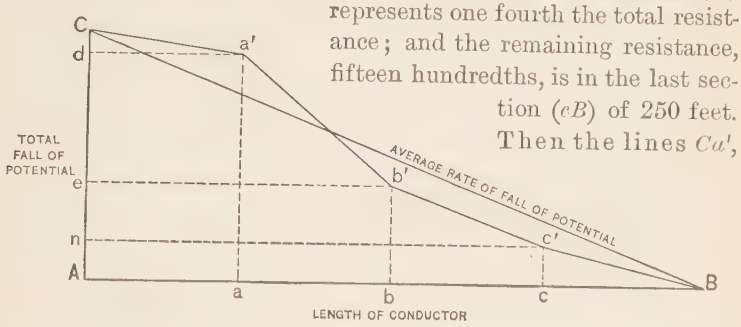


FIG. 336

$a'b'$, $b'c'$, and $c'B$ represent respectively the rate of fall of potential in each section of 250 feet. The fall in each section is proportional to the resistance, and again to this is proportional the work performed by the current in each section.

The above facts may be verified experimentally as follows: PQ (Fig. 337) is a fine German-silver wire one meter long, stretched along a board over a metric scale; B is a battery; G, a mirror galvanometer; K, a key for closing the circuit; and R, a high resistance. When the free end of the wire S is applied to any point of the wire PQ, the circuit is shunted through G and R. By this means an extremely small portion of the current will be shunted through the galvanometer. This will not perceptibly change the potentials of the points P and S. Observations

are made by placing the free end of the wire S at different points along the wire and reading the deflection produced at each point. Now if the potential differences are proportional to the resistances, i.e. to the lengths of wire between the points P and S, it follows that this resistance should have a constant ratio to the difference of potential between the same points. But the difference of potential is expressed by the current which it sends through the galvanometer, so that this current (which is indicated by the deflection of the galvanometer needle) should be proportional to the distance between P and S.

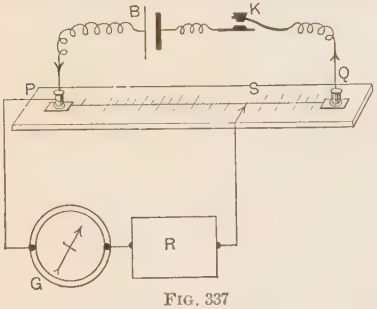


FIG. 337

SECTION XII

MAGNETS AND MAGNETISM

484. Law of magnets. The word magnet is supposed to have been derived from the name of an ancient district in Asia Minor called Magnesia, where was discovered at an early period a mineral (now known as the magnetic oxide of iron) which possesses the property of attracting iron. The term *lodestone*, or "leading stone" (inasmuch as it takes a definite position north and south), has been given to these natural magnets.

Suspend by fine threads in a horizontal position two stout darning needles which have been drawn in the same direction (e.g. from eye to point) several times over the same pole of a powerful electro-magnet. These needles, separated a few feet from each other, take positions parallel with each other, and both lie in a northerly and southerly direction with the points of each turned in the same direction.

That point in the Arctic zone of the earth towards which magnetic needles point is called the north magnetic pole of the earth.

That end of a needle which points toward the north magnetic pole of the earth is called the *north-seeking, marked,* or + *pole*: this is the end that is always *marked* for the purpose of distinguishing one from the other. That end of the needle which points southward is called the *south-seeking, unmarked,* or − *pole.*

Bring both points near each other; there is a mutual repulsion. Bring both eyes near each other; there is a mutual repulsion. Bring a point and an eye near each other; there is a mutual attraction.

Like poles of magnets repel, unlike poles attract each other.

485. Magnetic transparency and induction. Interpose a piece of glass, paper, or wood between the two magnets. These substances are not themselves perceptibly affected by the magnets, nor do they in the least affect the attraction or repulsion between the two magnets.

FIG. 338

Substances that are not susceptible to magnetism are said to be *magnetically transparent.* When a magnet causes another body, in contact with it or in its neighborhood, to become a magnet it is said to *induce* magnetism in that body. As attraction and never repulsion occurs between a magnet and an unmagnetized piece of iron or steel, it must be that the magnetism induced in the latter is such that opposite poles are adjacent; that is, a *N* or + pole induces a *S* or − pole next itself, as shown in Fig. 338.

FIG. 339

486. Polarity. Strew iron filings on a flat surface, and lay a bar magnet on them. On raising the magnet it is found that large tufts of filings cling to the poles, as in Fig. 339, especially to the edges; but the tufts diminish in size from each pole towards the center, where none are found.

Magnetic attraction is greatest at the poles, and diminishes towards the center, where it is nothing; i.e. *the center of the bar is neutral.* This dual character of the magnet, as exhibited at the opposite extremities of a magnet, is called

polarity. If a magnet be broken, each piece becomes a magnet with two poles and a neutral line of its own.

487. Retentivity and resistance. It is more difficult to magnetize steel than iron; on the other hand, it is difficult to demagnetize steel, while soft iron loses all its magnetism as soon as it is removed from the influence of the inducing body. That quality of steel by which it resists the escape of magnetism which it has once acquired is called its *retentivity*. The greater the retentivity of a magnetizable body, the greater is the *resistance* which it offers to becoming magnetized. *The harder steel is, the greater is its retentivity.* Hence, hard steel is used for *permanent* magnets. Hardened iron possesses considerable retentivity; hence the cores of electro-magnets should be made of the *softest* iron, that they may acquire and part with magnetism instantaneously.

488. Forms of artificial magnets. Artificial magnets, including permanent magnets and electro-magnets, are usually made in the shape either of a straight *bar* or of the letter U, according to the use to be made of them. If we wish, as in the experiments already described, to use but a single pole, it is desirable to have the other as far away as possible; then, obviously, the bar magnet is most convenient. But if the magnet is to be used for lifting or holding weights, the U-form is far better, because the attraction of both poles is conveniently available. Magnets, when not in use, ought always to be protected by armatures (*A*, Fig. 340) of soft iron; for, notwithstanding the retentivity of steel, they slowly part with their magnetism. But when an armature is used the opposite poles of the magnet and armature being in contact with each other, i.e. *N* with *S*, they serve to bind each other's magnetism. Thin bars of steel can be more thoroughly magnetized than thick ones. Hence if several thin bars (Fig. 340) be laid side by side, with their corresponding poles turned in the same direction and then screwed together, a very powerful magnet is the result. This is called a *compound magnet*.

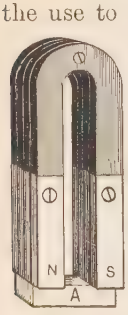


FIG. 340

SECTION XIII

MAGNETIC LINES OF FORCE. THE MAGNETIC CIRCUIT

489. Magnetic lines of force. These lines are easily studied by the use of iron filings. The field of force around a magnet is shown by placing a paper over it, dusting filings upon the paper, and tapping it. The filings take symmetrical positions, form curves between the poles of the magnet or magnets, and show that *the lines of force connect the opposite poles of the magnet.* The fact is that each filing, when brought within the magnetic field, becomes a magnet by induction, and of necessity tends to take a definite position which represents the resultant of the forces acting upon it from

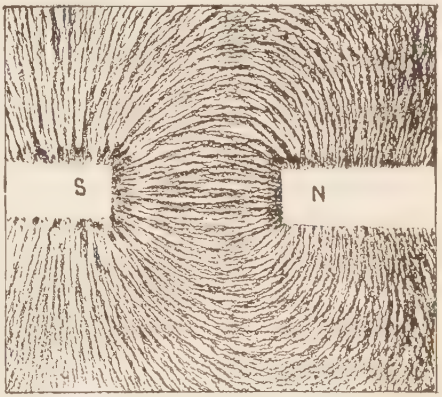


FIG. 341

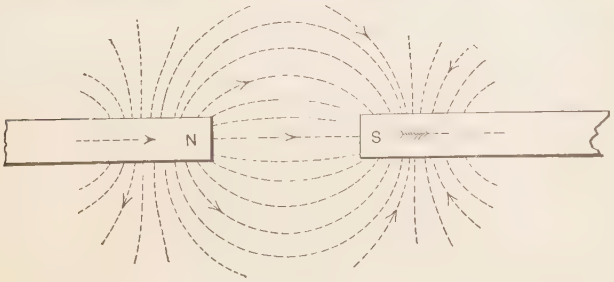


FIG. 342

each pole of the system. Fig. 341 represents a magnetic field photographed from a specimen paper, and Fig. 342 is a graphical

representation of the same. In this illustration the unlike poles of two magnets are placed opposite each other. Fig. 343 is a diagram of paths of lines of force of a bar magnet, and Fig. 344 of a horseshoe magnet.

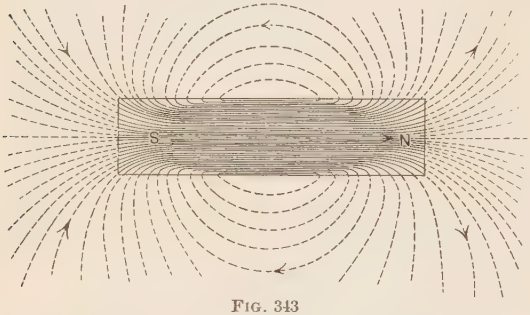


FIG. 343

490. Magnetic circuit. A line of force is assumed arbitrarily to start from the *N* pole and to pass through the surrounding medium (e.g. the air), entering the magnet by the *S* pole, and completing its path through the magnet itself to its starting point (the *N* pole), thus forming a complete circuit (Fig. 343). These lines do not all emerge, however, from the extremities. A multitude of lines start from all parts of the magnet and enter at corresponding points on the other side of its central or neutral line. No magnetic line of force can exist without completing its own circuit,[1] and lines of force never cross or merge into one another; consequently a magnet cannot have a single pole.

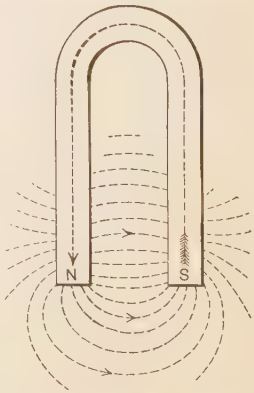


FIG. 344

Lines of force possess several peculiar characteristics. One is that in air and most other mediums they tend to separate from one another, but at the same time tend to become as short as possible. The strain is as if these lines were stretched elastic threads endowed

[1] Herein exists a notable difference between these and electrostatic lines of force. Every electrostatic charge is bound, i.e. has an opposite and equal charge somewhere. To this its lines of force go ; but there is no circuit, — there is only a connection.

with the property of repelling one another as well as of shortening themselves ; in other words, there is tension along the lines and pressure at right angles to them. Air is a poor conductor for lines of force, or its *permeability*[1] is low; on the other hand, iron has high permeability for lines of force, and if a piece of iron be brought within a magnetic field, a portion of the lines of force will crowd together into it, leaving their normal paths through the air for a medium of greater permeability.

FIG. 346

491. Line of force the axis of ether whirls. A line of force is supposed to represent the axis of a series of ether whirls. Fig. 345 gives a crude pictorial representation of the supposed constitution of the whirls of an electro-magnetic line of force. A series of curtain rings might be strung upon a stretched thread and caused to rotate around it. This would give some idea of the hypothesis ; the thread would give the direction of the line of force and its conventional representation as a simple line. The perfectly circular line of force is such as those surrounding a wire carrying a current, and represented in Fig. 346. Its mechanical analogue is seen in a smoke ring such as is sometimes caused to rise from the bowl of a tobacco pipe by skillful operation. It is well illustrated by the rings arising from the spontaneous combustion of phosphureted hydrogen in air. These show the whirling, rotary motion around the circular axis of the ring. Such are called vortex rings.

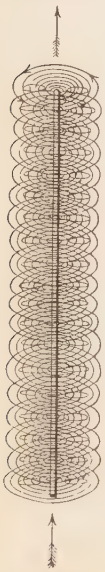


FIG. 345

492. Attraction and repulsion between magnetic poles. If the *N* pole of one magnet be placed opposite the *S* pole of another (Fig. 342), the lines of force issuing from the former will enter the latter, and, tending to shorten, will produce *attraction*. If the similar ends be opposed (Fig. 347), the lines of force will be turned

[1] *Permeability* is a quality christened by Lord Kelvin.

away from each pole in all directions, and will complete their circuits independently. Thus becoming parallel, they will repel one another; for this reason like magnetic poles repel each other.

493. Equipotential surfaces. The magnetic potential at any point in a magnetic field is the quantity of work that would have to be done in bringing a + pole of unit strength from an infinite distance to that point.

If a magnetized particle be moved either toward or from a magnetic pole, work is done either by or against the attracting or repelling force, and the particle is said to be moved from a point where the magnetic potential has one value to a point where it has another value. Imagine an N pole isolated from its

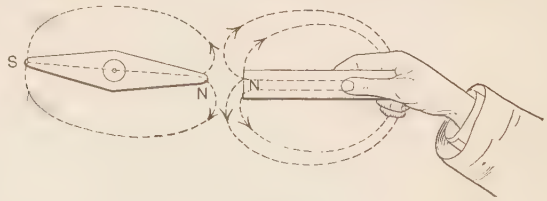


FIG. 347

companion S pole. We may suppose that all points at equal distances from this pole are at the same potential, and these points joined form a spherical surface. The potential at every point of this surface (being the same) may be represented by V. Within and without this lie successive spherical *equipotential surfaces* over each of which the potential is constant, but the potential at each surface differs from that at any of the others. No work is required to move a quantity of magnetism from one point on an equipotential surface to another point on the same surface. Equipotential surfaces are not necessarily spherical. But whatever be their form, the lines of force are always at right angles to the equipotential surfaces.

494. Tubes of force. Suppose AB (Fig. 348) to be a section of an equipotential surface. Lines of force pass through the surface, those grazing its edges cutting off an area $A'B'$ from another

equipotential surface. The space comprised between these equipotential areas and bounded laterally by lines of force is called a *tube of force*. This space may be supposed to be filled with a bundle of lines of force. Now the intensity of the magnetic force at the potential $A'B'$, as compared with that at AB, is in inverse proportion to the magnitudes of their respective areas.

The intensity of magnetic forces at any two points may be compared by stating the relative numbers of the lines of force which pass through equal units of area of the equipotential surfaces containing the points compared. The fewer these lines per unit area, the less the local intensity of the force. The strength of field between the poles of a magnet may be expressed in dynes, but now it is more usual to express this strength by the number of lines of force per square centimeter, each line of force representing a dyne.

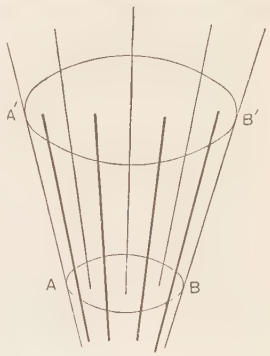


FIG. 348

For example, if the strength of the magnetic field, or the force on a unit pole, be ten dynes at any point, then ten lines of force are said to pass through an area of 1 square centimeter held perpendicularly to these lines.

495. Law of inverse squares. By obvious experimental methods the law of inverse squares may be demonstrated as applied to magnetism, viz.: *The force exerted between two magnetic poles varies inversely as the square of the distance between them.*

The magnitude of the force in any case is numerically equal to the product of the strength of the poles divided by the square of the distance between them;

$$f = \frac{mm'}{d^2}.$$

The C.G.S. unit pole is that pole which repels an equal pole placed a centimeter away with force of one dyne.[1]

[1] In practice it is impossible to obtain a single isolated pole; the total quantity of (+ and −) magnetism in any magnet is, algebraically reckoned, zero.

496. Definitions. We are now in a position to understand the following definitions: A portion of space throughout which magnetic effects are exerted by a distribution of magnetism is called a *magnetic field*. At each point of the field a pole of intensity, m, is acted on by a definite force. The *intensity of a magnetic field at a given point* is equal to the force in dynes with which a unit pole would be acted upon at that point. Let H denote the intensity of field at any point; then the force actually exerted at that point on a pole of strength m is mH.

A line of magnetic force is a line drawn in such a manner that the tangent to it at each point is in the direction of the resultant magnetic force at that point.

The *magnetic potential* at any point is the work that must be done against the magnetic forces in bringing up a positive unit magnetic pole from a point at an infinite distance to the given point. The *difference of magnetic potential* between two points in the field is the work done in transferring a unit positive magnetic pole from one of these points to the other, against magnetic forces.

Let m be the quantity of magnetism at one of the poles of a magnet, and l the distance between the poles[1]; the *magnetic moment* of the magnet is the product ml.

The *intensity of magnetization* is the ratio of the magnetic moment of a magnet to its volume.

A magnetizable body placed in a magnetic field becomes magnetized in the direction of the lines of force of the field. Its magnetism is called *induced magnetism*, and the action itself is called *magnetic induction*. The magnetism retained by a magnetic body after it has been withdrawn from the field is called *residual magnetism*.

[1] In a steel bar magnet the poles are not strictly at the extremities, and hence the *magnetic length* is a little less than the actual length of the magnet. For most bar magnets the magnetic length is about 0.83 of the actual length. The magnetic length of a horseshoe magnet is the shortest distance between its poles.

SECTION XIV

TERRESTRIAL MAGNETISM

497. The earth a magnet. A *dipping needle* is a long, light magnet so supported that it can revolve in a vertical plane. Indifferent equilibrium is first established in the steel needle, so that if placed in a horizontal (or any other) position it will rest in that position. Then it is strongly magnetized. Afterward it will take the horizontal position only at the magnetic equator of the earth.

Place such a needle over the positive pole of a bar magnet (Fig. 349). The needle takes a vertical position with its negative pole down. Slide the supporting stand along the bar; the negative pole gradually rises until the stand reaches the middle of the bar, where the needle becomes horizontal. Continue moving the stand toward the negative pole of the bar; after passing the middle of the bar the positive pole begins to dip, and the dip increases until the needle reaches the end of the bar, when the needle is again vertical with its positive pole down.

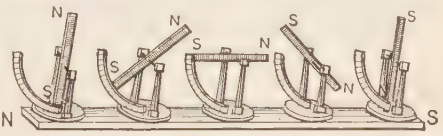


FIG. 349

If the same needle be carried northward or southward along the earth's surface, it will dip in the same way as it approaches the polar regions, and be horizontal only at or near the equator.

Support a small pane of window glass on a table by placing under the glass near its corners four slices of cork about one eighth of an inch thick. Beneath the center of the glass on the table place a circular disk of magnetized steel. Sift iron filings upon the upper face of the glass through a fine wire sieve. Gently tap the glass at convenient points with the end of a lead pencil. The filings arrange themselves in lines radiating from each pole.

Suspend a small magnetized cambric needle by a fine thread at its center and carry it around the disk (Fig. 350). The needle

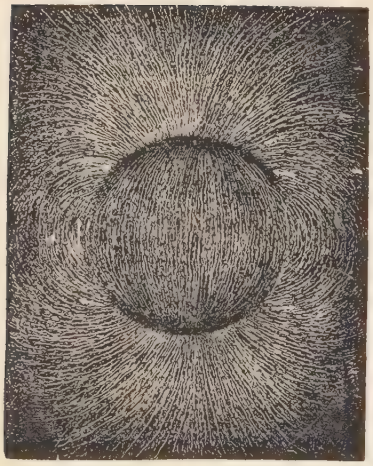


FIG. 350

passes through all the phases stated above, so that we may fancy the disk to be the earth, and study therefrom, in a general way, the changes that the needle undergoes as it is carried around the earth in a northerly or southerly direction.

The last experiment presents a true exhibition on a small scale of what the earth does on a large one, and thereby presents one of many phenomena which lead to the conclusion that the *earth is a magnet*. In other words, these phenomena are just what we should expect if a huge magnet were thrust through the earth near the axis of rotation, as represented in Fig. 351, having its *N* pole near the *S* geographical pole, and its *S* pole near the *N* geographical pole; or if the earth itself were a magnet.

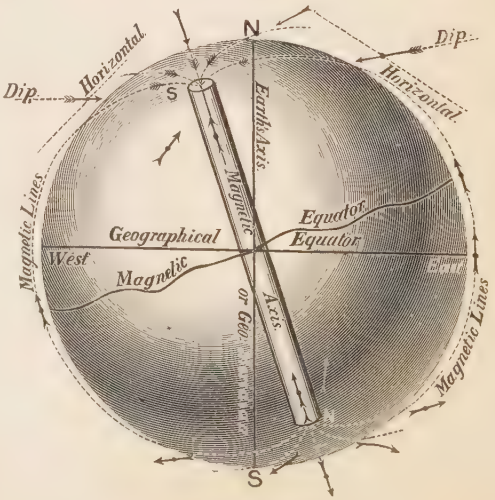


FIG. 351

498. Magnetic poles of the earth. It will be seen that there are two places where the needle points directly to the center of the disk. A point was found on the western

coast of Boothia, by Sir James Ross, in the year 1831, where the dipping needle lacked only one sixtieth of a degree of pointing directly to the earth's center. The same voyager subsequently reached a point in Victoria Land where the opposite pole of the needle lacked only 1° 20′ of pointing to the earth's center.

It will be seen that, if we call that end of a magnetic needle which points north the *N* pole, we must call that magnetic pole of the earth which is in the northern hemisphere the *S* pole, and vice versa (see Fig. 351). Hence, to avoid confusion, many careful writers abstain from the use of the terms *north* and *south poles*, and substitute for them the terms *positive* and *negative* or *marked* and *unmarked poles*.

499. Variation of the needle. Inasmuch as the magnetic poles of the earth do not coincide with the geographical poles, it follows that the needle does not in most places point due north and south. The angle which the vertical plane through the axis of a freely suspended needle makes with the geographical meridian of the place is known as the *angle of declination*. In other words, the angle of declination is the angle formed by the magnetic and geographical meridians. This angle differs at different places. The magnetic axis of a needle is a straight line connecting its two poles.

We can easily find, as did Columbus, the declination at any place by the following method: Set up two sticks so that a string joining them will lie in the same vertical plane with the Pole Star; the string will lie in the geographical meridian. Place a long magnetic needle over the string; the angle between the needle and the string is the required declination. If great accuracy be required, allowance must be made for the fact that the star is not exactly over the pole, but appears to describe daily around it a circle whose diameter is at the present time about $2\frac{1}{2}°$.

500. Isogonic curves. These are lines connecting all points of equal declination on the earth's surface. The line of no declination or isogonic of 0° (Fig. 352) commences at the *N* magnetic pole about latitude 70°, longitude 96°, passes in a southeasterly

direction across Lake Erie and western Pennsylvania, and enters the Atlantic Ocean near the boundary between the Carolinas. Pursuing its course through the south-polar regions, it reappears in the eastern hemisphere and crosses western Australia, the Caspian Sea, and thence to the Arctic Ocean. There is also a detached line of no declination inclosing an oval area in eastern Asia and the Pacific Ocean. In the eastern (or Atlantic) hemisphere,

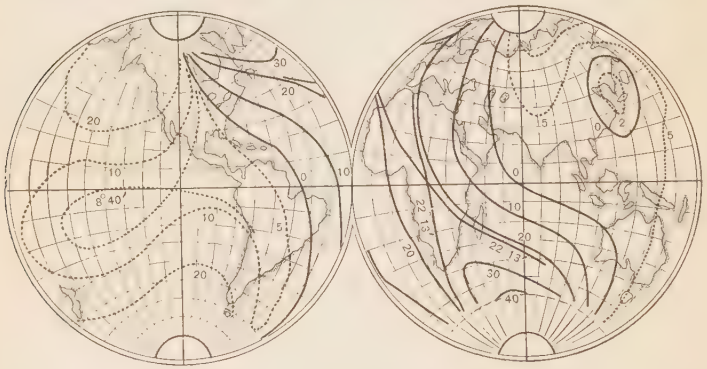


FIG. 352

bounded by the line of no declination, the declination is westward, as indicated by continuous lines in the figure. In the western (or Pacific) hemisphere the declination is eastward, as indicated by dotted lines.

The magnetic poles are not fixed objects that can be located like an island or a cape, but are constantly changing. They appear to swing something like a pendulum, in an easterly and westerly direction, each swing requiring centuries to complete it. The north magnetic pole is now on its westerly swing.

501. Inclination or dip. We have seen that in the northern hemispheres the lines of force tend downward and northward. A magnetic needle thus tends to place itself so that its axis points downward and to the magnetic pole of the earth. The angle which the axis of a freely suspended magnetic needle makes with the horizontal plane is called the *inclination* or *dip*

of the needle. Fig. 353 represents a dipping needle such as is used in determining magnetic *inclination*, and Fig. 354 represents a declinometer for determining the declination. *A* is a mounted telescope for sighting some astronomical object, e.g. the north star. Its axis is leveled by the spirit level *B*.

The line passing in an easterly and westerly direction around the earth along which the lines of force (or needle) are horizontal, i.e. at which the dip is zero, is the *magnetic equator*. It does not coincide with the geographical equator. The lines roughly parallel to the magnetic equator, along which the dip is equal, are the magnetic parallels. These are lines along which equipotential surfaces cut the surface of the earth.

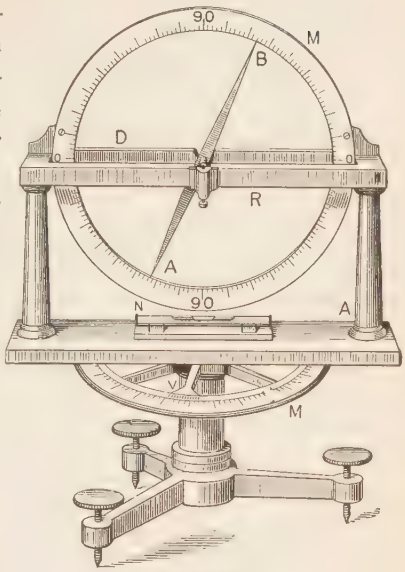


FIG. 353

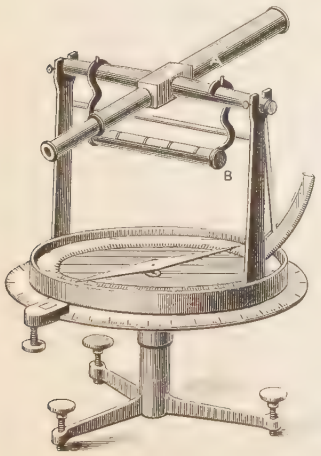


FIG. 354

We have before noted the fact that lines of force are always at right angles to equipotential surfaces, and consequently at the magnetic poles where the dip is 90° the equipotential surfaces are tangent to the earth's surface.

502. Intensity. The force, expressed in absolute measure, with

which the earth's magnetism urges a unit magnetic pole is the intensity of the earth's magnetic field[1] at the place.

The earth's action on a needle is a mechanical couple, the effect of which is to cause only a rotary motion. This is what is meant when the earth's action on the needle is said to be directive only.

503. Connection between the sun and the earth's magnetism. Magnetic storms, or disturbances of the earth's magnetism coincident with outbreaks of sun spots and solar storms, point to an undoubted connection or sympathy between the sun and the earth's magnetism; but of the nature of this connection our knowledge is as yet very limited.

SECTION XV

MAGNETIC RELATIONS OF THE CURRENT. ELECTRO-MAGNETS

504. Magnetic field due to a circular current. If a wire be bent into the form of a circle of about 10 inches diameter and placed vertically in the magnetic meridian, and a cardboard be placed at right angles to the circle so that its horizontal diameter is coincident with the upper surface of the cardboard, and a very strong current be sent through the wire in the direction indicated by the arrowhead in the wire, iron filings sifted upon the card will arrange themselves as shown in Fig. 355. And if a freely suspended test needle be carried inside and outside the circle, the several positions taken by the needle, as indicated in the figure by arrows, corroborate the directions of the lines of force as indicated by the filings.

[1] If the inclination be found, and the horizontal component of the intensity of the earth's field acting upon a unit pole be known, we have the data required for determining by parallelogram of forces the whole intensity of the earth's magnetic field in the direction of the lines of force at any point. Such a measurement is of importance in refined work, since it consists essentially in determining the couple which must be exerted on a needle by the earth's magnetic force in order to balance that produced by the current. For methods of determining these quantities, see any complete laboratory manual.

If the direction of the current be reversed, the direction of the needle will be reversed wherever it may be placed.

The direction indicated by the N pole of the magnetic needle placed anywhere in the field is called the *positive* direction of the lines of force.

505. Magnetic current. In fact, when a current traverses a wire (or other conductor) lines of force encircle the electric current at right angles to it. "Every conducting wire is surrounded by a sort of magnetic whirl. A great part of the energy of the so-called electric current in the wire consists in these external magnetic whirls. To set them up requires an expenditure of

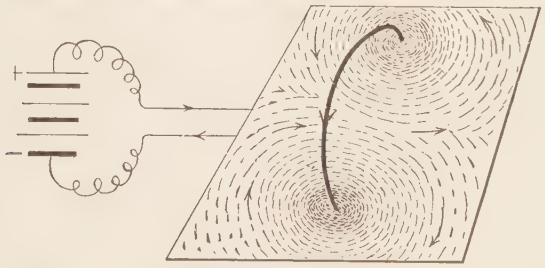


FIG. 355

energy; and to maintain them requires a constant expenditure of energy. It is these magnetic whirls which act on magnets, and cause them to set, as galvanometer needles do, at right angles to the conducting wire" (S. P. Thompson).

It may be demonstrated that a law analogous to Ohm's law is applicable to the magnetic whirl or "flux." Let C = the strength of the magnetic flux (or field) expressed in lines of magnetic force; M = the force which gives rise to the magnetic flux, called the *magneto-motive force;* and R = the resistance to the magnetic flux, called reluctance; then $C = \frac{M}{R}$. The electric current and its encircling lines of force always coexist, and one varies directly as the other when there is no magnetic substance near the wire. The direction of the encircling lines of force with reference to the electric current may be illustrated by the use of

a corkscrew. The direction of the electric current corresponds to the propulsion of the point of the corkscrew when entering a cork, and the direction in which the screw is turned or the hand is twisted in propelling it corresponds to the direction of the lines of force.

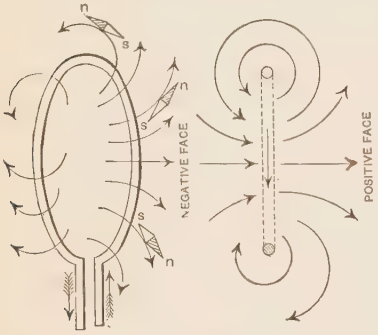


FIG. 356

If a circle of wire bearing a very strong current be freely suspended and a pole of a very strong bar magnet be presented to one of its faces, the circle will be attracted or repelled according to the nature of the pole and the direction of the current. We may consider a circular current as if it were a very short magnet, one face of the circle being the north end and the other face the south end, as represented in Fig. 356.

Observe (Fig. 355) that the directions of the filings near the center of the circle lie nearly parallel with its axis, but outside to the right and left of the axis the filings lie in curves around each wire.

506. Solenoid. An insulated wire wound into a helix of considerable length as compared with its diameter is called a *solenoid*.[1] It is evident that the intensity of the magnetic field must be greatly increased by the joint action of the many current turns. The magnetic field within the solenoid is nearly uniform in strength, and the lines of force to within a short distance of its ends are parallel with its axis, as shown in Fig. 357.[2]

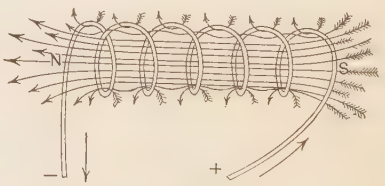


FIG. 357

[1] Faraday first applied the term *solenoid* to a system of circular currents parallel with one another.

[2] An open solenoid of a single layer is here given, in order the better to show the directions of the several lines of force.

507. Magnetic polarity of electro-magnetic solenoid. Fig. 358 represents a small battery floating on water. The leading wire of the cell is wound into a horizontal solenoid. After the cell is floated it will slowly take a position so that the axis of the solenoid will point north and south like a magnetic needle. Hold, say, the *S* pole of a bar magnet near that end of the solenoid which points north; the solenoid is attracted by the magnet. Hold the *N* pole of the magnet near the north-pointing end of the solenoid; the magnet repels the solenoid.

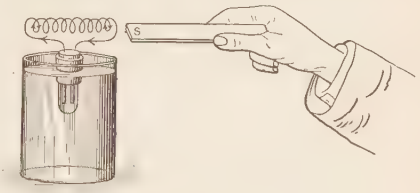


Fig. 358

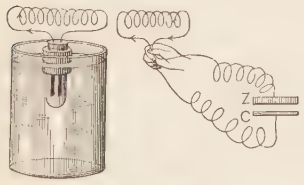


Fig. 359

Repeat the above, using in place of the bar magnet another solenoid (Fig. 359); there will be a repetition of the same phenomena as obtained with the bar magnet. Introduce a rod of soft iron into the solenoid held in the hand, thereby making of it an electro-magnet; the only change observed is that the force of attraction and repulsion is greatly increased.

Place the wire of another battery over and parallel with the coil (Fig. 360), so that the two currents will flow in planes at right angles to each other. The coil is deflected like a magnetic needle (Fig. 361).

Reverse the direction of the current above and the deflection is reversed.

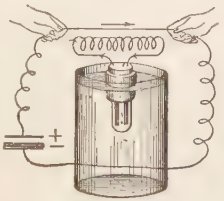


Fig. 360

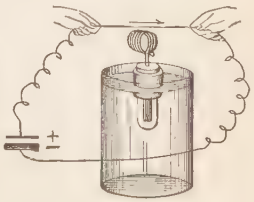


Fig. 361

We thus prove that a solenoid bearing a current possesses polarity as if it were a magnet, and that there can be produced

by a current-bearing solenoid a magnetic field of the same character as that produced by a permanent magnet. There is no essential difference between a permanent magnet, a current-bearing solenoid, and an electro-magnet, except that the last may be made much stronger than either of the others.

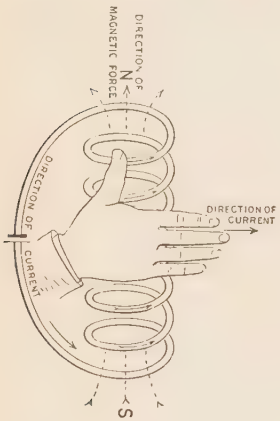


FIG. 362

Given the direction of the current in a solenoid, to find the N and S poles of the solenoid, and vice versa.

RULE 1. *Place the palm of the right hand against the side of the solenoid so that the fingers will point in the direction of the current passing through the windings* (as shown in Fig. 362); *the thumb will point in the direction of the N pole of the solenoid or electro-magnet.*[1]

RULE 2. *Ascertain the N pole of the solenoid or electro-magnet with a magnetic needle, and place the palm of the right hand upon the solenoid so that the outstretched thumb points in the direction of the N pole; the fingers will point in the direction in which the current passes in the windings.*

SECTION XVI

ELECTRODYNAMICS. AMPÈRE'S THEORY OF MAGNETISM

508. Mutual action of currents. We have hitherto discussed the direction of the magnetic field due to a straight current, have determined the direction which a test needle takes in virtue of the action of the current field, and have learned to

[1] The following suggestion will often prove of practical value: that is the south pole of a helix where the current corresponds to the motion of the hands of a watch, $\overrightarrow{S}$, and that is the north pole where the current is in the reverse direction, $\overleftarrow{N}$.

regard an electric current as producing north and south polarity along the whole circuit of the current-bearing wire. That is, if we suppose that a test needle be moved up or down just back of the current-bearing wires (Fig. 363), the *N* and *S* poles will take the positions indicated by *n* and *s*. We may readily premise from inspection of the polarity developed, that if the wires were so suspended as to be free to move either toward or from each other, the pair of wires in which the currents flow parallel to each other and in the same direction, *A*, would attract each other, and the pair of wires in which the currents flow in opposite directions, *B*, would repel each other; but if the currents be inclined to each other, as in Fig. 364, they will tend to move into a position in which they will be parallel and in the same direction. That such actually takes place may be shown by the following experiments.

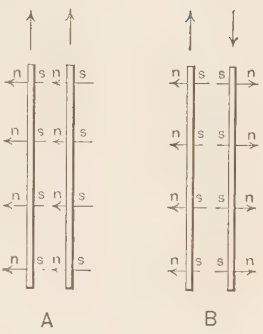


FIG. 363

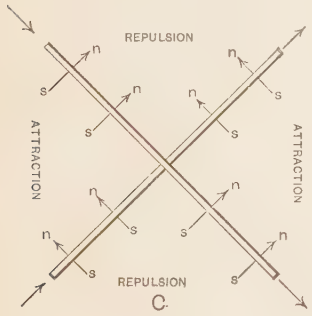


FIG. 364

Fig. 365 represents a portion of a divided circuit. The lower ends of the wires dip about one sixteenth of an inch into mercury, and the wires are so suspended that they are free to move toward or from each other. Send a current of a battery of three or four cells in multiple arc through this divided circuit. The two portions of the current travel in the same direction and parallel with each other, and the two wires at the lower extremities move toward each other, showing an attraction.

FIG. 365

Make the connections (Fig. 366) so that the current will go down one wire and up the other. They repel each other.

Send a current through the spiral wire represented in Fig. 367. Here the current flows nearly parallel with itself, and the attraction causes the coil to contract and to be lifted out of the cup of mercury below. But the instant it leaves the mercury the circuit is broken, the current and attraction cease, and the wire dips into the mercury again. Thus vibratory motion of the coil is produced.

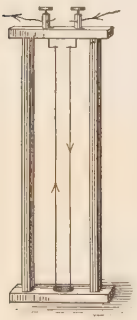


FIG. 366

In the experiment with the floating cell and current-bearing wire placed over and parallel to the solenoid (Fig. 360), a careful examination will disclose the fact that not only do the planes in which the current flows in the coil tend to become parallel to the current above, but that the current in the upper half of the coil, where the influence due to proximity is greatest, tends to place itself so as to flow in the same direction as that of the current above.

509. Ampère's laws. FIRST LAW. *Parallel currents, if in the same direction, attract one another: and if in opposite directions, they repel one another.*

SECOND LAW. *Currents that are not parallel tend to become parallel and flow in the same direction.*

A little reflection will show that the observed motion is the expression of *a tendency on the part of any movable current to cut lines of magnetic force at right angles*, the direction of motion being reversed when the direction either of the lines of force or of the current is reversed. This principle is of immense importance from an industrial aspect. The most important outcome of its application is the dynamo motor to be treated later.

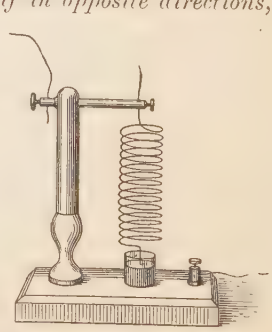


FIG. 367

510. Ampère's theory of magnetism. This celebrated theory briefly stated is that *magnets and solenoid systems are fundamentally the same;* that magnetism is simply electricity in rotation, and that a magnetic field is a sort of whirlpool of electricity. Not, of course, that a steel magnet contains an electric current circulating round and round it as does an electro-magnet, but that every molecule of iron, steel, or other magnetizable substance is the seat of a separate current circulating round it continuously and without resistance, and thus every molecule is a magnet.

According to the theory, in an unmagnetized bar these currents lie in all possible planes, and, having no unity of direction, they neutralize one another, and so their effect as a system is zero. But if a current of electricity or a magnet be brought near, the effect of the induction is to turn the currents into parallel planes, and in the same direction, in conformity to Ampère's second law. If the retentivity be strong enough, this parallelism will be maintained after the removal of the inducing cause, and a permanent magnet is the result.

Intensity of magnetization depends on the degree of parallelism, and the latter depends on the strength of the influencing magnet. When these currents have become quite parallel the body has received all the magnetism that it is capable of receiving, and is said to be saturated. Although the currents really circulate around the individual molecules, yet the resultant of these forces is essentially the same as if a superficial sheet of currents circulated around the body as a whole.

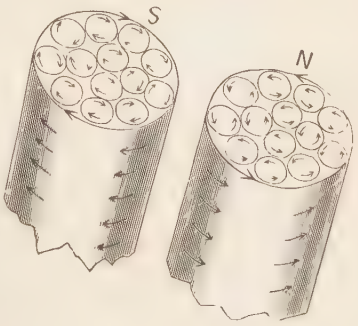


FIG. 368

Fig. 368 represents sections of a cylindrical magnet, and the included circles represent the circulation of the several currents

around the molecules lying in these sections. It will be seen that the currents at the contiguous sides of any two of these circles move in opposite directions, and therefore must neutralize each other; while the currents that pass next the circumference of the magnet are not so affected.

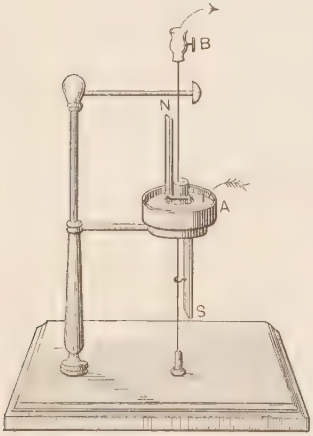


FIG. 369

511. Rotation of a magnetic pole round a current, and of a current round a magnetic pole. A current and a magnetic pole neither attract nor repel each other, but tend to rotate about each other, the action being at right angles to the line joining them. Hence a magnetic pole free to move will rotate round a current. This may be shown experimentally with apparatus like that shown in Fig. 369. The magnet NS is bent so that it may be pivoted on its middle point, the current being brought to an annular mercury cup A by means of a wire which dips into the mercury, leaving through the screw cup B. When a strong current is passing, the magnetic pole N rotates steadily, and by reversing the direction of the current the direction of rotation of the pole is reversed.

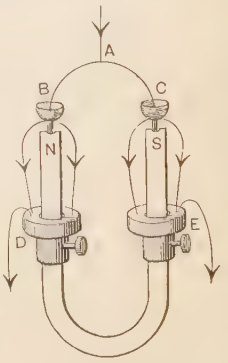


FIG. 370

The rotation of a current round a pole may be shown by pivoting a wire bent in the form of an inverted letter U (A, Fig. 370) on the top of a vertical horseshoe magnet. The divided current passes through the mercury cups B and C, and leaves by the annular cups D and E, which surround the magnet lower down. The cups B and C and the wires passing from them to the cups D and E are so pivoted upon the extremities

of the magnet as to be free to rotate around its poles. When a strong current is passed through the circuit the wires will rotate in opposite directions.

The hypothetical currents that circulate round a magnetic molecule we shall call *amperian currents*, to distinguish them from the known current that traverses the solenoid. In strict accordance with this theory, the poles of the electro-magnet are determined by the direction of the current in the helix. The inductive influence of the electric current causes the amperian currents to take the same direction with itself, as represented in Fig. 371.

By the amperian theory the earth's polarity is accounted for by assuming it to be girdled by electric currents, called *earth currents*, in planes approximately parallel to the equator. A person standing on the Arctic magnetic pole of the earth would, if

FIG. 371

the currents were visible to him, see them (more properly their resultant, a current sheet) circulating round him towards his right from east to west, or in the same direction as the sun appears to him to go round the earth. According to Ampère's theory it is the tendency of the nearer portions of the earth's currents and the amperian currents circulating round a magnetic needle to coincide in direction and to be parallel; that causes the needle to point north and south.

However well adapted this theory may be to explain most of the known phenomena of magnetism, it should be borne in mind that physicists of this generation value the theory rather as a help to the imagination and memory than as a true statement of the facts. It is nearer the truth to say that the molecules are polarized *as if* currents were circulating around them; of the actual existence of such currents we know nothing.

SECTION XVII

ELECTRO-MAGNETIC INDUCTION

512. Description of apparatus. *A* (Fig. 372) is a short coil of coarse wire (i.e. the wire which it contains is comparatively short), and has, of course, little resistance. *B* is a long coil of fine wire having many turns. Coil *A* is in circuit with two cells in multiple arc. This circuit we call the *primary circuit*, the current in this circuit the *primary* or *inducing current*, and the coil the *primary coil*. Another circuit, having in it no battery or other means of generating a current, contains coil *B* and a sensitive galvanometer. This circuit is called the *secondary circuit*, the coil the *secondary coil*, and the currents which circulate through this circuit are called *secondary* or *induced currents*.

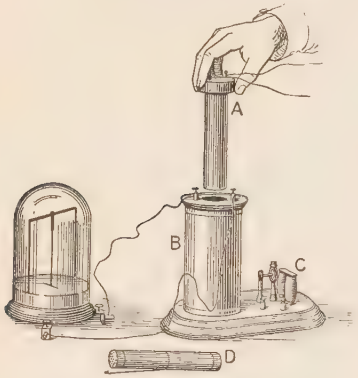


FIG. 372

After all the connections are made, and a current is established in the primary circuit, and the needle is brought to zero, lower the primary coil quickly into the secondary coil, watching at the same time the needle to see whether it moves, and, if so, in what direction. Simultaneously with this movement there is a movement of the needle, showing that a current must have passed through the secondary circuit. Let the primary coil rest within the secondary, until the needle comes to rest. After a few vibrations the needle settles at zero, showing that the secondary current was a temporary one. Now, watching the needle, quickly pull the primary coil out; another deflection in the opposite direction occurs, showing that a current in the opposite direction is caused by withdrawing the coil.

It is evident that in this case the current does not by its mere presence cause an induced current, but that a *change* in the relative positions of the two circuits, one of which bears a current, is necessary.

Place the primary coil within the secondary. Open the primary wire at some point and then close the circuit (Fig. 373) by bringing in contact the extremities of the wires. A deflection is produced. As soon as the needle becomes quiet break the circuit by separating the wires; a deflection in the opposite direction occurs.

The same phenomena occur when the primary current is by any means suddenly strengthened or weakened.

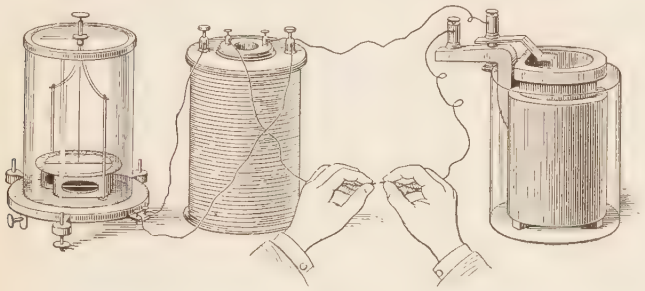


FIG. 373

An examination of the direction of these currents enables us to state the facts as follows: starting or increasing the strength of the primary current, or moving the primary nearer while the current is steady, produces a transitory current in the *opposite* direction in the secondary circuit. Stopping or diminishing the strength of the primary current, or moving the primary away while the current is kept steady, causes a transitory current in the *same* direction in the secondary circuit.

It is evident, therefore, that the conditions under which a current in the primary coil can cause a current in a neighboring secondary depend upon some *change* either in the strength of the primary current or in the relative positions of the primary and secondary circuits.

Introduce the bundle of soft-iron wires *D* (Fig. 372), called the *core*, into the primary coil, and make and break the primary circuit as before. The deflections are now very much increased.

Substitute a person for the galvanometer in the secondary circuit, the person grasping some metallic handles made for the purpose and used as electrodes. The person experiences at the instant of making and breaking a peculiar sensation in his wrists and arms, called a *shock*.

Introduce into the primary circuit the automatic make-and-break piece *C* (Fig. 372). Remove the core from the primary coil. Let a person grasp the electrodes of the secondary circuit. This person experiences a series of shocks which seem to him almost, if not quite, continuous. These shocks can be intensified to suit the pleasure of the person who is receiving them, by gradually lowering the core into the primary coil.

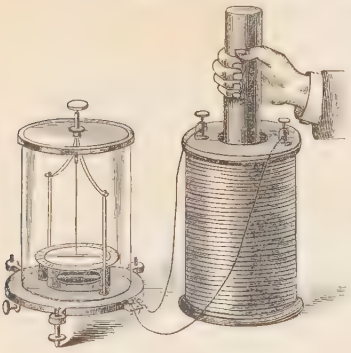


FIG. 374

Reflecting that you have hitherto found a coil of wire having a current passing through it acting as a magnet, you have now an opportunity to try the converse, i.e. to see whether a magnet may not take the place of a current-bearing coil. Introduce suddenly a bar magnet (Fig. 374) into the secondary coil; a deflection is produced. Withdraw it and an opposite deflection occurs.

The act by which the primary, or a magnet, causes a current in a neighboring secondary is called *magneto-electric induction*.

513. Law governing E.M.F. of induced currents. *In any induced current the E.M.F. at any instant is proportional to the rate of change in the number of lines of force passing through the circuit at that instant.* If there be no change in the number of lines there is no induced E.M.F., however rapid the motion may be.

Introduce a long bar magnet NS (Fig. 375) into a short coil of wire C, connected to a galvanometer G. Place the coil halfway between the poles of the magnet and move it rapidly, say, one centimeter toward either pole ; no movement of the galvanometer needle occurs, although all the lines of force of the magnet pass through C.

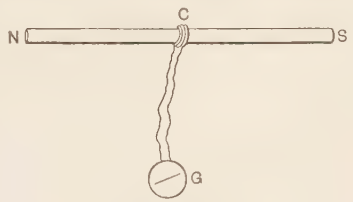


Fig. 375

Now place C near one of the ends of the magnet ; a similar motion produces a large deflection.

By means of an induction coil (§ 519) a current of a few amperes circulating in the primary under an E.M.F. of not more than 10 or 20 volts can be caused to yield currents in the secondary urged by an E.M.F. of many thousand volts.

514. Faraday's law of induction. If a change of relative position or change of strength of the primary current cause a change in the number of lines of force passing through the secondary, an electro-motive force is set up in the secondary proportional to the *rate* at which the number of lines of force included by the secondary is varying.

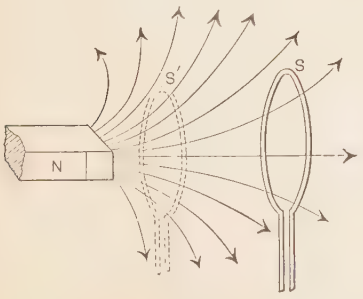


Fig. 376

Consider the case of induction by a magnet. Let S (Fig. 376) be a secondary circuit and N a magnet projecting a certain number of lines of force through the circuit. If S be moved nearer to the magnet, say to S', a much greater number of lines of force of the magnet pass through the circuit than when in its former position, owing to the divergence of the lines as they recede from the pole.

We may now understand, in part, the reason why a core of soft iron so greatly increases the induced current. It acts like a

lens in focusing or concentrating more lines of force from the magnetic inducer through the aperture of the secondary, and therefore any movement makes a greater rate of change, and hence a greater induced electromotive force.

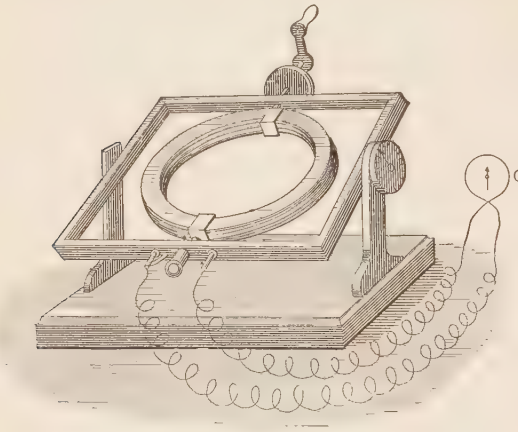


FIG. 377

515. Earth induction. Call to mind that the earth itself is a great magnet, and that its lines of force pass through our atmosphere from pole to pole, and it will be easy to conceive that the mere motion of a coil of wire about an axis properly placed is all that is necessary to produce a current. Such a coil with a galvanometer G in circuit is represented in Fig. 377. The rotation of the coil across the magnetic lines of force of the earth effects a change of the number of lines of force passing through it, as may be understood by inspection of Fig. 378, and this creates temporary currents in the coil.

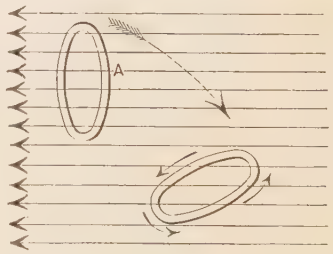


FIG. 378

By examination of Fig. 378 it will be seen that there are in each complete rotation of the coil two points (as A in the figure) where the coil incloses a maximum number of lines of force. In this position the induced current vanishes, for at this instant the number of lines is neither increasing

nor diminishing. As the coil in its rotation approaches these points, the number of lines of force increases, and after leaving it the number diminishes. This will evidently cause a change in the direction of the induced current twice during each revolution.

516. Lenz's law. Recurring to the primary and secondary circuits we remark that the motion of the one or the other may be in arcs of circles or in any way, yet the motion may always be resolved so as to give a resultant indicating approach or recession. The law by which the direction of the induced current is determined is known as Lenz's law, and may be expressed as follows: *In all cases of induction the direction of the induced current is such as to oppose the motion which produces it.* Thus approach develops an opposite current, since opposite currents resist approach, while recession develops a current of similar direction, since similarly directed currents attract one another and thus resist separation.

517. Mechanical energy transformed into electric energy, and vice versa. It is, then, apparent that the current developed in the secondary circuit is at the expense of mechanical energy, and thus mechanical energy is transformed into electric energy.[1]

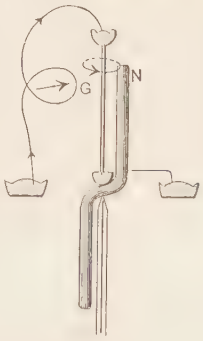


FIG. 379

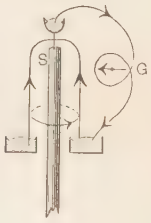


FIG. 380

Returning to the apparatus (Figs. 369 and 370) in which we have the movement of a magnet pole in the field of a current, and of a current in the field of a magnet, if we replace the battery by a sensitive galvanometer, it is evident from the above discussion that on rotating

[1] The student might have been able to prophesy Lenz's law by reasoning thus: Suppose coil B approaches circuit A, we know (1) that electrical energy appears in B; therefore from the doctrine of conservation of energy we know (2) that work must have been done; hence, if work has been done, there must have been a repellent force between A and B.

the magnetic pole (Fig. 379) or the conductor (Fig. 380), in other words, on reversing the operations indicated in § 511, currents will traverse the circuits, and their presence may be detected by the deflection of the needles of the galvanometers G. If we examine the direction of these induced currents, we shall discover that they are always opposite to the current which would actually cause the rotation. Since the induced current is opposite in direction to the current which would cause the motion, it is evident that the electro-magnetic effect of the induced current is to oppose the motion taking place in the field in conformity with Lenz's law.

We have seen that the same apparatus may be used either to transform electric energy into mechanical, or to transform mechanical energy into electric.

518. Self-induction; "extra currents." Not only does a current at starting and stopping or changing strength act on neighboring conductors, generating currents in them, but it acts upon *itself* by a process which is called *self-induction*. A current starting or increasing creates an oppositely directed current, not only in its neighbor but *also* in its own wire. A current does not start instantaneously; it takes a *certain time* — usually very short — to rise to its full strength. In other words, the circular lines of magnetic force round a straight current do not spring into existence instantaneously, but expand gradually like the widening ripples produced when a stone is dropped into still water. But when started it tends to persist, so that if its circuit be suddenly broken, it does not stop instantaneously. The lines of force gradually collapse, but the point of interest is that this collapse gives rise to an electrical *push*, or E.M.F., far greater than that which maintained the current, and this sudden drive forward of electricity in the wire at the instant the circuit is broken causes the *spark* seen on breaking a circuit; and the more sudden the break the more violent the spark. If a current pass through the helix of an electro-magnet, owing to the permeability of the iron a far larger number of lines of force traverse its circuit than if the core were removed; and hence, at the stoppage of the current

a correspondingly greater impulse operates in the wire and creates a correspondingly more powerful spark. For a similar reason the self-induction is much greater in a coil of wire than if the same wire were laid out straight.

The self-induction at breaking a circuit is somewhat analogous to the blow which a high-pressure-service water tap experiences when a flow of water is suddenly arrested by turning the tap. The water momentum often bursts the pipe. Similarly self-induction often breaks through the insulation of field magnets of dynamos when the current is suddenly arrested in them.

The two effects — the delay at making circuit, and the momentum at breaking — are frequently called "extra current" effects, but they are now more commonly spoken of as manifestations of *self-induction.*

The action of self-induction is to hinder the sudden rise and fall of current strength in a wire. Hence in circuits of large self-induction it is impossible to make very sudden changes in the strength of a current flowing through it. This hinders telephonic transmissions through long wires and long coils, and renders ocean-cable telegraphing a comparatively slow operation. It renders the changes of currents in the armatures of dynamos more sluggish than they would otherwise be.

519. Induction coils. If a core of iron, or, still better, a bundle of wires (*AA*, Fig. 381) be inserted in the primary coil, it is evident that it will be magnetized and demagnetized every time the primary is made and broken. The starting and cessation of amperian currents in the core in the same direction as the primary current, and simultaneously with the commencement and ending of the primary current, greatly intensifies the secondary current. To save the trouble of making and breaking by hand, the core is often utilized in the construction of an automatic make-and-break piece. A soft-iron hammer *b* is connected with the steel spring *c*, which is in turn connected with one of the terminals of the primary wire. The hammer presses against the point of a screw *d*, and thus, through the screw, closes the

circuit. But when a current passes through the primary wire the core becomes magnetized, draws the hammer away from the screw, and breaks the circuit. The circuit broken, the core loses its magnetism, and the hammer springs back and closes the circuit again. Thus the spring and hammer vibrate, and open

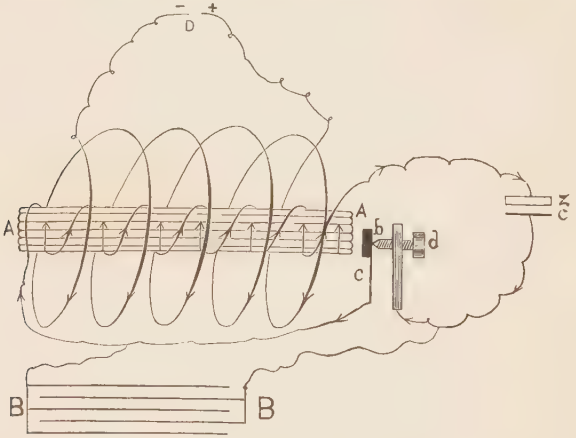


FIG. 381

and close the primary circuit with great rapidity. An instrument made on these principles is called an *induction coil.*

520. Ruhmkorff's coil. This instrument has the important addition to the parts already explained of a *condenser BB.* This consists of two sets of layers of tinfoil separated by paraffined paper; the layers are connected alternately with one and the other electrode of the battery, as the figure shows, so that they serve as a sort of expansion of the primary wire. When the circuit is broken the extra current tends to jump across at *b*, and to vaporize the points of contact, forming a bridge with the vapor of metal which would prolong the time of breaking. But when the condenser is attached the extra current finds an escape into it easier than to jump across at *b*, so the vaporizing of the contact is avoided, and the time of breaking being much shortened, the secondary is much more intense.

The primary helices of induction coils consist of comparatively few turns of coarse insulated wire; but the secondary helices contain many turns of very fine wire, insulated with great care. The secondary current is, at breaking, as we ought to expect from the extreme rapidity with which the primary circuit is broken, distinguished from the primary, or galvanic current, by its vastly greater E.M.F., or power to overcome resistances. A coil constructed for Mr. Spottiswoode of London has two hundred and eighty miles of wire in its secondary coil. With five Grove cells this coil gives a secondary spark forty-two inches long, and perforates glass three inches thick. Many brilliant experiments may be performed with these coils.

Connect a battery of two or more cells in multiple arc with a Ruhmkorff coil. Bring the electrodes of the secondary coil within from one fourth of an inch to several inches of each other, according to the capacity of the instrument. A series of sparks in rapid succession pass from pole to pole.

Introduce a Geissler tube *A* into the secondary circuit. These tubes contain rarefied gases of different kinds. Platinum wires are sealed into the glass at each end to conduct the electric current through the glass. The sparks become diffused in these tubes so as to illuminate the entire tubes with an almost continuous glow. Observe that the electrodes are separated from each other much more widely than would be admissible in air of ordinary density, showing that rarefied gases offer less resistance than dense gases. Gases have been so highly rarefied, however, that an electric current would not pass.

Since the discovery of the Röntgen rays in 1895, the demand for induction coils of high power and efficiency has resulted in a great improvement in the plan and working of this instrument. Coils delivering streams of huge sparks 50 centimeters or more in length are now made and sold in large numbers. They are used to operate X-ray tubes, for electro-therapeutic work, and in wireless telegraphy.

SECTION XVIII

DYNAMO-ELECTRIC MACHINES

521. Dynamo defined. The greatest single advance in industrial life made during the present century is the invention and perfection of the *dynamo-electric machine.*

The dynamo is a device for changing mechanical energy into electric energy. In the most improved types of dynamos this is done with a loss of less than five per cent of the energy.

522. Principle of the dynamo. Moving a wire across a space in which there are magnetic lines, so as to cut across those lines, sets up magnetic whirls around the moving wire, or, in other words, generates a so-called current of electricity in that wire. It is necessary, however, that the moving conductor should so cut the lines of force as to alter the number of lines of force that pass through the circuit of which the moving conductor forms a part.

The illustrations of electro-magnetic induction given in the preceding section suggest the explanation of the action of a dynamo. As we have seen, it depends upon the principle that when lines of magnetic force are cut by a wire a difference of potential is produced in the wire, and hence a current flows through the wire, if its ends be connected so as to form a closed circuit. This was illustrated in connection with Fig. 376, and is illustrated by the following experiment.

Connect a flat coil of about two inches in diameter, having several turns of wire, with a delicate galvanometer, and rotate the coil at one of the poles of a strong magnet on an axis at right angles to the axis of the magnet and the lines of force, as illustrated in Fig. 382.

The horizontal arrow *a* indicates the direction of the magnetic lines of force, the horizontal arrow *b* the direction of motion of the end of the coil of wire, and the vertical arrow *c* the direction of the current induced in the coil of wire from the movement of

the coil across the field of magnetic force in such a manner as to cut the lines of force.

If the coil be moved rapidly in front of the magnet, the current is stronger, and hence the E.M.F. must be greater than if it be moved slowly.

Also if the number of turns of wire be increased, the E.M.F. is correspondingly increased, as will be shown by the increased strength of current. The increase in strength of current will not be so great as the increase of the E.M.F. unless the galvanometer resistance is very large as compared with that of the coil.

Every portion of the wire that cuts lines of force must, then, evidently develop E.M.F., and the total E.M.F. is the sum of all that is induced in different parts of the coil.

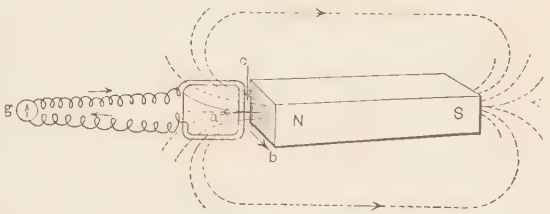


FIG. 382

If the strength of the magnet be increased, the number of lines of force, and consequently the strength of current, will be increased. Again, if the coil be of high resistance, the current will be feeble; if of low, the current is stronger.

We may continue our experiment still further by inserting a bar or disk of soft iron into the coil and again moving the end of the coil through the field of force in front of the north pole of the magnet. A very decided increase in the strength of current is observed. If, further, another bar magnet be placed so that its south end faces the other end of the coil, and the coil be fixed at its center while its two ends are made to rotate past the two poles, more lines of force are cut and greater E.M.F. is developed, as is seen from the increased strength of the current.

The same effect can be obtained by using a single magnet of horseshoe form, and placing the coil between its two opposite poles.

We have now found that the strength of current and E.M.F. depend upon (1) *the rapidity of motion of the wire through the field;* (2) *the number of turns of the wire;* and (3) *the number of lines of force cut, or the strength of field.*

A powerful electro-magnet is preferable to a permanent magnet, as its strength, or *magnetic density*, can be made much greater.

523. Relation existing between the direction of the lines of force, the motion of the coil, and the direction of the current. This relation may be conveniently remembered by placing the thumb, forefinger, and middle finger of the right hand all at right angles to one another, as in Fig. 383, and imagining the forefinger to indicate the direction of the magnetic lines of force (FORe, FORce), the thumb that of the motion of the conducting wire (thuMb, Motion), and the middle finger that of the direction of the induced current (mIddle, Induced). If the hand, with the fingers in this relative position, be held so that the direction of the forefinger coincides with the direction of the lines of force (as indicated by a test needle), and the thumb points in the direction of motion of the part of the conductor under consideration, the middle finger will indicate the direction of the induced current.

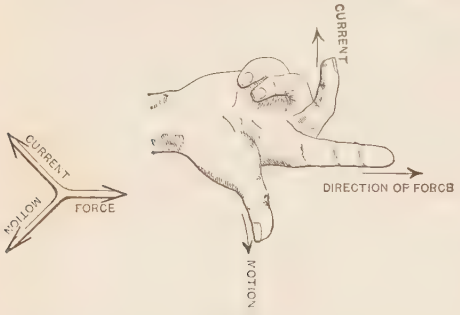


FIG. 383

524. The dynamo. We are now prepared to study the action of the dynamo. Our magnet, which is commonly an electro-magnet, is called the *field magnet*, and our coil or series of coils of wire, which is generally made to move in front of the poles of the field magnet, is called the *armature*. The armature is that part of the electric circuit in which the induced current is generated. Like the battery, it may be considered as the source of

the current. The number of lines of force passing through a circuit may in general be changed in two ways: either (1) by moving the circuit through a field in which the density of the lines of force varies, as represented in Fig. 384; or (2) by rotating the plane of the circuit so as to change the angle which it makes with the direction of the field, thus increasing or decreasing the number which the circuit incloses (Fig. 328). A simple form of dynamo in diagram is illustrated in Fig. 385. A large mass or bar of soft iron of the U-form, surrounded with a coil of insulated wire, and terminating in the *pole pieces* N and S, forms the *field magnet*. The armature consists of a single rectangular loop of wire, which is fixed to a horizontal axis, and terminates in two rings of metal, a and b, which are fixed to the axle, but insulated from it.

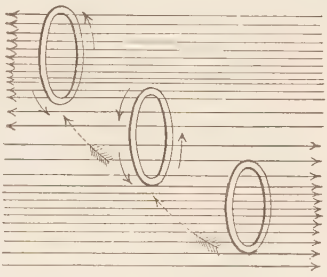


FIG. 384

When a current passes through the field coils, and the core becomes magnetized, lines of force will cross and fill the space between the pole pieces of the field magnet. As these lines are cut by the horizontal parts of the rotating wire, an E.M.F. is generated in these parts, and a current flows in the direction indicated by the arrows.

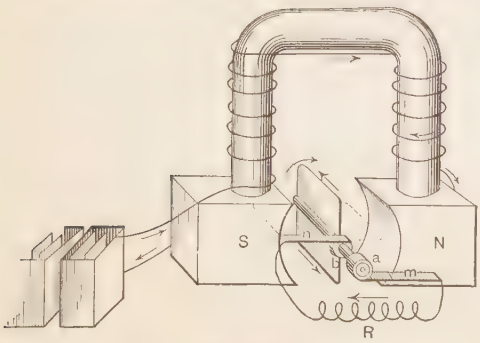


FIG. 385

Apply the preceding rule for determining the direction of the current by letting the forefinger of the right hand take the direction of the magnetic lines of force, the thumb the direction of motion at right

angles to the lines of force, and the middle finger the direction of the induced current. The end portions of the loop do not cut lines of force, and therefore no E.M.F. is generated in them, and they are *dead* wire, — simply serving as conductors to complete the circuit. A metallic or carbon brush m touches and carries off the current from the lower horizontal segment of the rectangular coil. This current flows through the external resistance R, and completes the circuit through the brush n, to the ring b, and the upper half of the loop. The current will continue to flow in this direction while the loop moves through one half of a revolution. Since the lines of force are cut in the opposite direction in the next half revolution, the current will be reversed in the armature wire and also through the external circuit. Thus with each half revolution of the armature a reversal of the current takes place. This, then, would be called an *alternating current dynamo.*

525. The commutator. The alternating current is not adapted to all uses, and for many purposes it is desirable to have the current continuously flowing in the same direction. To accomplish this a *commutator* is attached to the axis of the armature.

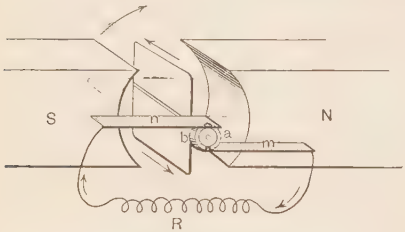


FIG. 386

In Fig. 386 the two brass rings a and b are replaced by a single brass tube divided into two parts by cutting it lengthwise. These two segments are attached to but insulated from the axis, and are connected with the separate ends of the armature wire. When the plane of the armature coil is perpendicular to the field of force passing from N to S, no lines of force are being cut, and hence no E.M.F. is developed and no current flows through the loop. But the instant it moves out of the perpendicular in the direction of the arrow, lines of force will be cut, and as the lower segment of the loop is moving upward past the pole S, and the other segment is moving downward in front

of the pole N, a positive current flows from the loop through the segment a, the brush m, the resistance R, the brush n, and the strip of the commutator b. During the next half of a revolution the lines of force will be cut from an opposite direction by each of the horizontal segments of the armature loop, and hence the current will be reversed. But the segment b of the commutator will now be in contact with the brush m; and although the current is reversed in the armature it will flow off at the brush m as before. Inasmuch as no E.M.F. is developed when the plane of the loop is perpendicular to the lines of force, it is at this point that the brushes pass from one segment to the other. If the segments were to leave the brushes while the coil was cutting lines of force, it is evident that a spark would be formed if the coil were revolving with great velocity. The maximum spark would be given off if the segments left the brushes when the loop is parallel to the lines of force.

Thus by means of the commutators and brushes reversal of the current is prevented in the external circuit, although the current in the armature reverses with each half revolution. This arrangement would constitute a *direct-current* dynamo. We may have two turns of wire before connecting with the commutator strips, giving twice the E.M.F., or three turns, giving three times the E.M.F.; i.e. the E.M.F. will be proportional to the number of turns of wire in the coil. Again, instead of having only one coil we may have two or any number of coils, each separate from the others, and terminating in strips or segments which are on opposite sides of the commutator. Generally the coils are connected in series, thus making any segment a terminal of one coil and the beginning of the next.

526. Eddy currents. It was observed above that with a mass of iron within our coil a greater current was produced when the coil was moved through the magnetic field. So, as in Fig. 392, the coils of wire may be wound around a cylinder or drum of iron, winding along the length of the drum and over the ends. From what we have previously seen it was evident that differences of

potential will be set up in the iron, and, as it is a good conductor, currents will flow through the iron unless prevented, and heat it. These *eddy currents* (commonly called "Foucault currents") are prevented by making the armature core laminated, i.e. having it made up of iron disks or plates insulated from one another. It must be noticed that the armature current, because of its action on the core, tends to distort the magnetic field in the direction of the rotation. This distortion necessitates more or less change of position, or *lead*, of the brushes on the commutator, in the direction of the rotation of the motion of the armature.

527. The Gramme armature. The machine we have described has the *drum* or *Siemens armature*, and is now more commonly used. Another form of machine frequently employed has the *ring* or *Gramme armature*. In this form of armature the core consists of an iron ring or hollow cylinder instead of a closed cylinder or drum, and the wire is wound round this ring instead of over it. The ring may be composed of a bundle of soft-iron wires, as shown in Fig. 387 (where a portion of the ring is cut away), surrounded by sectional coils of what is virtually an endless wire. A wire from each section is carried to and connected electrically with a section of the commutator.

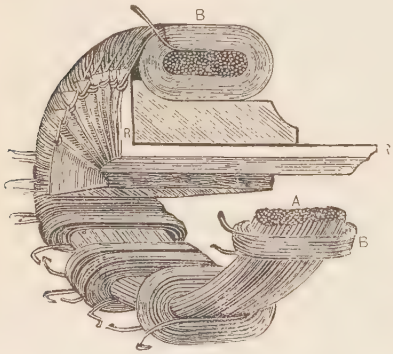


FIG. 387

Fig. 388 is a skeleton diagram of a generator of this kind; and Fig. 389 is a portion of the same, showing lines of force traversing the field pieces and the armature. The lines are supposed not to be distorted by the motion of the armature. When the coils with the segments are connected in series it is evident that those coils moving downward in front of the pole piece N (Fig. 388) develop E.M.F., urging the current away from

one brush and towards the other. At the same time those coils moving upward in front of the pole piece S also have currents urged towards the upper brush. Thus the armature is virtually divided into two equal parts, each half having currents flowing to the upper brush. The brushes must therefore be placed on opposite sides of the commutator in such a position that a line connecting them will be perpendicular to the lines of force. They will then lead the current from the armature where the potential difference is a maximum and from the coils that are in the field of least action.

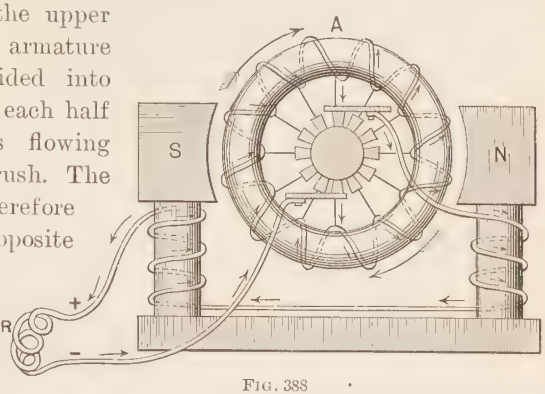


FIG. 388

528. Classes of dynamos. Dynamos in general may be divided into different classes according to the method by which their

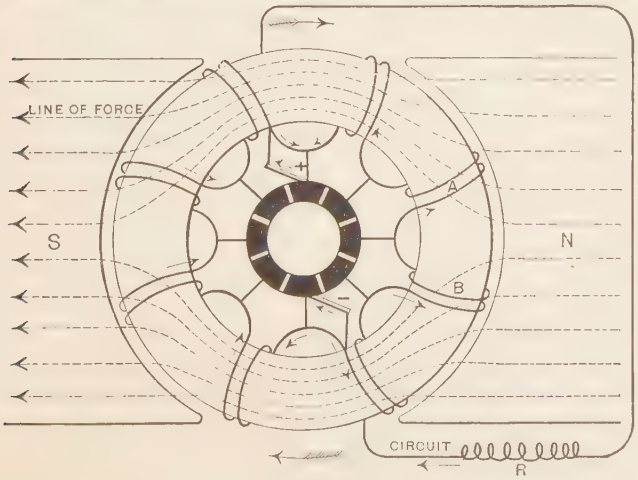


FIG. 389

field magnets are excited. Fig. 390 illustrates a *magneto-electric* machine, where the *field* magnet is a permanent steel magnet. This form of machine is seldom used, since a permanent steel magnet cannot be made as powerful as an electro-magnet having a soft-iron core of equal mass.

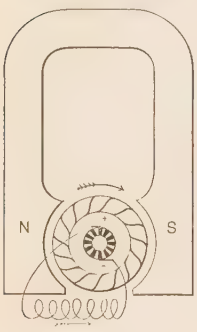


FIG. 390

Fig. 385 illustrates a *separately excited dynamo*, where the field-magnet coils receive their currents from a separate generator, e.g. a battery, and not from the armature coils. Since an alternating-current dynamo does not produce a constant magnetic field, alternating dynamos in general are separately excited.

Fig. 388 is a *series dynamo*, where the coils of the field magnet are joined in series with the armature so that the entire current passes through these coils.

Fig. 391 illustrates a *shunt machine*, where the field coil serves as a shunt to the external circuit. *A* is the main wire and *a* is the shunt wire. In the shunt machine only a part of the current generated in the armature passes through the field coils.

A dynamo is said to be "self-exciting" when the whole or any part of the current which is produced is used to magnetize the field magnets. Such are the Edison incandescent dynamos.

The fields, after being once excited from any source, e.g. another dynamo, always retain a little residual magnetism, so that when the armature begins to rotate a slight current is at once induced in it. This strengthens the field, and the stronger field reacts to increase the current.

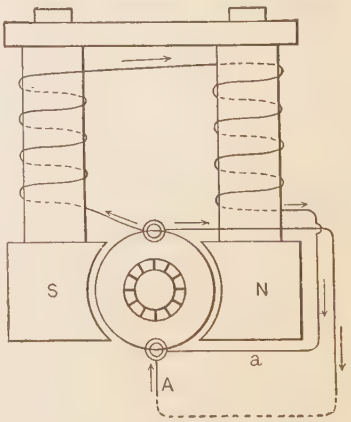


FIG. 391

More than one set of coils may be used on the field magnets, and these coils may receive currents from different sources. Dynamos employing these are called *compound-wound* machines. They may be arranged for *constant potential*, or *constant current*. A combination of the series and separately excited methods of winding, or of series and shunt, gives *constant potential*; while a combination of shunt and separately excited methods gives *constant current*.

529. Fuses. For the protection of electrical apparatus of various kinds from dangerously large currents, it is customary to insert into the line at suitable places short wires of some fusible material so adjusted in size that an overload will melt them, thus opening the circuit. Without some such protection a careless or accidental short circuit in a line would frequently cause the burning out of an armature or give rise to other damage. The fuse wires are now usually inclosed in cartridge tubes which can be easily slipped into contact sockets constructed to receive them.

530. Circuit breaker. As these fuses are often too slow of action, a *circuit breaker* is now almost always put into the line at some convenient place. It is a combined solenoid and electro-magnet so arranged as automatically to open the circuit either when the line becomes overloaded or when the current for any reason is reduced to zero. It operates very quickly when called upon, and so protects completely all instruments in circuit.

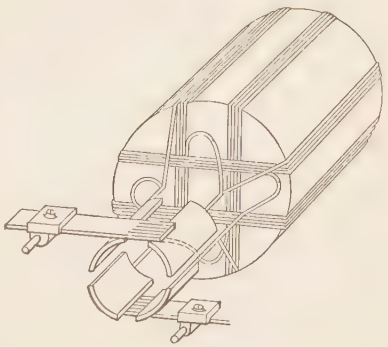


FIG. 392

531. Classes of armatures. (1) In *ring armatures* the coils are wound round a ring-shaped core, as shown in Fig. 388.

(2) In *drum armatures* the coils are wound longitudinally over a cylinder or drum, as in Fig. 392.

(3) In *pole* or *radial armatures* the coils are wound on separate poles that project radially from a cylinder (Fig. 393).

In alternating-current dynamos, in order to obtain the rapid reversals of currents sometimes desired in opposition to resistance offered by self-induction, a number of poles of alternate polarity are employed.

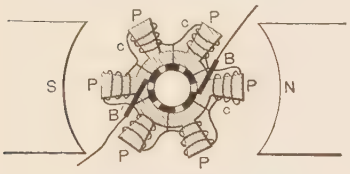


FIG. 393

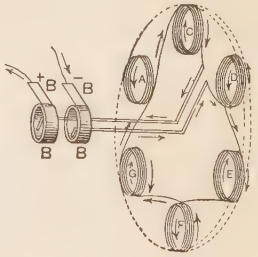


FIG. 394

The separate coils may be coupled either in series or in multiple arc. When low E.M.F. is desired, as for incandescent lamps in multiple arc, the separate coils are united in multiple arc; but where great E.M.F. is required they are connected in series, as shown in Figs. 393 and 394.

(4) *Disk armatures* are usually composed of a number of separate coils set side by side in the circumference of a disk (Fig. 394).

SECTION XIX

THE ELECTRIC MOTOR

532. Reversibility of the dynamo. This subject has already been touched upon, and it only remains to treat it a little more in detail. If a current from an external source, e.g. a battery or another dynamo, be passed through the armature and field magnet of a direct-current dynamo, it will excite the armature and make of it an electro-magnet, and will also excite the fields. The current will enter at the terminals and will pass through the commutator into the armature. The relation of

parts is such that in doing this it will develop N and S poles in parts of the periphery of the armature distant from the N and S poles of the fields. Hence a stress will be set up between the armature and the poles of the field magnet tending to move the former a little in the opposite direction to that in which it is compelled to move when generating a current. But as soon as it has turned a short distance, the action of the commutator shifts the current, and new poles are established in the armature back of the first and in the same relative positions which they at first occupied. The armature continues to rotate as the new poles are developed, and the action goes on so long as a current is supplied. Obviously, if there were no commutator the poles of the armature would be fixed, and it never could rotate through a greater angle than 180°.

It is evident, then, that if two dynamos be connected by wires in the same circuit, and if the armature of one be rotated, the armature of the other will rotate in a reverse direction as soon as the current transmitted from the first attains a certain intensity.

If in a separately excited direct-current dynamo, or in a magneto-dynamo, or in a series dynamo, we substitute a dynamo or other current generator for the resistance, and the current be made to flow through the armature in the same direction as when generating a current as shown by the arrows, the armature will tend to rotate in the opposite direction from that indicated by the barbed arrows. If the current from our generator flows in the opposite direction from that shown by the arrows, the armature will rotate in the same direction as the barbed arrow. If again we use a shunt dynamo (Fig. 391), placing a generator in the main circuit A with a current flowing into the armature in the direction of the arrow, the current will divide at the negative brush, a part going through the field-magnet coils, but in a direction opposite to that in which it flows when the dynamo is used as a generator. Hence the polarity of the field magnet is reversed, and the armature will rotate in the same direction when receiving a current as when generating a similar current.

It is now evident that the dynamo is a reversible machine, in which mechanical energy can be changed directly into electrical energy or electrical energy into mechanical energy. When the dynamo is used for the latter transformation, it is commonly known as an *electric motor*. In other words, a modern motor is a dynamo reversed. The discovery of the reversibility of the dynamo is considered to be one of high importance. The reversibility leads to some curious results. For example, when a car on an electric railway is descending a hill, its motor, instead of driving the car, might be driven by it and thereby become a dynamo and send a current into the line to drive or help drive other cars in the same circuit. It would be possible for one car of an electric system, in running down a steep hill, to have its mechanical energy absorbed by its motor acting as a dynamo (and thus serving as a brake to retard its motion), and thus to draw another car of the same system up a hill miles distant.

533. The action of the dynamo motor. This may be understood by referring to Fig. 395, and imagining a generator to replace the external resistance *R*. Suppose the current from the generator enters at the brushes and flows in the loop in the direction of the arrows; then the upper face of the loop will have *S* polarity and the under face, *N* polarity. Then, by the mutual action between this field and that of the magnet *NS*, a rotation of the loop will take place clockwise till it comes into a vertical position. When it reaches this position, however, the brushes are so arranged with reference to the commutator segments that the current in the loop — and hence its polarity — is reversed. Even if there were only one loop its inertia would be sufficient to carry it by this critical position, and the loop would continue to rotate in the attempt again to bring its field parallel to that

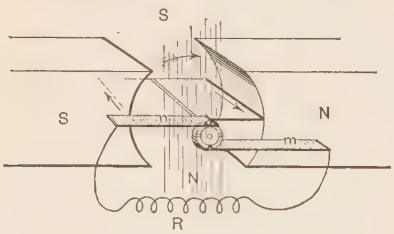


FIG. 395

of NS; but as a matter of fact the other loops in the armature are never in the critical position at the same time as the one considered, and those on each side of it conspire to produce a continuous rotation in the same direction.

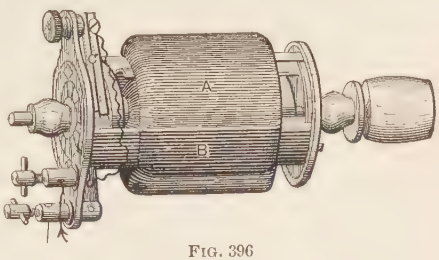


FIG. 396

If the armature contain a soft-iron core, as is usually the case, the intensity of the field will be much greater and the mechanical effect correspondingly increased.

Fig. 396 represents a modern form of motor weighing only two or three pounds, and capable, when worked with four or five Bunsen cells, of operating a sewing machine or running a small saw. It consists of a movable coil within a fixed coil. The wires of each coil are wound on an iron framework, the two opposite edges of the iron being north and south poles when the current is passing.

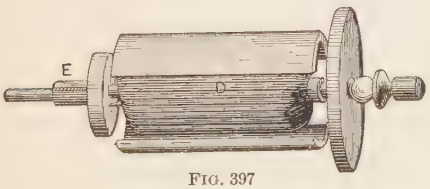


FIG. 397

The inner coil is furnished with a commutator, which reverses the current as soon as opposite poles of the inner and outer coils are opposed. A represents the outer coil of wires, B one pole of the fixed electro-magnet made by it, and C the commutator by which the inner coil has the current reversed at each half revolution. Fig. 397 shows the inner coil D, whose terminals are attached to the two halves of the spindle E, which are carefully insulated from each other. In Fig. 398 the

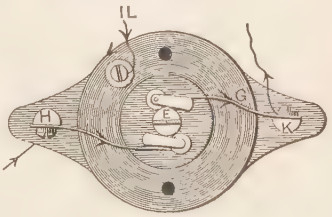


FIG. 398

commutator is shown in plan. The current enters the inner coil through the spring H, which carries a friction roller working on the commutator E; after traversing the coil it returns to the upper half of E, and thence passes by the spring G to K, from K through the outer coil to L, and from L back to the battery.

The dynamo as a generator and the dynamo as a motor have already revolutionized electrical economics and relegated the battery to an honored position among things of the past. The electric motor is now extensively used in large towns and cities in factories where power is not continuously needed. Its widest application at present is in the propulsion of street cars. The current is generated by dynamos at some central power house and thence distributed to the motors at various points on the circuit.

SECTION XX

THE TRANSFORMER

534. The induction coil reversible. An induction coil is in a certain sense a reversible machine. If a current of considerable strength circulate under small E.M.F. in the primary, then variations in its strength give rise to very weak currents of exceedingly high E.M.F. in the secondary. Conversely, if we cause to circulate in the *secondary* weak currents under very high E.M.F. by their fluctuations there will be generated in the primary strong currents of small E.M.F. We do not in either case create electric energy. Electric activity (or power) is the product of two factors, current and electro-motive force. The induction coil enables us to increase one of these factors at the expense of the other, and to transform electric energy in form much as a mechanical machine (e.g. a lever) enables us to convert a quantity of work which consists of a small stress exerted through a great distance into a large stress exerted through a small distance.

535. The transformer. This instrument is merely an induction coil used to change the relation of the number of volts to the

number of amperes of any current. In a *perfect* transformer the number of watts in the primary equals the number of watts in the secondary.

The Ruhmkorff coil as ordinarily used may be regarded as a "step up" transformer from low potential to high potential. But if the coil of long thin wire be used as the primary, it becomes a "step down" transformer from high potential to low potential.

If we have a primary consisting of two thousand turns of wire and a secondary of one thousand turns, we get in our secondary half the voltage of the primary, but twice the number of amperes. In general, if we could concentrate all the lines of force of the primary upon the secondary we should have the following relation: *volts in primary* $\times$ *amperes in primary* = *volts in secondary* $\times$ *amperes in secondary.*

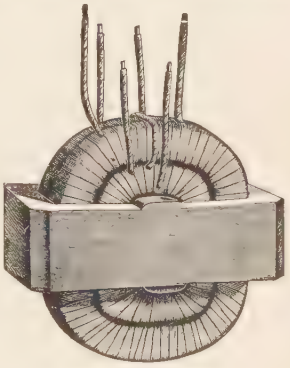


FIG. 399

Fig. 399 represents the coils of a transformer used in the incandescent-lamp service, which when in use is inclosed in an iron case. These transformers are usually supported on the street poles.

The transformer is applied in the welding of metals, i.e. to fuse the ends of metals that are to be joined together, where many hundred or even many thousand amperes of current, and only a fraction of a volt, would be required for an instant.

A still wider application of transformers is in the transmission of electric activity.

Since the rate at which a current performs work equals the volts times the amperes ($E \times C$), then according to Ohm's law the work done per second by a current passing through a conductor equals C^2R. That is, when the current strength is doubled there will be four times as much energy transformed per second. We see, then, that to transfer electric energy to a

great distance it may be desirable to have a high E.M.F. with a small current passing through the mains, and then to reduce the E.M.F. and increase the current by a transformer at the place where the energy is to be used. By this means the expense involved in the copper conductors is much reduced.

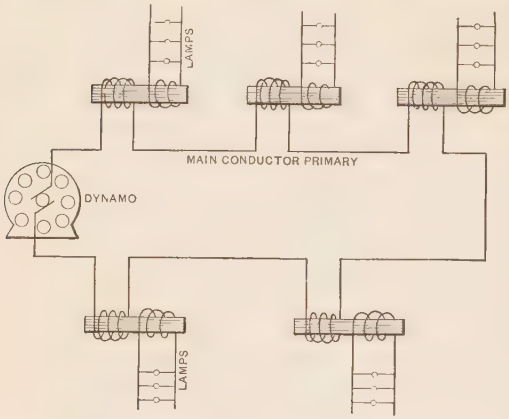


FIG. 400

For electric lighting in private houses transformers are used to bring down the high potential of the mains to the safe limit of about 100 volts.

The transformers may be arranged in series, as shown in Fig. 400, in which case the same primary current passes through all of them, and from each secondary circuit leads may be taken off to work local lights. Or they may be arranged in multiple arc (Fig. 401), in which case each primary current is a bridge across from main to main of the dynamo which supplies the alternating current of high E.M.F. In both cases the lamps are joined in multiple arc.

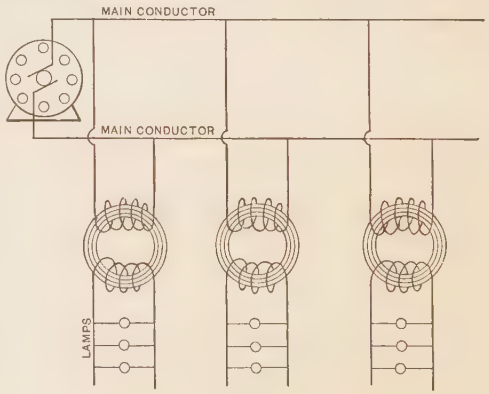


FIG. 401

SECTION XXI

SECONDARY OR STORAGE BATTERIES

536. Reversibility of electrolysis. If water be decomposed for a time between neutral electrodes such as platinum plates and then the battery or other generator be withdrawn from the circuit and replaced by a sensitive galvanometer, a deflection of the needle shows that a transitory current flows in the opposite direction to the primary or electrolyzing current. It is evident that the electrolyzing current polarizes the electrodes in the electrolyte, and that energy is thus stored in the cell. When the wires are joined this polarization causes a current to flow during an appreciable period, and the platinum electrodes become depolarized. The electrical energy of the cell is converted into chemical potential energy by overcoming the E.M.F. of the decomposing cell and separating the compound. Polarization sets up a counter E.M.F. It is precisely this polarization which we have to contend with in nearly all voltaic cells, and which we seek to neutralize by means of depolarizing substances.

Devices for thus storing up energy by electrolysis, and liberating it when desired in the form of electric current, are called *storage* or *secondary batteries*, and sometimes *accumulators*. Note that the process is an electrical storage of energy, not a storage of electricity. The energy assumes the form of chemical potential energy, and there is really no more electricity in the cell when it is fully charged than at the commencement of the operation.

If, instead of platinum electrodes, two plates of lead covered with a coating of red lead, or, better, the positive plate (which is the positive electrode when the cell is being charged) covered with a paste of red lead and sulphuric acid, and the negative plate with a paste of litharge and sulphuric acid, be used as electrodes, dipping as before into sulphuric acid, and the electrodes be connected with a powerful generator, the positive electrode becomes by electrolysis peroxidized by the oxygen which is liberated,

while the negative is deoxidized by the hydrogen. The plates may remain in this state indefinitely. Hence the storage battery is a very convenient means of accumulating energy at one time or place, and using it at some other time or place. For example, energy may be stored during the daytime; and this energy, reconverted into electric energy, may feed incandescent lamps at night at any convenient place. Or these batteries, having been charged by a dynamo, may be transported to lecture halls, workshops, electric cars, etc., where powerful currents may be needed. The E.M.F. of these batteries may be multiplied many-fold by joining them in series. The E.M.F. of a single cell similar to the above is about 2.2 volts. The internal resistance of a cell whose surface of electrodes is 300 square centimeters is about 0.006 ohm. Their low resistance constitutes one of their chief virtues as generators. An idea of the capability of a storage battery may be obtained from the statement that a battery capable of furnishing one horse power for five hours weighs 500 pounds; it will supply twelve ordinary incandescent lamps of sixteen candle power each for five hours. It will then require to be recharged.

SECTION XXII

ELECTRICAL TRANSMISSION OF POWER

537. Essentials for the transmission of activity electrically. The electrical transmission of power or activity demands (1) a source of energy (e.g. a water-power or steam engine) and a dynamo; (2) a line of wire to serve as a conductor of electricity; and (3) a motor. The current generated in the dynamo reaches the motor (which may be many miles away) and causes a rotation, and the activity is communicated therefrom to machinery.

538. Limitations. Activity of any desired magnitude may be transmitted, provided the conductor be of proper size for the current employed. The rate at which energy is expended in the motor is proportional to the potential difference at its terminals,

and to the current strength. If the dynamo give a high potential difference, less current will be required to furnish a required activity. The smaller current heats the wire less; consequently the wire may be smaller as the motor works at a higher difference of potential. The size of the wire that should be used depends wholly upon the strength of the current to be transmitted. For example, the entire energy of Niagara Falls might be transmitted by an ordinary telegraph wire were it not that the enormous potential difference required would cause a leakage of current at every possible avenue of escape, and therefore make the system a menace to every one near it.

539. Electric railways. These furnish the most familiar illustrations of transmission of electric activity. The current from a dynamo stationed at some "electric plant" is conveyed by a trolley wire or "third rail" running along the road. Each car carries one or more motors, one of whose terminals connects with the trolley and the other with the rails through the wheels. One of the terminals of the dynamo is also carried to the rails. Cars on an electric railway are usually worked in multiple arc. The trolley wire and the rails being regarded as parallel conductors, the car motors serve as bridges from one to the other, like the rungs of a ladder.

540. Advantages of electrical transmission of power. The advantages of this method of transmission of activity over that by means of belts, shafts, compressed air, etc., are many and important. For example, it is attended usually with much less waste of energy. An electrical conductor is flexible; it can be bent and carried round corners and tapped wherever wanted. It is motionless, though transmitting large quantities of energy. It transmits energy through very long distances. In some places in the mining regions of the Western States this is the *only* practicable way of supplying activity to the crushers and stamps of the mills. These could not be driven with profit by steam engines on account of the insurmountable difficulties of transporting coal to these localities.

SECTION XXIII

THERMOELECTRIC CURRENTS

541. Heat energy transformed directly into electric energy. Join to one terminal of a sensitive galvanometer an iron wire, and in the other a copper, or, better, a German silver wire. Twist the other ends of the wire together, and at their junction heat them in a flame; a deflection of the needle shows that a current of electricity is traversing the wire. Place a piece of ice at their junction; a deflection in the opposite direction shows that a current now traverses the wire in the opposite direction.

Take a strip of sheet copper about fifteen inches long and three fourths of an inch wide, and a strip of zinc of the same dimensions. Lay them one upon the other, fold over each end upon itself for about half an inch, and hammer the joints flat, so that they shall hold together quite firmly. Then separate the two strips into a somewhat elliptical or rectangular shape, as shown in Fig. 402. Cut a hole through the center of one of the strips and pass the wire support of a magnetic needle through it. Place the band in the magnetic meridian parallel with the needle. Direct a flame against one of the junctions, and note the deflection, and determine the direction in which the current traverses the band, i.e. whether the current passes through the heated junction from the copper or from the zinc strip. These currents are named, from their origin, *thermoelectric* current. The apparatus required for the generation of these currents is very simple, consisting merely of bars of two different metals joined at one extremity, and some means of raising or lowering the temperature at their junction, or of raising the temperature at one extremity of *the pair* and lowering it at the other; for the

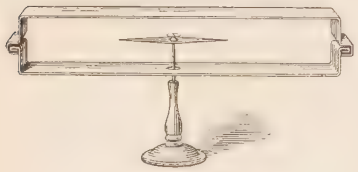
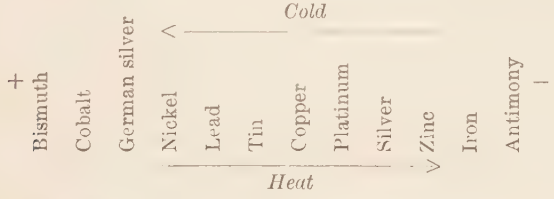


FIG. 402

electro-motive force, and consequently the strength of the current, is nearly proportional to the difference in temperature of the two extremities of the pair. The strength of the current is also dependent, as in the voltaic pair, on the *thermoelectro-motive force* of the metals employed. The following *thermo-electric series* is so arranged that if the mean temperature of the two junctions be near the ordinary temperature of the air, those metals farthest removed from each other give the strongest current when combined; and the current passes, when they are heated at their junction, from the one first named to that succeeding it. The arrows indicate the direction of the current at the heated and the cold ends respectively. At high mean temperatures the current may be reversed.



542. Thermoelectric batteries and thermopiles. The E.M.F. of the thermoelectric pair is very small in comparison with that of the voltaic pair; hence the greater necessity of combining with one another a large number of pairs in series. This is done on the same principle and in the same manner that voltaic pairs are united, namely by joining the + metal of one pair to the − metal of another. Fig. 403 represents such an arrangement. The light bars are bismuth, and the dark ones antimony. If the source of heat be strong and near, one face may be heated much hotter than the other, and a current equal to that from an ordinary galvanic cell is often obtained. Such contrivances for generating electric currents are called *thermoelectric batteries*. They are seldom used, inasmuch as the best of them transform

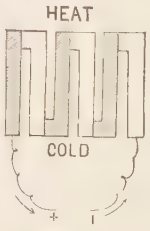


FIG. 403

less than one per cent of the heat energy given out by the source of heat. Furthermore, the E.M.F. of thermic piles is generally so small that any considerable external resistance makes the current extremely weak.

If the source of heat be feeble or distant, the feeble current may serve to measure the difference of temperature between the ends of the bars turned toward the heat and the other ends, which are at the temperature of the air. The apparatus, when used for this purpose, is called a *thermopile* or *thermomultiplier*. A combination (Fig. 404) of as many as thirty-six pairs of antimony and bismuth bars, connected with a very sensitive galvanometer, constitutes an exceedingly delicate *thermoscope* and *thermometer*. Changes of temperature that would not produce a perceptible variation in an ordinary thermometer can by the use of a thermoelectric pile be made to produce large deflections of the galvanometer needle. Heat radiated from the body of an insect several inches from the pile may cause a sensible deflection.

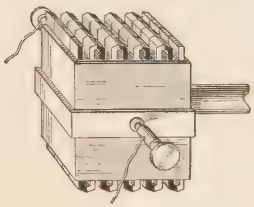


FIG. 404

SECTION XXIV

ELECTRO-MAGNETIC THEORY OF LIGHT; ELECTRIC RADIATION

543. Maxwell's theory of light. In 1865 Maxwell propounded the theory that light is the result of electro-magnetic disturbances of rapidly alternating character in the ether, such as would result from its being set in local strains and being released from them, that the vibrations which constitute light are electrical vibrations, and that light waves are electrical or electro-magnetic waves.

Justification for this theory is found in the fact that electrical discharges such as that of a Leyden jar have been proved to be *oscillatory*, and give rise to radiations, i.e. ether waves; that these radiations or electrical disturbances travel with approximately

the same speed as light ; and that they are susceptible of the same changes and modifications as light. Furthermore it may be shown that a magnetic field may rotate the plane of polarization of light, and an electrostatic field may change plane-polarized into elliptically polarized light and produce double refraction. These modifications might be expected if light itself consists of electro-magnetic disturbances.

The Maxwellian theory of light may now be considered as completely verified by the wonderful experimental researches made by the late Dr. Hertz while a professor at Karlsruhe.

Hertz showed experimentally that the electrical ether is wonderfully like if not identical with the ether which transmits light waves. By rapidly charging and discharging a conductor he causes the ether upon it to surge to and fro. This agitates the surrounding dielectric ether, and the disturbance travels in waves. The speed of these waves he determines to be the same as the speed of light waves. He finds that these waves may interfere, be reflected from metal mirrors, be refracted by lenses and prisms, and are susceptible of diffraction effects. He has shown that many optical experiments can be electrically performed by substituting dielectrics for transparent bodies, and conductors for opaque bodies. "So that we have now a real undulatory theory of light, no longer based on an analogy with sound, and its inception and early development are among the most tremendous of the many achievements of the latter half of the nineteenth century. The whole domain of optics is now annexed to electricity, which has thus become an imperial science" (Lodge).

The importance of these experiments in a scientific sense can scarcely be overestimated, in so far as they teach us to refer electrostatic and electro-magnetic phenomena to the intervention of the same all-pervading medium. This medium forms the vehicle by which energy passes through space from one body to another, and to which we now must probably look for a knowledge of the process by which one body is enabled to attract another, as well as of facts concerning the ultimate constitution

of matter. The term *radiant energy* is continually acquiring new scope in physics.

A prophecy. "The conclusions at which we have arrived, that light is an electrical disturbance, and that light waves are excited by electric oscillations, must ultimately, and may shortly, have a practical import. Our present systems of making light artificially are wasteful and ineffective." (It is estimated that not more than five per cent of the energy put into an incandescent lamp is useful for illumination.) "We want a certain range of oscillation between 700 and 400 trillion vibrations per second; no other is useful to us, because no other has any effect on our retina; but we do not know how to produce vibrations of this rate. . . . We want a small range of rapid vibrations, and we know no better than to make the whole series leading up to them. It is as though, in order to sound some little shrill octave of pipes in an organ, we were obliged to depress every key and every pedal, and to blow a young hurricane. . . .

"Any one looking at a common glowworm must be struck with the fact that not by ordinary combustion nor yet on the steam engine and dynamo principle is that easy light produced. Very little waste radiation is there from phosphorescent things in general." (Professor Langley has found that light of one candle power from a Cuban firefly can be produced by an expenditure of one fourth hundredth of the energy required for a one-candle-power gas jet.) . . .

"Solar radiation consists of waves of all sizes, it is true; but solar radiation has innumerable things to do besides making things visible. The whole of its energy is useful. In artificial lighting nothing but light is desired; when heat is wanted it is best obtained separately, by combustion. So soon as we clearly recognize that light is an electrical vibration, so soon shall we begin to beat about for some mode of exciting and maintaining an electrical vibration of any required degree of rapidity. When this has been accomplished the problem of artificial lighting will have been solved" (Lodge).

SECTION XXV

SOME USEFUL APPLICATIONS OF ELECTRIC ENERGY: ELECTRIC LIGHT

The applications of electric energy to industrial uses are so numerous and varied that the limits of an ordinary text-book on general physics can do little justice to the subject, and, indeed, a description of the various appliances in use is of too technical a character to come properly within the scope of a school or college course. Public libraries are now well provided with both scientific and popular works relating to every industrial application.

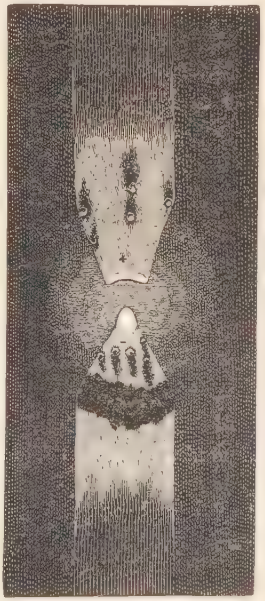


FIG. 405

544. Electric light. If the terminals of wires from a generator of sufficient E.M.F. be brought together, and then separated 1 or 2 millimeters, the current does not cease to flow, but volatilizes a portion of the terminals. The vapor formed becomes a conductor of high resistance, and, remaining at a very high temperature, produces intense light. The light rivals that of the sun both in intensity and whiteness. The heat is so great that it fuses the most refractory substances. Metal terminals quickly melt and drop off like tallow, and thereby become so far separated that the electro-motive force is no longer sufficient for the increased resistance, and the light is extinguished. Hence, pencils of carbon (prepared from the coke deposited in the distillation of coal inside of gas retorts), being less fusible, are used for terminals. Even carbon, however, slowly wastes away.

The light is too intense to admit of examination with the naked eye; but if an image of the terminals be thrown on a screen by means of a lens or a pin-hole in a card, an arch-shaped light is seen extending from pole to pole, as shown in Fig. 405.

The heated air containing the glowing particles of carbon forms what is called the *electric arc.*

The larger portion of the light, however, emanates from the tips of the two carbon terminals, which are heated to an intense whiteness, although some emanates from the arc. The + pole is hotter than the − pole, as is shown by its glowing longer after the current is stopped. The carbon of the + pole becomes volatilized, and the light-giving particles are transported from the + pole to the − pole, forming a bridge of luminous vapor between the poles. What we see is not electricity, but *luminous matter.*

The light of the ordinary street arc lamp has an intensity varying from one to two thousand candle power, or the combined intensity of from fifty to a hundred ordinary gaslights. To sustain such a light, about one horse power per lamp must be applied at the dynamo.

545. Electric lamp. It is apparent that the + pole is subject to a wasting away; so also the − pole wastes away, but not so fast. At the point of the former a conical-shaped cavity is formed, while around the point of the latter warty protuberances appear. When, in consequence of the wearing away of the + pole, the distance between the two pencils becomes too great for the electric current to span, the light goes out. Numerous self-acting regulators for maintaining a uniform distance between the poles have been devised. Such an arrangement is called an *electric arc lamp.* The movements of the carbons are accomplished automatically by the action of the current itself.

The difference between the arc lamps of the various inventors is a difference in the mode of adjusting or "feeding" the carbons. The following is a simple plan which is made clear by the diagram.

The current, entering at A (Fig. 406), divides at B into two branches, which pass around the bobbin C in opposite directions, one branch being a coarse wire of low resistance and in the same circuit as the carbons, and the other branch, SS, being a shunt of high resistance, connecting the terminals B and G. Inside the bobbin is a soft-iron core, F, which is attached to the upper carbon. When a current passes through the two branch circuits on the bobbin C they tend to magnetize the core in opposite directions, but the resistances and number of turns in the two circuits are so proportioned that the magnetic field due to the low-resistance branch is the stronger, and the core F is therefore drawn up into the bobbin, lifting the upper carbon and establishing the arc. Should the carbons become too widely separated, the resistance of the arc, and consequently of the coarse-wire circuit on C, increases, diminishing the current in C and increasing that in the shunt S. The field due to the shunt is therefore strengthened, and that due to the coarse wire diminished, allowing the core F to fall slightly, bringing the carbons nearer together. By the device of the two opposing fields, due to the coils on C being wound in opposite directions, the feeding of the lamp is done automatically, and the actual distance of the two carbons varies but little.

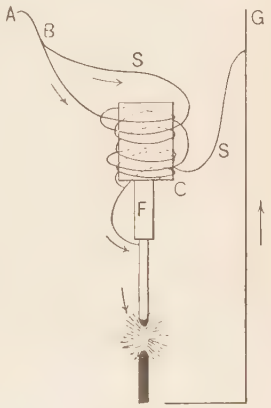


FIG. 406

546. The inclosed arc. As the presence of the oxygen of the air causes the carbons to be consumed very fast, it is customary to put a small glass case around the arc. The fit is not air-tight, so that the heat of the arc soon expands the inclosed air and a large part of it is forced out from the inclosure. The oxygen being by this means excluded, the life of the carbons is increased many-fold, and the time required for caring for the lamps much lessened.

547. Incandescent electric lamps. The incandescent (or "glow") light is produced by the heating of some refractory body to a state of incandescence by the passage of an electric current. Carbon filaments are now almost exclusively used in incandescent lamps. It is essential that the oxygen of the air be removed from these bulbs, otherwise the carbons would be quickly burned out; hence very high vacua are produced in the bulbs.

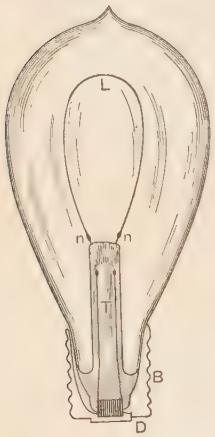


FIG. 407

Fig. 407 represents one of these lamps. The loop or filament of carbon *L* is joined at *nn* to two platinum wires which pass through the closed end of the glass tube *T*. One of these wires is connected with the brass ring *B*, and the other with the brass button *D* at the bottom of the lamp. When the lamp is screwed into its socket, connection is made with the line through pieces of brass in the socket which are insulated from each other.

An ordinary 16-candle-power lamp has a resistance (when hot) of about 220 ohms, the difference of potential at its terminals is about 110 volts, and it requires a current of about 0.5 ampere. Each lamp consumes about one twelfth of a horse power.

Incandescent lamps are usually introduced into the circuit in multiple arc, the current being equally divided by properly regulating the resistance between all the lamps in the circuit.

The customer pays by the kilowatt hour[1] for the electric energy consumed; a watt hour being the energy expended in

[1] A watt of electrical activity corresponds to $\frac{1}{746}$ horse power of mechanical activity; hence, if a lamp or motor take activity equivalent to $\frac{1}{746}$ horse power for one hour, it consumes one watt hour of energy, or 3600 joules.

It has been found that an incandescent lamp consuming 1000 watt hours of electrical activity gives about one tenth of the illumination given by 1000 feet of gas under ordinary circumstances; consequently, in order that the customer may pay for electric light at the same proportional rate for the same illumination that he would pay for gas, the price per thousand watt hours must be placed at one tenth of the price per thousand feet of gas.

one hour when the activity is one watt. A 16-candle-power lamp consumes about 55 watts, or about 3.5 watts per candle. A great many meters have been devised for registering the electrical energy consumed. Some of them are virtually small electric motors, which revolve at varying speed according to the strength of the current and record the total number of revolutions per month, somewhat as is done in a gas meter.

SECTION XXVI

USEFUL APPLICATIONS OF ELECTRICITY CONTINUED: ELECTROTYPING AND ELECTROPLATING

548. Electrotyping. This book is printed from electrotype plates. A molding case of brass, in the shape of a shallow pan, is filled to the depth of about one quarter of an inch with melted wax. A few pages are set up in common type, and an impression or mold is made by pressing these into the wax. The type is then distributed, and again used to set up other pages. Powdered graphite is applied by brushes to the surface of the wax mold to render it a conductor. The case is then suspended in a bath of copper sulphate dissolved in dilute sulphuric acid. The — pole of a galvanic battery or dynamo machine is applied to it; and from the + pole is suspended in the bath a copper plate opposite and near to the wax face. The salt of copper is decomposed by the electric current, and the copper is deposited on the surface of the mold. The sulphuric acid appears at the + pole, and, combining with the copper of this pole, forms new molecules of copper sulphate. When the copper film has acquired about the thickness of an ordinary visiting card, it is removed from the mold. This shell shows distinctly every line of the types or engraving. It is then backed, or filled in, with melted type metal, to give firmness to the plate. The plate is next fastened on a block of wood, and thus built up type high, and is now ready for the printer. Such a plate is of course very durable.

549. Electroplating. The distinction between electroplating and electrotyping is that with the former the metallic coat remains permanently on the object on which it is deposited, while with the latter it is intended to be removed. The processes are, in the main, the same. The articles to be plated are first thoroughly cleaned and suspended on the — pole of a battery, and then a plate of the same kind of metal that is to be deposited on the given articles is suspended from the + pole (Fig. 408). The bath used is a solution of a salt of the metal to be deposited. The cyanides of gold and silver are generally used for gilding and silvering. Many of the base metals require to be electrocoppered first, in order to secure the adhesion of the gold or silver. The dynamo has almost completely replaced the voltaic battery for electrotyping and electroplating purposes.

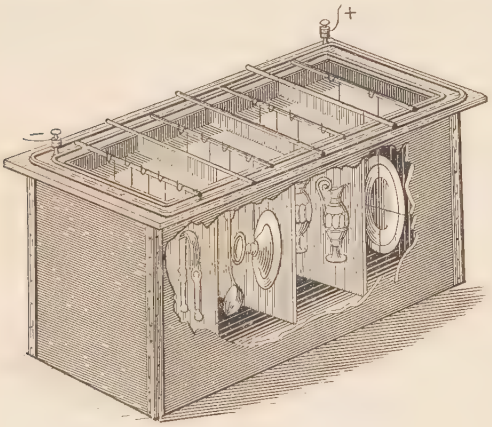


FIG. 408

SECTION XXVII

USEFUL APPLICATIONS OF ELECTRIC ENERGY CONTINUED: TELEGRAPHY

550. The telegraph. The word *telegraph* literally signifies *to write far away.* In its broadest sense it embraces all methods of communicating thought with great speed to a distance, by means of intelligible characters, sounds, or signs.

Fig. 409

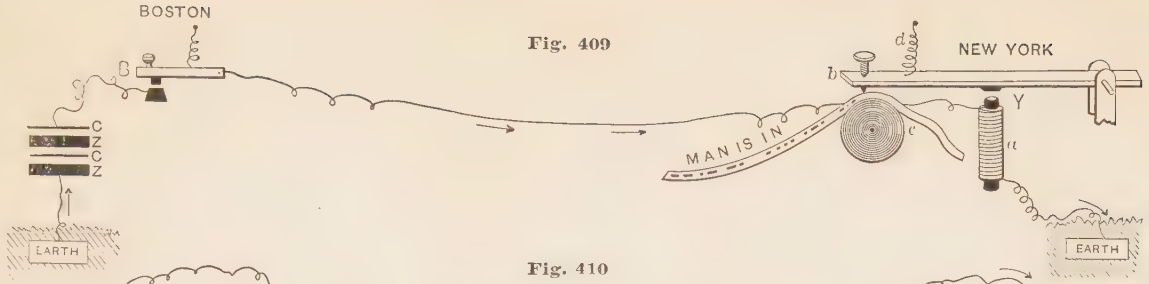
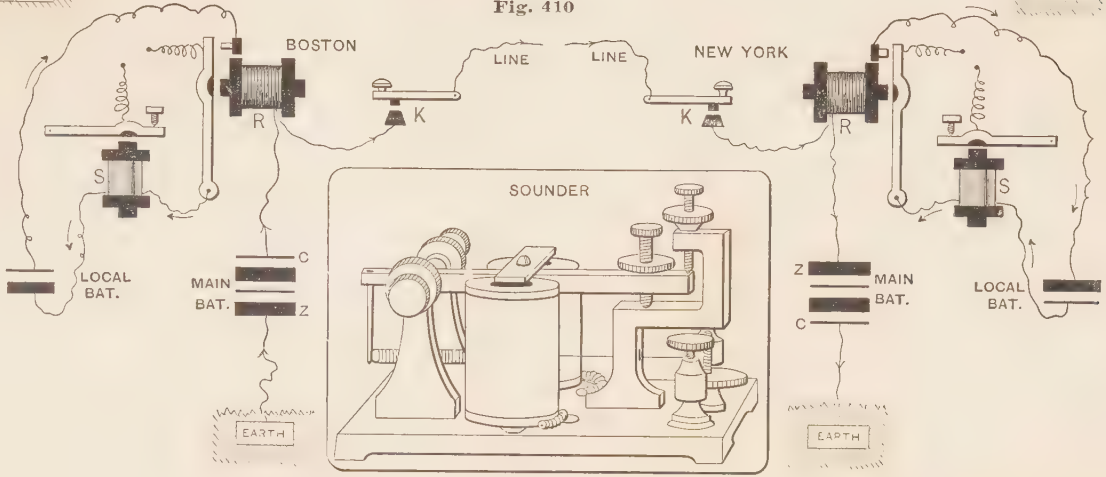


Fig. 410



First, it should be understood that, instead of two lines of wires (one to convey the electric current far away from the battery, and another to return it to the battery), if the distant pole be connected with a large metallic plate buried in moist earth, or, still better, with a gas or water pipe that leads to the earth, and the other pole near the battery be connected in like manner with the earth, so that the earth forms about one half of the circuit, there will be needed *only one wire* to connect telegraphically two places that are distant from each other. Furthermore, *the resistance offered by the earth to the electric current is practically nothing;* so that, disregarding the resistance of the ground connections, there is a saving of one half the wire and nearly one half the resistance, and consequently of about one half the battery power.

Let *B* (Fig. 409) represent the message sender, or operator's key; *Y*, the message receiver. It may be seen that the circuit is broken at *B*. Let the operator press his finger on the knob of the key. He closes the circuit, and the electric current instantly fills the wire from Boston to New York. It magnetizes *a*; *a* draws down the lever *b*, and presses the point of a style on a strip of paper *c* that is drawn over a roller. The operator ceases to press upon the key, the circuit is broken, and instantly *b* is raised from the paper by a spiral spring *d*. Let the operator press upon the key only for an instant, or long enough to count one: a simple *dot* or indentation will be made in the paper. But if he press upon the key long enough to count three, the point of the style will remain in contact with the paper the same length of time; and, as the paper is drawn along beneath the point, a short straight line is produced. This short line is called a *dash*. These dots and dashes constitute the *alphabet of telegraphy*. For instance, a part of a message, "man is in," is represented as printed in telegraphic characters on the strip of paper. The Roman letters above interpret their meaning.

551. The sounder. If the strip of paper be removed, and the style be allowed to strike the metallic roller, a sharp click is

heard. Again, when the lever is drawn up by the spiral spring it strikes a screw point above (not represented in the figure), and another click, differing slightly in sound from the first, is heard. A listener is able to distinguish dots from dashes by the length of the intervals of time that elapse between these two sounds. Operators generally read by ear, giving heed to the clicking sounds produced by the strokes of a little hammer. A receiver so used is called a *sounder*, a common form of which is represented in the lower central part of Fig. 410.

552. The relay and the repeater. The strength of the current is diminished, of course, as the line is extended and the number of instruments in the circuit is increased. Hence, a battery that would give a current sufficient to move the parts of a single sounder audibly, on a short line, would not move the same parts of many sounders on a long line with sufficient force to render the message audible. Resort is had to *relays* and *repeaters*.

In Fig. 410, *R* represents a relay and *S* a sounder. Suppose a weak current arrives at New York from Boston, and has sufficient strength to attract the armature of the relay at that station. This, as may be seen by examination of the diagram, will close another short circuit, called the *local circuit*, and send a current from a *local battery* located in the same office through the sounder at that station. The sounder, being operated by a battery in a circuit of only a few feet in length, delivers the message audibly. If it be desired that the message should go beyond New York, — for instance, to Philadelphia, — then we have only to suppose the local line at New York to be lengthened so as to extend to Philadelphia, and a powerful line battery to be substituted for the small local ; then the message that leaves Boston will be shifted from one circuit to the other at New York, and be delivered in Philadelphia without the intervention of any operator on the route. In this case a relay is called a *repeater*. The electro-magnets in relays are wound with long, thin wire, while those of sounders are wound with short, large wire. The main battery consists of many cells in series. It may be located

at either terminus, but is generally split in halves, and one half is placed at each terminus.

In the diagram the circuit is represented as open at both keys. When the line is not in use the circuit ought always to be left closed, by means of switches connected with the keys (not represented in the diagram), so that when the line is not "at work" an electric current is constantly traversing the wire. Sending a message, consequently, consists in interrupting this current by means of a key. Suppose that an operator at Boston wishes to communicate with New York. He first removes the switch on his key, which breaks the circuit and enables him to control the circuit with his key. He then manipulates his key so as to produce an understood signal, which will attract New York's attention. Every time that Boston presses on his key, every armature in his own office, and in the New York office, and at way stations, falls. Of course the message may be read at every station on the route.

TELEGRAPHIC ALPHABET

A	B	C	D	E	F
- —	— - - -	- - -	— - -	-	- — -
G	H	I	J	K	L
— — -	- - - -	- -	— - — -	— - —	— —
M	N	O	P	Q	R
— —	— -	- -	- - - - -	- - — -	- - -
S	T	U	V	W	X
- - -	—	- - —	- - - —	- — —	- — - -
Y	Z	&	,	?	.
- - - -	- - - -	- - - -	- — - —	— - - — -	- - — — - -

TELEGRAPHIC FIGURES

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SECTION XXVIII

USEFUL APPLICATIONS OF ELECTRIC ENERGY CONTINUED: TELEPHONY

553. The telephone. Fig. 411 represents a sectional and a perspective view of a simple form of this instrument. It consists of a steel magnet *A*, encircled at one extremity by a spool *B* of very fine insulated wire, the ends of which are connected with the binding screws *DD*. Immediately in front of the magnet is

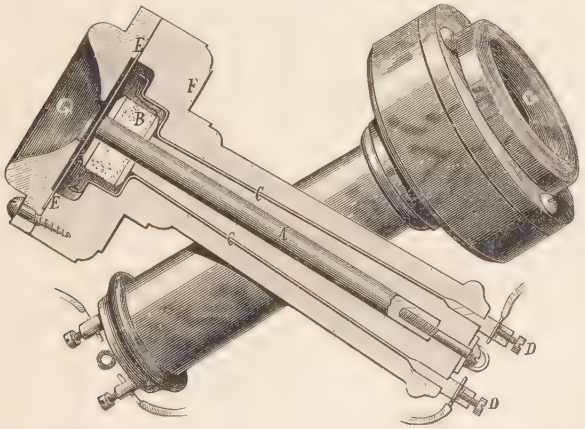


FIG. 411

a thin circular iron disk *EE*. The whole is inclosed in a wooden or rubber case *F*. The conical-shaped cavity *G* serves the purpose of either a mouthpiece or an ear trumpet. There is no difference between the transmitting and the receiving telephone; consequently either instrument may be employed as a transmitter, while the other serves as a receiver. Two magneto-telephones in a circuit are virtually in the same relation as a dynamo to a motor. The transmitter being in itself a diminutive dynamo, of course no battery is required in the circuit. Connect in circuit two such telephones, and the apparatus is ready for use.

When a person talks near the disk of the transmitter he throws it into rapid vibration. The disk, being quite close to the magnet, is magnetized by induction; and as it vibrates, its magnetic power is constantly changing, being strengthened as it approaches the magnet, and enfeebled as it recedes. This fluctuating magnetic force will of course induce currents in alternate directions in the neighboring coil of wire. These currents traverse the whole length of the wire, and so pass through the coil of the distant instrument. When the direction of the arriving current is such as to increase the intensity of the magnetic field of the receiver the magnet attracts the iron disk in front of it more strongly than before. When the current is in the opposite direction the disk is less attracted, and flies back. Hence the disk of the receiving telephone is forced to repeat whatever movement is imparted to the disk of the transmitting telephone. The vibrations of the former disk become sound in the same manner as the vibrations of a tuning fork or of the head of a drum.

The above is a description of the original and simplest form of the Bell telephone. It is apparent that the original energy (i.e. that of the voice) applied at the transmitter must, during its successive transformations and especially during its transmission in the form of electric energy through large resistances, become very much enfeebled, so that when it reappears as sound, the sound is quite feeble and frequently inaudible. The first grand improvement on the original consists in introducing a battery into the circuit, and so arranging that the voice, instead of being obliged to generate currents, shall be required to act only as the controlling force of a current already generated by the battery. It is evident that only a fluctuating or undulating current can produce the necessary vibrations in the disk of the receiver. The fluctuations are caused by a varying resistance in the circuit. The pupil must have learned by experience ere this that the effect of a loose contact between any two parts of a circuit is to increase the resistance and thereby weaken the current; but the effect of a slight variation in pressure is especially

noticeable when either or both of the parts are carbon. Fig. 412 illustrates a simple telephonic circuit in which are included a variable-resistance transmitter T, a magneto-receiver R, and a

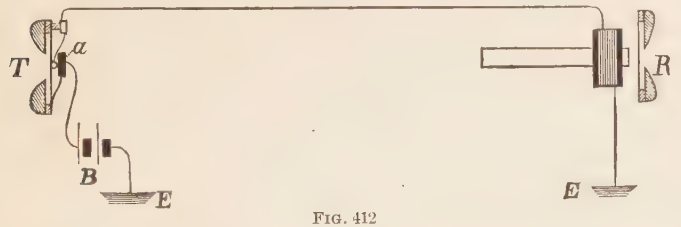


FIG. 412

battery B. One of the electrodes, a platinum point, touches the center of the transmitter disk; the other electrode, a carbon button a, is pressed by a spring gently against the platinum point. Every vibration of the disk, however minute, causes a variation in the pressure between the two electrodes and a corresponding variation in the circuit resistance. As the resistance changes, so changes the current strength, and so consequently

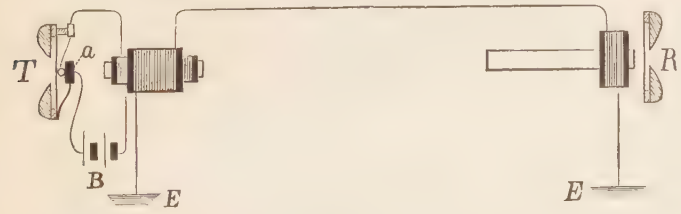


FIG. 413

changes the force with which the magnet in the receiver R pulls its disk. The varying tension between magnet and disk causes the latter to vibrate and reproduce sounds.

The next improvement of considerable importance consists in the adoption of an induction coil, which, we have learned, produces a current of much greater electro-motive force than is possessed by the original battery current. By its adoption we are able to converse over much longer distances, and since the

battery current traverses only a local circuit, as may be seen by reference to Fig. 413, a single Leclanché cell is generally sufficient to operate it. The currents induced by the fluctuating primary current traverse the line wire and generate sonorous vibrations in the disk of the receiver in the same manner as in the original telephone.

554. Microphone. In Fig. 414 *A* and *B* are buttons of carbon; the former is attached to a sounding board of thin pine wood,

FIG. 414

the latter to a steel spring *C*, and both are connected in circuit with a battery and a telephone used as a receiver. The spring presses *B* against *A*, and any slight jar will cause a variation in the pressure and corresponding variations in the current strength.

By means of this instrument, called the *microphone*, any *little sound* (as its name indicates), such as the ticking of a watch or the footfall of an insect, may be reproduced at a considerable distance, and be as audible as though the original sounds were made close to the ear. As in the telephone transmitter, the effect is increased by introducing a small induction coil, and putting the receiver in the secondary circuit.

SECTION XXIX

USEFUL APPLICATIONS OF ELECTRICITY CONTINUED

555. The bolometer. The bolometer,[1] also called a *thermic balance*, is an instrument devised by Langley for measuring minute quantities of radiant energy. It is based upon the fact that when equal conductors of the electrical current are at the same temperature their conductivities are equal, and the current of a battery can be equally divided between them; while, if unequally heated, their conductivities are unequal, and the difference can be detected by a sensitive galvanometer. By the substitution of thin sheets of metal for the wires ordinarily used as conductors, so as to take up and part with radiations with great rapidity, an instrument is produced capable of measuring such minute changes of temperature as a hundred-thousandth of a degree centigrade. It has achieved results of immense importance in astronomical and optical investigations, especially in the study of the distribution of heat energy in the solar and other spectra. It has revealed much hitherto unknown concerning the infra-red region of the spectrum. It has demonstrated that the spectrum, instead of being confined to rather less than an octave, really covers far more.

SECTION XXX

HIGH-FREQUENCY CURRENTS

556. Of the various branches of electrical investigations now in progress perhaps the most interesting and most promising is that relating to alternating currents.

Tesla's work has been especially with alternating currents of very high frequency and enormously high potential. In his experiments he operates an induction coil either with a specially

[1] Bolometer, *ray measurer.*

constructed alternating dynamo capable of giving many thousands of reversals of current per second, or by disruptively discharging a condenser through the primary. By the latter means there may be produced in the secondary circuit a vibration of a frequency of many hundred thousands or even millions per second. The discharge produced by this means is quite different from the series of sparks produced by the ordinary induction coil. Herein is opened to the experimenter a field as yet quite unexplored.

If two straight wires terminate the secondary coil and run parallel to each other for a short distance, the discharge appears in the form of powerful brushes and luminous streams issuing from all points of the wires. The apparatus is usually inclosed in a wooden box and completely immersed in oil, in order that the insulation may be more nearly perfect. When an ordinary low-frequency discharge is passed through moderately rarefied air, the air assumes a purplish hue. If by some means the intensity of the molecular or atomic disturbance be increased, the color of the gas changes to white. A similar change occurs in air at ordinary pressure when agitated by electric impulses of very high frequency.

The chief interest in investigations along this line seems to lie in the *possibilities* they offer for the production of an efficient illuminating device. The present means of producing artificial light is woefully inefficient and wasteful. Who can say that it is not to be by means of alternating currents of high frequency and high potential that we shall soon vie with the firefly in economical illumination ?

QUESTIONS AND PROBLEMS

1. (*a*) Describe and explain an ordinary multiplying galvanoscope. (*b*) Describe a coil of a galvanoscope suited to a circuit of high resistance, and give your reason.

2. You wish to make of an iron rod an electro-magnet with a certain marked end as an *N* pole. Explain the method by a diagram.

3. What happens to a fixed bar of soft iron if a current passes above and near it and at right angles to it ? Make a diagram.

4. At a distance of 10 cm., what force does a magnetic pole whose strength is 25 units exert upon a pole whose strength is 10 units? *Ans.* 2.5 dynes.

5. Upon what conditions does the strength of the current furnished by a dynamo depend?

6. If a current be sent through the armature of a dynamo, what happens? Why?

7. How much work is done in maintaining a current of 10 amperes for one hour in a circuit against a resistance of 50 ohms?

8. A current of 30 amperes flows through a wire. What heat is developed in an hour in a section of the wire whose ends differ in potential by 90 volts?

9. A difference of potential of 18.5 volts is maintained at the terminals of a wire of 0.5 ohm resistance. What is the activity of the circuit in joules per second?

10. What distinction should be made between electricity and electrification?

11. What is the strength of a current which, falling 15 volts, yields 0.002 horse power?

12. When would you wind an electro-magnet with fine wire?

13. Upon what does the E.M.F. furnished by a dynamo depend?

14. A dynamo feeds 20 arc lamps in series; the average resistance of each, including the lead wires, is 4.82 ohms, and the dynamo resistance 8.2 ohms. What current does the dynamo furnish with an E.M.F. of 560.4 volts?

15. A current of 10 amperes deflects the needle of a tangent galvanometer 64°. What is the strength of a current which will deflect the same needle 70°?

16. How many coulombs are required to deposit 15 g. of zinc?

17. In what time will a current of 1 ampere decompose 10 g. of water?

18. A Daniell battery maintains a current of 1 ampere for 6 hr. (*a*) What quantity of zinc will be consumed? (*b*) What quantity of copper will be deposited upon the negative plate of each cell?

19. In an electrolytic cell 4 g. of copper were deposited in one half hour. (*a*) What was the average strength of current? (*b*) If an average difference of potential of 3 volts were maintained between the terminals of the cell, what would be the average activity of the cell?

20. If the difference of potential between the terminals of an arc lamp supplied with a 10-ampere current be 50 volts, what is the power consumed in the lamp?

21. A coil of platinum wire whose resistance is 4 ohms is submerged in 2 kg. of water at 20° C. and a current of 10 amperes is maintained in the coil. In what time will the temperature of the water be raised to 100° C. (losses by radiation, etc., being disregarded)?

22. (*a*) Compare the activity of a 10-ampere current with an E.M.F. of 40 volts with the activity of a 40-ampere current with an E.M.F. of 10 volts.

(*b*) Suppose that these currents flow through wires having the same resistance, how will the quantities of heat developed in the two wires compare? (*c*) Suppose that it is desired to transmit energy to a great distance, with which current would there be the greater waste of energy, provided there were no leakages and the wires were of the same kind, size, and length? (*d*) Which current would be the most difficult to insulate? (*e*) With which current is a greater saving in the cost of conductors possible?

23. A Bunsen cell has an E.M.F. of 1.9 volts and a resistance of 0.4 ohm. Its plates are joined first by a 2-ohm wire, and second by a 28-ohm wire. (*a*) What is the current in each case? (*b*) What quantity of heat is generated in the wire in each case? (*c*) How many times as much heat is generated in one case as in the other? Why?

24. Let 200 incandescent lamps be arranged in multiple arc. Each lamp is rated at 110 volts and 200 ohms. (*a*) What is the resistance of all the lamps in multiple arc? (*b*) What current passes through each lamp? What horse power is expended on their lighting?

25. Let 25 cells, each 2 volts and 1 ohm, be arranged in series on an external circuit of 250 ohms resistance. The efficiency of the battery (i.e. the ratio of resistance of the external circuit to the total resistance of the circuit) is about 91 per cent. What work does the battery do in that circuit?

26. An electric lamp has a resistance of 45 ohms; it is connected to a street main by leads of 3 ohms resistance. What portion of the heat is wasted in the house circuit?

27. Compare the quantities of heat developed in a 100-volt 200-ohm lamp and in a 35-volt 30-ohm lamp.

28. A difference of potential of 5.5 volts is maintained at the terminals of a wire of 0.1 ohm resistance. (*a*) What current flows? (*b*) What is the activity of the circuit in joules per second?

SECTION XXXI

WIRELESS TELEGRAPHY

557. The method. In this section wireless telegraphy means that method of telegraphy which has been built up during the last few years from the researches of Maxwell and Hertz. A sending station is equipped with an apparatus for generating and transmitting electric waves, and a receiving station is equipped with apparatus for detecting these waves. The phenomena to be studied, then, have to do primarily with wave motion. We now know that there is an intimate relation between electricity

and light; indeed, electric waves are of the same nature as light waves, are propagated by the ether, and can be reflected, refracted, and made to interfere with one another. They differ from light waves only in length.

558. The theory. As has just been stated, when any conductor or condenser is rapidly charged and discharged electric waves are produced in the ether; also when a condenser is discharged once, the discharge is vibratory in character like the motion of a pendulum, dying down only after a finite interval. The period of electric oscillation depends on three electrical constants characteristic of the particular circuit in hand, just as the period of a vibrating string depends on the various constants of the string. The electrical quantities involved are the resistance, the capacity, and the inductance or self-induction of the circuit; that is, when a current is set up it generates a magnetic field in the surrounding medium, which creates a counter E.M.F. to be overcome. This increases with the number of reversals of the current per second.

In a simple discussion of wireless telegraphy the resistance may be omitted from the equation of the period without serious error. In that case $T = 2\pi\sqrt{LC}$, where L is the inductance and C the capacity. A good mechanical analogue for capacity is found in the reciprocal of the tension of a stretched string, and for inductance in the inertia of the string. It is clear, then, that the "tune" of an electrical circuit may be regulated at will.

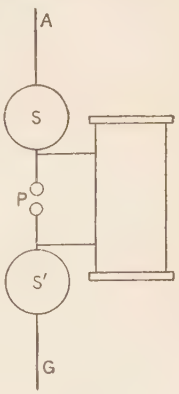


FIG. 415

559. The apparatus. One form of generator used in wireless telegraphy is the induction coil of the Ruhmkorff type. Fig. 415 illustrates a simple arrangement. S and S' are conducting spheres in the secondary circuit, with the small spark gap P. The capacity of the circuit may be increased by approaching the spheres nearer together or may be decreased by separating them. By this adjustment the period of oscillation

may be regulated. Hertz received the waves by a very simple device, – just a circle of wire R with a short gap P' in it (Fig. 416). When a spark passes at P one will also occur at P', provided the two circuits are so adjusted as to be electrically in tune. If one of the spheres S be connected with a long wire A, extending vertically into the air, and the other S' be connected to the ground G, while the receiving apparatus is modified as in Fig. 417, A being an air and G a ground wire, we have a simple system practical for short distances. The air and ground wires were introduced by Marconi. In the Hertzian experiments the sending apparatus was called the *oscillator*, while the detector was called the *resonator*.

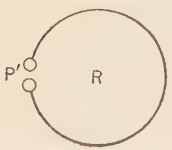


FIG. 416

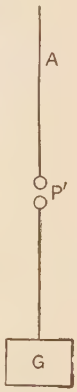


FIG. 417

560. The coherer. This was discovered by Branley and has proved itself a very valuable assistance in the detection of electro-magnetic waves. It consists of a short glass tube TT (Fig. 418) having a bore of a couple of millimeters and containing between two silver plugs PP a few loosely packed metal filings F. Nickel with a few per cent of silver has been found very sensitive. Wires extend from the plugs out of the tube at each end, by means of which the instrument can be included in a receiving circuit in place of the spark gap P'. This arrangement has the property of being a nonconductor *except* when electric oscillations are produced in the vicinity. From the effect of the resulting waves the coherent filings *cohere* to a slight extent and become conducting. After the waves have ceased it is only necessary to give the glass of the coherer a slight mechanical shock in order to cause the filings to *decohere*, thereby becoming again a nonconductor. In practice a local battery circuit is shunted round the coherer containing a relay, which in turn operates an automatic

FIG. 418

device for decohering the instrument as soon as it closes the circuit. Thus the Morse signals may be received and read by ear from the relay or a sounder in the usual way. To send them a telegraph key is included in the primary of the induction coil, and is operated exactly as in ordinary telegraphy. While the key is closed sparks are passing across the gap at P, which by their oscillating character set up intermittent wavelets in the ether, a single train being represented by Fig. 419.

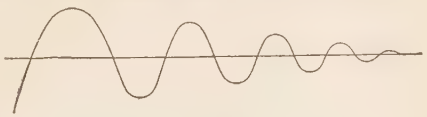


Fig. 419

These wave trains move through the ether with the speed of light in all directions, and corresponding oscillations are set up in the resonator and interpreted as described above.

Other Detectors of Hertzian Waves

561. Cymoscopes. The general term *cymoscope* has been applied to all devices for detecting electric waves. Besides the coherer already described, there are numerous detectors, the principle of each being more or less implied by the name. The mention of only a few of these must suffice. The *magnetic* cymoscope was developed by Rutherford about 1896. The *thermal* detector depends on the heating of a very fine wire by the oscillations set up by the waves. The microphonic detector and the electrolytic detector are other very sensitive forms.[1]

SECTION XXXII

THE RÖNTGEN PHENOMENA

562. Radiography. If a glass bulb be exhausted to approximately one one-millionth of an atmosphere, a condition of things is attained such as was investigated and described by Crookes

[1] For fuller details the student is referred to "The Principles of Electric Wave Telegraphy," by A. J. Fleming.

in 1879. The mean free path of the residual gas molecules in this bulb is greater than the dimensions of the bulb. Gas in this state was called by Crookes *radiant matter*, since, when under the influence of an electric discharge, it presents properties quite different from those of ordinary gas. It has now been shown beyond question, by the most ingenious experiments of Crookes, Lenard, Rutherford, Perrin, J. J. Thomson, and others, that the carriers of the charges of electricity in a vacuum tube are far smaller than the atoms of residual gas, in fact, have a mass of only about one one-thousandth of that of a hydrogen atom, — and that these particles are of the same mass whatever the gas used. These particles, when excited, are supposed to start from the *kathode*, or negative terminal of the bulb, and to travel in straight lines and with great speed till they strike the walls of the bulb or other surface placed athwart their path. Such surfaces usually fluoresce more or less brilliantly, with a color depending upon the substance which receives the impact. German glass, of which most bulbs are made, fluoresces green. The streams are known as the *kathode rays* of a Crookes tube.

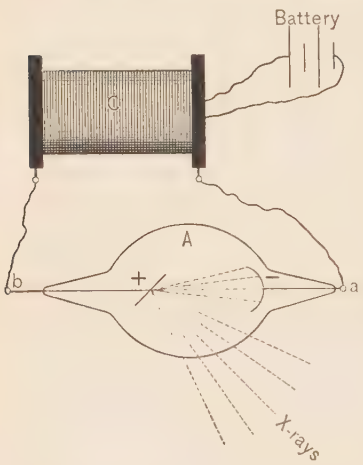


FIG. 420

The usual method of exciting these bulbs is to connect them by wires to the terminals of the secondary of an ordinary induction coil, or to the conductors of a powerful Holtz machine.

Let A (Fig. 420) represent an exhausted glass bulb. Hermetically sealed into this bulb are the electrodes a and b; the former usually consists of a concave disk of aluminum, and the latter of a flat disk of platinum. Let a be connected to the negative and

b to the positive pole of an induction coil *C*. The discharge consists of faint streamers which proceed in straight lines from the negative electrode, as indicated in the figure. These streams are, as stated above, the kathode rays.

The remarkable discovery of Professor W. C. Röntgen, of Würzburg, in the early part of the winter of 1895–1896, consisted in the somewhat accidental detection of a nonluminous radiation emanating from an excited Crookes bulb. He enveloped with black cardboard a bulb through which kathode rays were passing and, though all light due to the fluorescence of the glass was cut off, he noticed that crystals of barium platinocyanide in the darkened room were rendered luminous. Appreciating the significance of his discovery, he began at once to investigate the effect of the "new kind of rays" on photographic films. Chancing to use a box of weights to hold a film in place in one of his experiments, he found, on development, that the wood of the box left little impression, while shadows of the metal weights appeared upon the photographic surface with startling distinctness. Since this first step the applications of the discovery have been most rapid. It was quickly found by Röntgen and others that numerous substances, such as the metals, glass, bone, and many crystals, are rather opaque to the new form of energy; while many other substances, notably wood, paper, flesh, rubber, water, etc., are relatively transparent. The retina of the eye is quite insensitive to these rays; the eye, placed in their path, sees nothing. It soon became possible to produce excellent shadow pictures of the bones of the hand or foot.

The name *radiograph* for such pictures was suggested independently by Lodge, of England, and Goodspeed, of this country. Röntgen called the new radiation the "X-rays," since he was unable to state its nature. Three different theories have been advanced, — the corpuscular theory, the ultra-violet theory, and the longitudinal ether-wave theory. The first assumes the rays to consist of highly electrified material particles projected with great speed from within the bulb quite through the glass walls. The

ultra-violet theory supposes the energy to be ether-wave motion of the same character as light, but of far less wave length. The last mentioned of the three theories was at first favored by Röntgen. Indeed, it has been known for a long time that, besides the transverse vibrations which account for the phenomena of light, it is possible that longitudinal vibrations should exist in the ether, and it has been argued by some physicists that they must exist. At the present time the theory generally accepted is that advanced by Stokes, namely that X-rays consist of transverse *pulses* in the ether, as distinguished from trains of waves like those of light. The wave length is very short.

In the bulb the kathode-ray energy probably is transformed into X-ray energy at the point of impact on the platinum disk. By improved apparatus sharply defined radiographs of the wrist bones may be taken with an exposure of a second or less, and the bones in the chest of a man with an exposure of a few seconds.

The applications of radiography thus far lie chiefly in the fields of surgery, anatomy, and pathology. It is easy to locate foreign metallic bodies in the human subject, such as bullets, needles, and the like, and also to examine injured bones and joints, fractures, dislocations, malformations, and the like.

A very important and useful addition to the apparatus alluded to above is an instrument called a *fluoroscope*. It consists of a dark box with an opening at one end to look into. On the opposite side, within, is spread some fluorescent material, preferably barium platinocyanide. Any body opaque to X-rays placed on the outside of the fluorescent screen, and between it and the Crookes bulb, casts a shadow upon the screen which may be viewed by looking in at the opening; for example, one can see a shadow of the bones of his own hand; i.e. the radiographic surface glows with fluorescent light, except at those portions shaded from the X-rays by the bones of the hand. The fluoroscope enables one easily to detect a false diamond from a genuine one, the latter being quite translucent to the rays while the false one is usually opaque.

REVIEW QUESTIONS AND PROBLEMS

1. Show that Newton's first law of motion suggests a definition of force.

2. Explain that the second law of motion suggests a way of measuring forces.

3. Discuss the formula $ft = mv$.

4. A force of 80 dynes acts upon a mass of 20 grams for 10 seconds. What velocity does it produce?

5. A constant force acting upon a mass of 25 g. causes it to move 10 m. in 4 sec. What is the magnitude of the force in dynes?

6. How long must a force of 5 dynes act upon a body in order to give it a momentum of 3000 units (the unit of momentum being that of a gram moving at the rate of 1 cm. per second)?

7. Discuss the expression "centrifugal force."

8. Express the acceleration due to gravity in our latitude in meters per minute.

9. A force of 10 lb. acts upon a mass of 25 lb. What acceleration is produced?

10. Find the value of g at a place where the seconds pendulum is 0.994 m. long.

11. Find the work done by a force of 60 dynes acting through a distance of 4 m.

12. A stone of mass 8 kg. falls at a place where $g = 980.4$ cm. per second. What is its kinetic energy in ergs at the end of 10 seconds?

13. What should be the horse power of an engine intended to pump 200 gal. of water per minute to a height of 50 yd.? (Assume 1 gal. $\doteq$ 10 lb.)

14. A body has a mass of 1 kg., and its density is five times that of water. What is its volume?

15. The radii of two spheres are 2 cm. and 3 cm. respectively, and their masses are 200 g. and 250 g. respectively. Compare their densities.

16. (*a*) Find the pressure in dynes per sq. cm. caused by the weight of a column of mercury 1 m. high; (*b*) in grams per sq. cm.

17. Determine the available water pressure (in pounds per square inch) in a laboratory which is supplied from a tank at a height of 45 ft.

18. State how, with a mercury manometer, you would determine the pressure of water at a tap.

19. A cube, the edge of which is 1 dm., is suspended in water with its upper surface 1 m. below the surface of the water. Find the pressure on each of its faces.

20. (*a*) How is the height of the mercurial column of a barometer affected by changes of temperature? (*b*) by capillary action? (*c*) by the size of the tubes? (*d*) by differences in the value of g?

21. Calculate in dynes per sq. cm. the atmospheric pressure when the barometer stands at 78 cm.

22. What is the atmospheric pressure in pounds per square inch when the barometric height is 27.5 in. ?

23. The volume of a quantity of gas is measured when the barometer stands at 72 cm., and is found to be 646 cc. What would its volume be at the normal pressure (76 cm.) ?

24. A piston is in the middle of a closed cylinder 10 in. long, and there are equal quantities of air on each side of it. The piston is pushed until it is within an inch of one of the ends. Compare the pressures on the two sides of the piston.

25. A piece of wood floats on water. Oil is poured on the water until the wood is covered. State and explain the change of position of the wood that will occur with reference to the water.

26. An iron yard measure is correct at the temperature of melting ice. Express, as a fraction of an inch, its error at the temperature of boiling water.

27. A clock which keeps correct time at 25° C. has a seconds pendulum rod of brass. How many seconds a day will it gain if the temperature falls to 0° C. ?

28. (*a*) What will be the volume at 75° C. of a quantity of air which occupies 2.5 l. at 0°, the pressure being constant ? (*b*) At what temperature will its volume be 3 l. ?

29. At what temperature will the volume of a given mass of gas be double what it is at 30° C. ?

30. At 0° C. and under a pressure of 76 cm. a liter of dry air has a mass of 1.293 g. At what temperature will a liter of air have a mass of 1 g., the pressure being 72 cm. ?

31. Compare the volumes of equal masses of air (*a*) at 0° C. and 76 cm. pressure, and (*b*) at 10° and 85 cm. pressure.

NOTE. The density of dry air at 0° C. and 76 cm. pressure is 0.001293 g. per cc.

32. The cubical contents of a certain room are 750 m^3. Calculate the mass of dry air it will contain at 17° C. and 77 cm. pressure.

33. A piece of silver of mass 10.205 g. is heated to 101.9° C. and dropped into a calorimeter containing 81.34 g. of water, the temperature of which was raised from 11.09° to 11.71° C. The combined water equivalent of the calorimeter and thermometer employed is 2.91 g. Find the specific heat of silver.

34. A certain vessel holds 750 cc. of water at its maximum density. How much heat must be imparted to the water before it begins to boil under a pressure of 76 cm. ?

35. Equal masses of boiling water (pressure 76 cm.) and of mercury at 5° C. are mixed. What is the resulting temperature ?

36. How much ice at 0° will be melted by 400 g. of water at 90° C. ?

37. Explain clearly what is meant by the statements, "the specific heat of ice is 0.5" and "the latent heat of water is 80."

38. The density of ice is 0.91674 g. per cc. (*a*) What is the volume of a gram of ice? (*b*) A mixture of ice and water will diminish how much in volume for each gram of ice melted?

39. A vessel containing 50 g. of ice at $-10°$ C. is placed over a spirit lamp. How much heat will be required to change the ice to water vapor?

40. If 100 g. of boiling water (100°) be poured upon 20 g. of ice (0°), the temperature falls to 70° when the ice is just melted. What is the latent heat of fusion, assuming no heat to be lost?

41. Explain why the velocity of sound in air increases with the temperature, but is independent of the pressure.

42. Find the temperature at which the velocity of sound in air is 350 m. per second.

43. Find the length of a closed organ pipe which gives for its fundamental the note C (256 vibrations).

44. Two similar wires of the same length are stretched, one by a force of 4 lb., and the other by a force of 9 lb. What is the interval between the notes which they produce?

45. How can the number of beats per second made by two forks be calculated?

46. An observer sets his watch by the sound of a signal gun fired at a station 1500 m. distant. Find to a hundredth of a second the error due to distance, the temperature being 15° C.

47. State the laws for the frequency of vibration of stretched strings.

48. How much heat is generated when an inelastic body of mass 100 g., moving with a speed of 25 m. per second, is suddenly brought to rest?

49. Activity equal to 100 watts is entirely converted into heat. How much water can be heated in this manner through 1° in an hour?

50. A standard candle and a gas flame are placed 6 ft. apart, the gas flame being of 16 candle power. At what point between them must a screen be placed that it may be equally illuminated by each of them?

51. Find the image of an object placed in front of a concave spherical mirror, and show that the sizes of the image and object are as their distances from the center of the mirror.

52. An object is placed at a distance of 10 cm. in front of a convex mirror of 30 cm. focal length. Where will the image be formed?

53. A candle flame 1 in. long is 18 in. in front of a concave mirror whose focal length is 15 in. Find the position and size of the image.

54. The relative refractive index from air to water is $\frac{4}{3}$, and from air to glass $\frac{3}{2}$. Find the relative index from glass to water.

55. If an object at a distance of 3 in. from a convex lens has its image formed on the opposite side of the lens and magnified three times, what is the focal length of the lens?

56. Rays of light diverging from a point 8 in. in front of a lens are brought to a focus 20 in. behind it. What is the focal length of the lens?

57. A person whose distance of most distinct vision is 20 cm. uses a lens of 5 cm. focal length as a reading glass. (*a*) At what distance from a book must he hold it? (*b*) What will be its magnifying power?

58. Two small spheres are at a distance of 8 cm. apart; one has a charge of 12 electrostatic units, the other of 8. What is the magnitude of the force exerted between them?

59. Two small electrified bodies at a distance of 12 cm. apart are found to attract each other with a force of 6 units. The one has a positive charge of 32 electrostatic units. What is the charge of the other?

$$\left(f = \frac{qq'}{d^2}\right)$$

60. An incandescent lamp takes a current of 1.32 amperes, and the E.M.F. between its terminals is 66 volts. What is its resistance when hot?

61. Copper wire $\frac{1}{12}$ in. in diameter has a resistance of 8 ohms per mile. What is the resistance of a mile of copper wire $\frac{1}{36}$ in. in diameter?

62. In what time will a current of 0.5 ampere deposit 8 g. of silver?

63. What is the strength of a current that deposits 1 mg. of zinc per minute?

64. The resistance of a circuit, including that of the battery, is 18 ohms. What change will be produced in the current through the battery if two points of the circuit, between which the resistance is 12 ohms, are connected by a wire of 4 ohms resistance?

65. What shunt is required to reduce to one one-hundredth the sensitiveness of a galvanometer of 396 ohms resistance?

66. A galvanometer of 45 ohms resistance is shunted by a resistance of 5 ohms. Find the resistance of the shunted galvanometer and the current which flows through it when a difference of potential of 2 volts is maintained between its terminals.

67. What activity in horse power is required to maintain a current of 4 amperes through a resistance of 37.3 ohms?

68. A current of 2 amperes is allowed to flow for half an hour through a coil of platinum wire of 6 ohms resistance immersed in 750 g. of water. What elevation of temperature will be produced by it?

69. Find the efficiency of a motor from the following tests:

E.M.F. at motor terminals	125.2 volts
Current in external circuit	100.9 amperes
Power applied	18.62 h. p.

NOTE. The efficiency of a motor is the ratio between the power developed by the motor and the electrical power absorbed by it.

70. The resistance of an incandescent lamp is 40 ohms, and the difference of potential between its two terminals is 45 volts. Determine the heat produced in it per hour.

71. Ninety incandescent lamps are placed in multiple arc, and a current of 40 amperes is distributed between them, the E.M.F. between the terminals being 120 volts. The resistance of the conductors is .2 ohm, and that of the dynamo .25 ohm, and the insulation resistance between the flow and return conductors is 1000 ohms. What horse power will be required for electrical work, and how much will be wasted?

APPENDIX

TABLES OF METRIC MEASURES

Measure of Length

1 millimeter (mm.) = .001 meter (m.) = .03937 inch.
1 centimeter (cm.) = .01 meter = .39371 inch.
1 decimeter (dm.) = .1 meter = about 4 inches.

1 Meter = 39.37079 inches = about 3 ft. $3\frac{3}{8}$ in.

1 dekameter (Dm.) = 10 meters.
1 hektometer (Hm.) = 100 meters.
1 kilometer (Km.) = 1000 meters = about $\frac{5}{8}$ mile.
1 myriameter (Mm.) = 10,000 meters = about $6\frac{1}{5}$ miles.

Measure of Surface

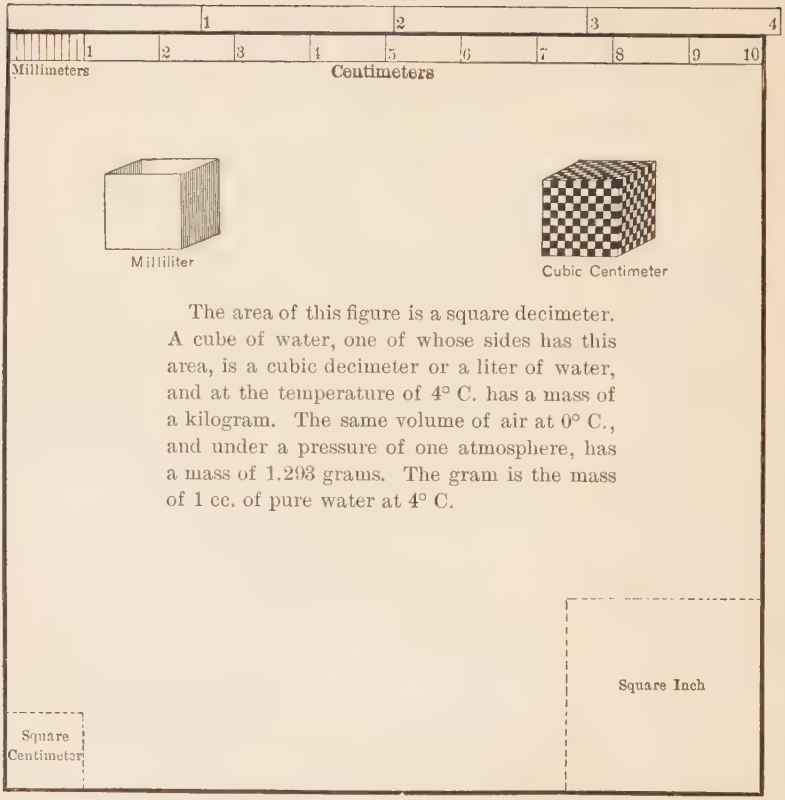
1 square millimeter (mm^2.) = .000001 square meter (m^2.) = .0015 sq. in.
1 square centimeter (cm^2.) = .0001 square meter = .1550 sq. in.
1 square decimeter (dm^2.) = .01 square meter.
1 centare = 1 square meter = 10.7643 sq. ft. = about $1\frac{1}{5}$ sq. yd.
1 are = 100 square meters.
1 hectare = 10,000 square meters = about $2\frac{1}{2}$ acres.

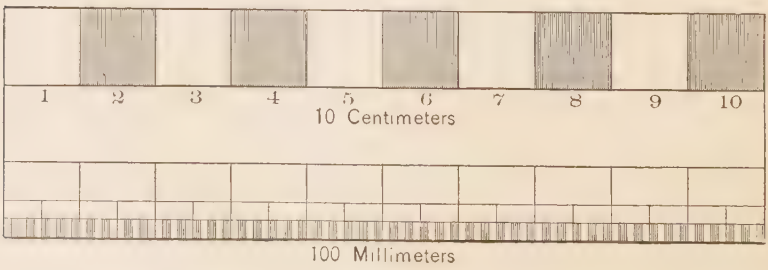
Measure of Volume

1 cubic millimeter (mm^3.) = .000000001 cubic meter (m^3.).
1 cubic centimeter (cm^3. or cc.) = .000001 m^3. = .061 cu. in.
1 cubic decimeter (dm^3.) = .001 m^3. = 1000 cm^3.
1 cubic meter = about 1.308 cu. yd.

Measure of Capacity

1 milliliter (ml.) = .001 liter (l.) = 1 cc. = .061 cu. in.
1 centiliter (cl.) = .01 liter = 10 cc.
1 deciliter = .1 liter = 100 cc.
1 Liter = 1000 cm^3. = 61.027 cu. in. = 1.0567 qt. (liquid measure).


Inches
1
2
3
4
1
2
3
4
5
6
7
8
9
10
Millimeters
Centimeters
Milliliter
Cubic Centimeter
The area of this figure is a square decimeter. A cube of water, one of whose sides has this area, is a cubic decimeter or a liter of water, and at the temperature of 4° C. has a mass of a kilogram. The same volume of air at 0° C., and under a pressure of one atmosphere, has a mass of 1.293 grams. The gram is the mass of 1 cc. of pure water at 4° C.
Square Inch
Square Centimeter


1
2
3
4
5
6
7
8
9
10
10 Centimeters
100 Millimeters

Measure of Mass and Weight

1 milligram (mg.) = .001 gram (g.) = .0154 grain.
1 centigram (cg.) = .01 gram = .1543 grain.
1 decigram (dg.) = .1 gram = 1.5432 grains.
1 Gram = 15.432 grains = .03527 av. oz.
1 dekagram (Dg.) = 10 grams.
1 hektogram (Hg.) = 100 grams.
1 kilogram (kg.) = 1000 grams = 2.2046 av. lb.
1 myriagram (Mg.) = 10,000 grams.

1 cu. ft. of water at 62° F. has a mass of 62.321 av. lb., or about 1000 av. oz. 1 cu. in. of water at 62° F. has a mass of .036 av. lb. Pressure in water per square foot increases at the rate of about 62.3 lb. for every foot of depth, or per square inch at the rate of about $\frac{1}{3}$ lb. for every foot of depth; or per square centimeter at the rate of 1 g. for every centimeter of depth, or 1 kg. for every ten meters of depth.

Table of Equivalent Values

1 in. = .0254 m. = 2.53995 cm. = about $2\frac{1}{2}$ cm.
1 ft. = .3048 m. = 30.48 cm. = about $30\frac{1}{2}$ cm.
1 yd. = .9144 m. = about $\frac{10}{11}$ m.
1 mile = 1609 m. = about 1.609315 Km.

1 sq. in. = 6.4514 cm^2.
1 sq. ft. = 929.01 cm^2.
1 sq. yd. = 8361.1 cm^2. = .83611 m^2.

1 cu. in. = 16.38618 cm^3.
1 cu. ft. = 28,316 cm^3.
1 cu. yd. = 764,526 cm^3. = about .76 m^3.

1 U.S. pint = 473 cm^3.
1 U.S. quart = 946 cc. = .946 l.
1 U.S. gallon (231 cu. in.) = 3784 cm^3.
1 gr. = .0647987 g.
1 av. oz. = 28.3494 g.
1 av. lb. = 453.59 g. = .45359 kg. = $\frac{5}{11}$ kg.

Reduction of Measures to and from the C.G.S. System

1 gram weight = 980 dynes (where $g = 980$ cm. per sec.).
1 av. lb. weight = 4.445×10^5 dynes (where $g = 980$ cm. per sec.).
1 kg. weight = 980,000 dynes (where $g = 980$ cm. per sec.).
1 lb. per sq. ft. pressure = 478.5 dynes per cm^2.
1 g. per cm^2. pressure = 980 dynes per cm^2.
1 atmosphere (76 cm. 0°) = 1,012,630 dynes per cm^2.
1 atmosphere (30 in. 0°) = 1,015,300 dynes per cm^2.
1 gram-centimeter = 980 ergs.
1 erg = 1 dyne-centimeter = .0000001 joule = $\frac{1}{980}$ gram-centimeter.
1 kilogrammeter = 98,000,000 ergs = 7.23314 ft. lb.
1 ft. lb. = 13,550,000 ergs.
1 foot-poundal = 421,402 * ergs.
1 joule = 10,000,000 * ergs.
1 horse power { 33,000 ft. lb. per min. = about 7.452×10^9 ergs per sec.
1 horse power { 75 kilogrammeters per sec. = 7.35×10^9 ergs per sec.
1 erg per sec. = .0000001 watt or volt-ampere.
1 watt = 1 joule per sec. = 10^7 ergs per sec. = 44.2394 ft. lb. per min.
1 gram-degree = 4.17×10^7 ergs.
1 calorie = 4.17×10^{10} ergs.
1 lb. degree F. = 1.051×10^{10} ergs = 772 ft. lb. = 1047.03 joules.

* Independent of acceleration of gravity (g).

Properties of Solids

	Specific Gravity 17°	Hardness	Expansion Coefficient 0°–100°	Melting Point	Specific Heat	Latent Heat of Fusion	Refractive Index
Agate	2.6	7	. . . . .	. . .	. . .	. . .	1.54
Alum	1.7	2+	. . . . .	. . .	. . .	. . .	1.45
Aluminum	2.7	3	.00002	700°	.21	. . .	. . .
Antimony	6.7	3	.00001	432°	.05	. . .	. . .
Arsenic	5.7	3	.000007	. . .	.08	. . .	. . .
Beech	.8	. . .	. . . . .	. . .	. . .	. . .	. . .
Bismuth	9.8	2	.000013	266°	.03	13	. . .
Boxwood	.9	. . .	. . . . .	. . .	. . .	. . .	. . .
Brass (cast)	8.3	3	.000019	900°?	.09	. . .	. . .
Brass (hard drawn)	8.5	. . .	. . . . .	. . .	. . .	. . .	. . .
Bricks	1.7	. . .	. . . . .	. . .	.2	. . .	. . .
Bronze	8.8	3	. . . . .	. . .	. . .	. . .	. . .
Canada balsam	1.07	. . .	. . . . .	. . .	. . .	. . .	1.53
Cherry	.7	. . .	. . . . .	. . .	. . .	. . .	. . .
Copper	8.8	3	.000017	1100°	.09	30	. . .
Cork	.24	. . .	. . . . .	. . .	. . .	. . .	. . .
Diamond	3.5	10	. . . . .	. . .	.14	. . .	2.47
Emerald	2.7	8	. . . . .	. . .	. . .	. . .	1.58
Feldspar	2.5	6	. . . . .	. . .	. . .	. . .	. . .
German silver	8.5	3	.00002	. . .	. . .	. . .	. . .
Glass (crown)	2.5	. . .	.000007	400°	.19	. . .	1.51
Glass (flint)	3.6	. . .	. . . . .	. . .	. . .	. . .	1.62
Gold	19.3	. . .	.000012	1050°	.03	. . .	. . .
Granite	2.7	. . .	. . . . .	. . .	. . .	. . .	. . .
Graphite	2.3	. . .	. . . . .	. . .	.20	. . .	. . .
Ice	.9	1.5	. . . . .	0°	.5	79.7	1.31
Iceland spar	2.7	. . .	. . . . .	. . .	. . .	{Ord. 1.66 Extr. 1.49	
Iron (cast)	7.2	6?	.000012	1500°	.11	. . .	. . .
Iron (wrought)	7.7	4	. . . . .	. . .	. . .	. . .	. . .
Ivory	1.9	. . .	. . . . .	. . .	. . .	. . .	1.53
Lead	11.3	2	.000028	326°	.03	5	. . .
Marble	2.7	3	. . . . .	. . .	.21	. . .	. . .
Mica	2.8	. . .	. . . . .	. . .	. . .	. . .	. . .
Paraffin	.9	. . .	. . . . .	55°	. . .	. . .	. . .
Platinum (wire)	21.4	. . .	.000008	1800°	.03	. . .	. . .
Quartz	2.6	7	. . . . .	. . .	. . .	. . .	1.54
Rock salt	2.2	2	. . . . .	800°	.21	. . .	1.54
Selenite	2.3	2	. . . . .	. . .	. . .	. . .	1.52
Silver	10.4	. . .	.000019	1000°	.05	24	. . .
Slate	2.8	. . .	. . . . .	. . .	. . .	. . .	. . .
Spermaceti	.9	. . .	. . . . .	44°	.08	. . .	1.54
Steel (tempered)	7.8	9?	.000013	. . .	. . .	. . .	. . .
Sulphur (native)	2.	. . .	. . . . .	115°	.17	9	2.03
Talc	2.6	1	. . . . .	. . .	. . .	. . .	. . .
Tallow	.9	. . .	. . . . .	40°	. . .	. . .	1.49
Tin	7.3	2	.000019	232°	.05	14	. . .
Zinc	7.1	3	.00003	360°	.09	28	. . .

Properties of Liquids

	Specific Gravity	Coefficient Expansion at 0°	Freezing Point	Boiling Point 760 mm.	Specific Heat	Refractive Index
Acid, nitric, 0°	1.5	.00111	− 47°	. . .	. . .	. . .
Acid, sulphuric, 0°	1.84	.00059	. . .	330°?	.34	1.43
Alcohol (grain), 0°	.81	.00106	. . .	78.2°	.59	1.36
Benzine, 20°	.87	.00118	4°	80°	.39	1.49
Carbon dioxide, 20°	1.37	. . .	. . .	. . .	. . .	. . .
Carbon disulphide, 15°	1.26	. . .	. . .	. . .	.23	1.64
Ether, 0°	.73	.00148	. . .	35°	.54	1.35
Glycerin, 0°	1.26	.0005	. . .	290°	. . .	1.47
Mercury, 0° (Regnault)	13.596	.00018	− 39°	350°	.034	. . .
Mercury, 15°–20°	13.558	. . .	. . .	. . .	. . .	. . .
Mercury, (solid), − 40°	14.3	. . .	. . .	. . .	. . .	. . .
Milk, 0°	1.032	. . .	. . .	. . .	. . .	. . .
Naphtha	.847	. . .	. . .	. . .	. . .	. . .
Oil of turpentine, 0°	.89	.00071	− 10°	160°	.43	1.47
Olive oil, 0°	.92	.00080	. . .	. . .	. . .	1.47
Sea water, 0°	1.026	. . .	. . .	. . .	. . .	. . .
Water, 0°	.999	. . .	0°	100°	1.00	. . .
Water, 4.07°	1.000	. . .	. . .	. . .	. . .	1.33
Water, 20°	.998	. . .	. . .	. . .	. . .	. . .
Water, 100°	.958	. . .	. . .	. . .	. . .	. . .

Specific Gravity of Gases and Vapors

(Standard: air at 0° C.; barometer, 76 cm.)

Air	1.0000	Hydrogen	0.0693
Ammonia	0.5367	Nitrogen	0.9714
Carbonic acid	1.5290	Oxygen	1.1057
Chlorine	2.4400	Sulphuretted hydrogen	1.1912
Hydrochloric acid	1.2540	Sulphurous acid	2.2474

THE C.G.S. SYSTEM OF UNITS

The C.G.S. system of units is the result of an attempt to express all quantities with which physical science deals in terms of the three following fundamental units:

A Unit of Length, the centimeter;
A Unit of Mass, the gram;
A Unit of Time, the second.

From these the following units are derived:

Unit of *Surface:* the square centimeter.

Unit of *Volume:* the cubic centimeter.

Unit of *Velocity:* the velocity of one centimeter per second.

Unit of *Acceleration:* the acceleration of one centimeter per second per second.

Unit of *Force:* the dyne; the force which, acting on a gram-mass, imparts to it a unit of acceleration.

Unit of *Work:* the erg or dyne-centimeter; the work done by a dyne working through one centimeter.

Unit of *Energy:* also the erg; since the energy of a body is measured by the amount of work it can do.

Unit of *Heat:* the calorie; amount of heat required to raise one gram of water from 4° to 5° C.

Unit of *Magnetic Strength.* A magnetic pole has unit strength when it repels a similar pole of equal strength, one centimeter distant, with the force of a dyne.

Unit of *Electric Current* (electro-magnetic system): a current of such strength that one centimeter of its circuit, bent so that every point of it is one centimeter distant from a unit magnetic pole, exerts upon this pole the force of a dyne.

Unit of *Electric Quantity* (electro-magnetic system): the quantity conveyed by a unit current in one second.

Unit of *Difference of Potential* (electro-magnetic system). Two points have a unit difference of potential when one erg of work must be expended to bring a unit of + electricity from one to the other against the electric force.

Unit of *Electric Resistance* (electro-magnetic system). A conductor possesses unit resistance when a unit difference of potential between its ends causes a unit current to flow through it.

ALPHABETICAL TABLE OF UNITS USED IN PHYSICAL SCIENCE

Activity. *See* Horse power and Watt.

Ampere: unit of electric current; 10^{-1} C.G.S. units; the current produced by the difference of potential of a volt through the resistance of an ohm.

Calorie: unit of heat; quantity of heat required to raise one gram of water from 4° to 5° C. *See* Heat, English unit of.

Capacity. *See* Farad.

Coulomb: unit of quantity of electricity; 10^{-1} C.G.S. units; quantity conveyed by a current of one ampere in a second.

Current. *See* Ampere.

Dyne: C.G.S. absolute unit of force.

Electro-motive force. *See* Volt.

Erg: C.G.S. absolute unit of work or energy.

Farad: unit of electric capacity; 10^{-9} C.G.S. units; the capacity of a condenser which can be charged to a potential of one volt by one coulomb.

Foot-pound: English unit of work; work required to raise one pound through one foot in opposition to the force of gravity.

Foot-poundal: F.P.S. absolute unit of work or energy; the work done or energy imparted by one pound working through one foot.

Force. *See* Dyne, Kilogram, and Pound.

Gram-centimeter: gravitation unit of work or energy $= g$ ergs.

Heat, English unit of: heat required to raise one pound of water from 32° to 33° F. *See* Calorie.

Horse power: English unit of activity; activity required to perform 550 foot-pounds of work per second.

Joule: electrical unit of work; 10^7 ergs. It is the work done in one second when the rate of working is one watt; in other words, that done in one second in maintaining a current of one ampere against a resistance of one ohm.

Kilogram: metric unit of mass, and also of force. *See* Pound.

Kilogrammeter: metric unit of work; work required to raise one kilogram of mass through one meter in opposition to the force of gravity.

Mass. *See* Kilogram and Pound.

Ohm: unit of electric resistance; 10^9 C.G.S. units.

Potential. *See* Volt.

Pound: English unit of mass; regarded as a *weight* it is used also as the unit of force, i.e. the force exercised on the mass of a pound by the earth.

Quantity. *See* Coulomb.

Resistance. *See* Ohm.

Volt: unit of electro-motive force; 10^8 C.G.S. units; equals about the E.M.F. of one Daniell cell.

Watt: unit of electric activity; 10^7 ergs per second; activity exerted by a current of one ampere against a difference of potential of a volt.

Watt-hour: electrical unit of work = 3600 joules.

Work. *See* Foot-pound, Kilogrammeter, Gram-centimeter, Erg, Joule, and Watt-hour.

TABLE OF RESISTANCE OF WIRE

Chemically pure, one meter long, one millimeter in diameter at 0°C. (Jenkin). *Also relative resistances* (Ayrton)

		Relative Resistances
Silver, annealed	.01937 ohm	1.000
Silver, hard drawn	.02103 ohm	1.086
Copper, hard drawn	.02104 ohm	1.086
Zinc, pressed	.07244 ohm	3.741
Platinum	.11660 ohm	6.022
Iron, annealed	.12510 ohm	6.460
Lead, pressed	.25270 ohm	13.050
German silver	.26950 ohm	13.920

Values of Practical Electrical Units in C.G.S. Electro-Magnetic Units

Electro-motive force	Volt $= 10^8$.
Resistance	Ohm $= 10^9$.
Current	Ampere $= 10^{-1}$.
Quantity	Coulomb $= 10^{-1}$.
Capacity	Farad $= 10^{-9}$.

Table of Trigonometrical Functions

Degree	Tangent	Sine	Degree	Tangent	Sine	Degree	Tangent	Sine
1	.017	.017	31	.601	.515	61	1.80	.875
2	.035	.035	32	.625	.530	62	1.88	.883
3	.052	.052	33	.649	.545	63	1.96	.891
4	.070	.070	34	.675	.559	64	2.05	.899
5	.087	.087	35	.700	.574	65	2.14	.906
6	.105	.105	36	.727	.588	66	2.25	.914
7	.123	.122	37	.754	.602	67	2.36	.921
8	.141	.139	38	.781	.616	68	2.48	.927
9	.158	.156	39	.810	.629	69	2.61	.934
10	.176	.174	40	.839	.643	70	2.75	.940
11	.194	.191	41	.869	.656	71	2.90	.946
12	.213	.208	42	.900	.669	72	3.08	.951
13	.231	.225	43	.933	.682	73	3.27	.956
14	.249	.242	44	.966	.695	74	3.49	.961
15	.268	.259	45	1.000	.707	75	3.73	.966
16	.287	.276	46	1.036	.719	76	4.01	.970
17	.306	.292	47	1.07	.731	77	4.33	.974
18	.325	.309	48	1.11	.743	78	4.70	.978
19	.344	.326	49	1.15	.755	79	5.14	.982
20	.364	.342	50	1.19	.766	80	5.67	.985
21	.384	.358	51	1.23	.777	81	6.31	.988
22	.404	.374	52	1.28	.788	82	7.12	.990
23	.424	.390	53	1.33	.799	83	8.14	.993
24	.445	.407	54	1.38	.809	84	9.51	.995
25	.466	.423	55	1.43	.819	85	11.43	.996
26	.488	.438	56	1.48	.829	86	14.30	.998
27	.510	.454	57	1.54	.839	87	19.08	.999
28	.632	.469	58	1.60	.848	88	28.64	.999
29	.554	.485	59	1.66	.857	89	57.29	1.000
30	.577	.500	60	1.73	.866	90	Infinite	1.000

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